EXTRACT BEFORE DETECT, N-SIGNAL COMPLEX APPROXIMATE MESSAGE PASSING APPLIED TO RADAR SIGNALS

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1. ABSTRACT

Standard radar processing chains compute coherent processing functions, estimate noise level and apply associated threshold. Then, the extraction function groups elementary detections to distinguish between the different targets and to estimate their position. In this paper, it is proposed to combine coherent filtering, thresholding and extraction into a single Compressed Sensing process named "Extract Before Detect". The proposed technique is based on Approximate Message Passsing (AMP), adapted to the structure of radar signals: N blocks of complex signals.

Index Terms— *Radar processing, Compressed Sensing, Approximate Message Passing, Complex, Block.*

2. INTRODUCTION

Standard radar processing chains combine

- Coherent processing : Digital Beam Forming, Pulse Compression, Doppler Filtering
- Noise estimation : Constant False Alarm Rate, Clutter Maps
- Thresholding : selection of hits where the signal exceeds the threshold deduced from noise estimation
- Extraction : grouping together the hits most probably associated to the same target, and computing the mean position of each group of hits.

The radar waveforms are often ambiguous in range, in Doppler or both. In this case, the radar transmits several bursts with different range and/or Doppler ambiguities, to solve ambiguities by combining the hits from the different bursts.

To do so, an extraction method consists in unfolding each hit over all possible ambiguities in the "unfolded domain", and to group the hits issued from the different bursts, in this domain. Whenever a group of hits has been identified, all the ambiguities of these hits are removed, and the algorithm proceeds to the next group of hits.

This method suffers from possible errors when grouping the hits. Due to ambiguities, echoes from multiple targets at different locations may produce close unfolded hits. In this case, a group of hits associated to a target can include hits from other targets. These hits are removed from their other ambiguous locations, including the right one, thus making more difficult the target detection process. To avoid such hit suppressions, the extractor combines a series of tests.

Another approach consists in considering the processing chain as a single global function, designed to meet a limited number of global objectives:

- to locate the targets in the unfolded domain
- to determine their complex amplitudes.

In this paper, we propose to apply this global approach by using a Compressed Sensing technique, based on the Complex Approximate Message Passing algorithm, CAMP [2].

In section 3, the global objectives of radar processing are written in terms of Compressed Sensing, and the characteristics of the structure of radar signals are analyzed: series of blocks of complex signals.

In section 4, the block-CAMP algorithm is described, using complex derivatives and matrix notations, and extended to any coefficient matrix. We will denote this algorithm by N-signal Complex Approximate Message Passing, N-CAMP.

In section 5, simulation results are obtained by applying this algorithm to ambiguous radar signals.

The section 6 summarizes the effectiveness of this approach.

3. COMPRESSED SENSING RADAR PROCESSING

3.1 Radar signals

Let us consider here that the transmitted signal consists of a sequence of bursts. The radar signal y^k received during burst k ($k = 1 \cdots K$), can be modeled as the sum of a noise n^k and I received echoes backscattered by the targets. Each received echo (index $i = 1 \cdots I$) is the product of the echo complex amplitude x_i^k and the echo "signature" $A^k\left(p_i^k\right)$ that depends on the echo position p_i^k and on the signal transmitted during burst k:

(1) $y^k = n^k + \sum_i A^k(p_i^k) \cdot x_i^k$

Echo position p_i^k contains several parameters like range, azimuth, elevation, radial speed. In this paper, no distinction is made between these parameters: echo position is considered as a whole. Moreover, it is assumed that the echo position does not vary during the set of bursts processed together (if an echo position would vary, one could describe the variation by a model of higher order whose parameters do not vary, and the set of parameters would take place of the echo position): $A^k(p_i^k) = A^k(p_i) = A_i^k$. Under this assumption, the received signal can be written as

(2) $y^k = n^k + \sum_i A_i^k \cdot x_i^k$

The received signal consists of a block of complex signals with several dimensions: the number of receivers, the number of coherent pulses and the number of range bins. Here the block of radar signals is assumed to be written in a vector whose length is the product of the block dimensions: A_i^k, n^k and y^k are vectors. As the dimensions of the blocks vary from burst to burst, the vector lengths also vary from burst to burst.

Finally, in this paper, it is assumed that the echo complex amplitude of a given target can vary from burst to burst. This arises for instance when the frequency transmitted by the radar changes from burst to burst.

3.2 Compressed Sensing

The proposed Compressed Sensing strategy considers the large set of "all possible" echo positions, and assumes each position is associated to a complex amplitude, but only few of them are non-zero:

(3) $y^k = n^k + \sum_j A_j^k \cdot x_j^k$

where the summation extends over all possible positions $j = 1 \cdots J$, and x_j^k is non-zero only if j is an index associated to a real target position, that is $\exists i : j = j(i)$ where $j(i)$ is the table that provides the correspondence between the target index and the position index.

Vectors A_j^k, $j = 1 \cdots J$ can be grouped in a matrix A^k, and associated amplitudes x_j^k, $j = 1 \cdots J$ in a vector x^k:

(4) $y^k = n^k + A^k \cdot x^k$

Without loss of generality, each column of the matrix is assumed to be normalized:

(5) ${A_j^k}^H \cdot A_j^k = \sum_a \left|A_{a,j}^k\right|^2 = 1\ \forall j$

Let us denote by $\ddot{x}^k$ the vector x^k reduced to its non-zero elements, and $\ddot{A}^k$ the matrix A^k reduced to the vectors associated to the non-zero elements of x^k. This means $\ddot{x}^k$ is the vector that contains the elements $x_i^k, i = 1 \cdots I$ and $\ddot{A}^k$ is the matrix that contains the vectors $A_i^k, i = 1 \cdots I$. This gives

(6) $A^k \cdot x^k = \ddot{A}^k \cdot \ddot{x}^k$

As the target position does not vary from burst to burst, non-zero locations remain at the same place over the K bursts. However, amplitudes vary from burst to burst, so $\ddot{x}^k$ is not constant over index k. In a similar way, the burst parameters (including the size of the observation vector) vary from burst to burst, so that $\ddot{A}^k$ is not constant over index k.

3.3 Radar processing objective in terms of Compressed Sensing

The aim of radar processing is to detect and to locate received echoes from observed signals, that is to find echo complex amplitudes (to detect) and to find associated positions (to locate). In terms of Compressed Sensing, this corresponds to determine, from observed vectors $y^k, k = 1 \cdots K$, the sparse vectors $x^k, k = 1 \cdots K$ such that their non-zero elements correspond to the same position indexes. Note also that the Compressed Sensing setting indeed applies here since the matrices A^k are rectangular matrices with much less rows than columns.

3.4 Extract Before Detect

The proposed Compressed Sensing processing produces at once the echo position and their amplitudes: it extracts the positions at the same time it detects. We therefore propose to denote this approach "Extract Before Detect" in analogy with Track Before Detect algorithms that produce tracks at the same time that they detect.

4. CAMP APPLIED TO MULTIPLE BURST SIGNALS

[3] extends AMP algorithm [1] to the case of N-signal. In this section, this algorithm is written in the case of complex signals, using complex notation (complex numbers and their conjugate) and matrix notations, while removing any restriction over some matrix properties.

First, let us notice that grouping

- observed signals $y^k, k = 1 \cdots K$ into a "long" vector y
- matrices $A^k, k = 1 \cdots K$ into a "long" matrix A
- echo amplitudes $x^k, k = 1 \cdots K$ into a "long" vector x

does not reveal any simple linear relationship between them, since complex amplitudes x^k change from burst to burst. Thus N-signal processing cannot be reduced to a longer 1-signal processing of the form $y = n + A \cdot x$.

4.1 N-signal complex soft threshold variation

[3] and [4] describe the Compressed Sensing block-soft thresholding operator applicable to a set of real signals $v = \cdots v^k \cdots$.

$$(7)\quad \eta(v;\lambda) = max\left(0; 1 - \frac{\lambda}{\|v\|_2}\right)\cdot v$$

The same expression applies to complex signals. In the case $\|v\|_2 > \lambda$, that is $|\eta(v;\lambda)| \neq 0$,

$$(8)\quad \eta^k(v;\lambda) = \left(1 - \lambda\cdot\left(\sum_m v^m \cdot v^{m*}\right)^{-\frac{1}{2}}\right)\cdot v^k,$$

its complex derivatives [6][7] are

$$(9)\quad \frac{\partial\eta^k(v;\lambda)}{\partial v^n} = \left(1 - \frac{\lambda}{\|v\|_2}\right)\cdot\delta(k-n) + \frac{\lambda}{2}\frac{v^{n*}\cdot v^k}{\|v\|_2^3}$$

$$(10)\quad \frac{\partial\eta^k(v;\lambda)}{\partial v^{n*}} = \frac{\lambda}{2}\frac{v^n\cdot v^k}{\|v\|_2^3},$$

and the soft threshold differential is

$$(11)\quad \eta^k(v+dv;\lambda) - \eta^k(v;\lambda) = \sum_n \frac{\partial\eta^k(v;\lambda)}{\partial v^n}\cdot dv^n + \frac{\partial\eta^k(v;\lambda)}{\partial v^{n*}}\cdot dv^{n*}$$

$$(12)\quad \begin{aligned}\eta^k(v+dv;\lambda) - \eta^k(v;\lambda) &= \left(1 - \frac{\lambda}{\|v\|_2}\right)dv^k + \frac{\lambda}{2}\cdot\frac{v^k}{\|v\|_2}\sum_n \frac{v^{n*}}{\|v\|_2}\cdot\frac{dv^n}{\|v\|_2}\\ &+ \frac{\lambda}{2}\cdot\frac{v^k}{\|v\|_2}\sum_n \frac{v^n}{\|v\|_2}\cdot\frac{dv^{n*}}{\|v\|_2}\end{aligned}$$

4.2 N-signal CAMP (N-CAMP)

N-signal CAMP is based on CAMP expressions [2] [5] applied to the K vectors of observed signals, while taking into account the fact that the non-zero entries are located at the same position index for all observed signals

$$(13)\quad v_j^{k,t} = \sum_b A_{bj}^{k*} z_b^{k,t-1} + x_j^{k,t-1}$$

$$(14)\quad x_j^{k,t} = \eta^k\left(v_j^t;\tau_t\right)$$

$$(15)\quad \begin{aligned}z_a^{k,t} &= y_a^k - \sum_j A_{aj}^k x_j^{k,t}\\ &- \sum_j A_{aj}^k\left(\eta^k\left(v_j^t + dv_{a,j}^t;\tau_t\right) - \eta^k\left(v_j^t;\tau_t\right)\right)\end{aligned}$$

Let us consider the burst associated to the longest observed signal (assuming only one burst is associated to the longest observed signal), and let us denote a_{max} the maximum index value of the observed signal and k_{max} the associated burst index. Expression (15) applied to index a_{max} contains

$$(16)\quad dv_{a_{max},j}^t = dv_{a_{max},j}^{1,t}\cdots dv_{a_{max},j}^{k,t}\cdots dv_{a_{max},j}^{K,t}$$

For all burst indices but k_{max}, the length of vector $dv_{\cdot,j}^{k,t}$ is smaller than a_{max}, so expression (16) is not defined. In this expression, $dv_{a_{max},j}^t$ should be restricted to the only vector that gives sense, $dv_{a_{max},j}^{k_{max},t}$. More generally, expression (15) should be based only on the variations associated to burst index k, $dv_{a,j}^{k,t} = -A_{aj}^{k*} z_a^{k,t-1}$ [2] :

$$(17)\quad dv_{a,j}^t = \left(0,\cdots,0,-A_{aj}^{k*} z_a^{k,t-1},0,\cdots,0\right)$$

$$(18)\quad \begin{aligned}z_a^{k,t} &= y_a^k - \sum_j A_{aj}^k x_j^{k,t}\\ &- \sum_j A_{aj}^k\begin{pmatrix}\eta^k\left(v_j^t + \left(0,\cdots,0,-A_{aj}^{k*} z_a^{k,t-1},0,\cdots,0\right);\tau_t\right)\\ -\eta^k\left(v_j^t;\tau_t\right)\end{pmatrix}\end{aligned}$$

$$(19)\quad \begin{aligned}z_a^{k,t} &= y_a^k - \sum_{j,\left|x_j^{k,t}\right|\neq 0} A_{aj}^k x_j^{k,t}\\ &+ \left(\sum_{j,\left|x_j^{k,t}\right|\neq 0}\left|A_{aj}^k\right|^2\left(1 - \frac{\tau_t}{\left\|v_j^t\right\|_2} + \frac{1}{2}\frac{\tau_t}{\left\|v_j^t\right\|_2}\left|\frac{v_j^{k,t}}{\left\|v_j^t\right\|_2}\right|^2\right)\right)\cdot z_a^{k,t-1}\\ &+ \left(\sum_{j,\left|x_j^{k,t}\right|\neq 0} {A_{aj}^k}^2\frac{1}{2}\frac{\tau_t}{\left\|v_j^t\right\|_2}\left(\frac{v_j^{k,t}}{\left\|v_j^t\right\|_2}\right)^2\right)\cdot {z_a^{k,t-1}}^*\end{aligned}$$

4.3 N-CAMP matrix notation

Using matrix notations, we obtain the following set of equations:

$$(20)\quad v^{k,t} = {A^k}^H z^{k,t-1} + x^{k,t-1}$$

(21) $u^{k,t} = \frac{v^{k,t}}{\left\|v^t\right\|_2}$ [term by term operations over index j]

(22) $w^t = \frac{\tau_t}{\left\|v^t\right\|_2}$ [term by term operations over index j]

(23) $x^{k,t} = Max(0; 1 - w^t)v^{k,t}$ [term by term operations over index j]

$$(24)\quad \begin{aligned}z^{k,t} &= y^k - \dddot{A}^{k,t}\dddot{x}^{k,t}\\ &+ \left(\left(\dddot{A}^{k,t}\circ{\dddot{A}^{k,t}}^*\right)\cdot\left(1 - \dddot{w}^t + \frac{1}{2}\dddot{w}^t\circ\dddot{u}^{k,t}\circ{\dddot{u}^{k,t}}^*\right)\right)\circ z^{k,t-1}\\ &+ \left(\left(\dddot{A}^{k,t}\circ\dddot{A}^{k,t}\right)\cdot\left(\frac{1}{2}\dddot{w}^t\circ\dddot{u}^{k,t}\circ\dddot{u}^{k,t}\right)\right)\circ {z^{k,t-1}}^*\end{aligned}$$

4.4 Real signals, single burst

In the case of real signals, and a single burst, this expression reduces to

$$(25)\quad z^t = y - \dddot{A}^t\dddot{x}^t + \left(\left(\dddot{A}^t\circ{\dddot{A}^t}^*\right)\cdot\underline{1}\right)\circ z^{t-1}$$

and, if all matrix coefficients have the same modulus, that is if $\left|A_{aj}\right| = \frac{1}{\sqrt{N_y}}$, where N_y is the number of elements of y, then this expression becomes

$$(26)\quad z^t = y - \dddot{A}^t\dddot{x}^t + \frac{N_{\ddot{x}}^t}{N_y}z^{t-1}$$

where $N_{\ddot{x}}^t$ is the number of non-zero elements of x^t.

This expression differs a bit from those given by [1][2]. The reason is that these references suppose that the sum of the squared lines of matrix A are constant (exactly or statistically). In this paper we do not make such an hypothesis about matrix A, we rather only assume its columns are normalized (5).

4.5 Coherent Extractor

As no restrictive hypothesis is made about matrix A, one can envisage to apply N-CAMP to radar signals already processed by standard coherent functions (Digital Beam Forming, Doppler Filtering, Pulse Compression). In this case, the processed signal is

$$(27)\, y'^k = B^k \cdot y^k = A'^k \cdot x^k$$

where B^k is the processing matrix and $A'^k = B^k \cdot A^k$.

A'^k columns contain the set of filter responses to a given echo position. Their energies are maximum in the filter matched to echo position, lower in adjacent filters (main lobe edge), and much lower in farther filters (side lobes). Figure 1 shows matrix A'^k in the case where the observed signal is a set of Doppler filters, and target position is the radial speed, lying in $\left[-2\, V_{amb}^k ; 2\, V_{amb}^k\right]$ where V_{amb}^k is the ambiguous speed. 2 columns are highlighted, associated to 2 target speeds. The corresponding reduced matrix $\ddot{A}'^k$ is shown on the right side of Figure 1.

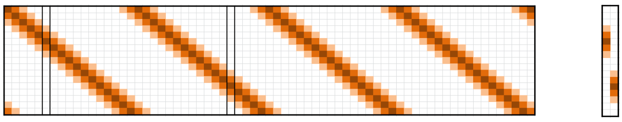

Figure 1 : Multidiagonal matrix (full and reduced matrix)

The squared sum of matrix $\ddot{A}'^k$ lines corresponds to the energies obtained in all observed filters, that are "high" in filters matched to real echo positions, and lower in other positions. One sees it is important, in this case, not to assume the lines have a constant energy.

Finally, as the lines close to echo positions are much more energetic than the lines "far away" from them, one probably misses only a small information amount not to observe those "far away" lines. In other words, one could process by the Compressed Sensing technique the only filters that detected, plus some adjacent filters.

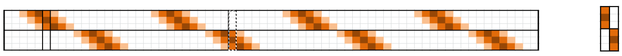

Figure 2 : Matrix reduced to its "lines of interest"

Standard radar processing chains work in this way. They apply coherent filters, detect and transmit only the detected signals to the extractor.

The interest to process only the pre-detected signals by a Compressed Sensing function is that it replaces a series of tests over hit energies (standard extraction) by an extraction method based on complex signals. In this case, Compressed Sensing acts as a Coherent Extractor.

5. SIMULATION RESULTS

N-CAMP expressions have been applied to radar signals received in one range bin during 5 successive bursts made of 17, 19, 21, 23 and 25 pulses respectively. All the bursts have the same Doppler resolution, $\partial v = \frac{\lambda}{2T_r} = 10 m/s$. Their ambiguous speeds are 170, 190, 210, 230 and 250 m/s.

N-CAMP algorithm is used to detect and to locate the echoes on the radial velocity range [-1000 m/s ; 1000 m/s] with a 5 m/s step (oversampling by a factor 2). Note that target radial speeds fit this grid.

5.1 Scenario 1, targets separated by 1 ambiguous speed

The following parameters were chosen for this first scenario.

Target 1: 60 m/s, mean SNR = 50 dB, Swerling 2.

Target 2: 275 m/s, mean SNR = 20 dB, Swerling 2.

The speed difference between both targets is 215 m/s, that is close to one ambiguous speed.

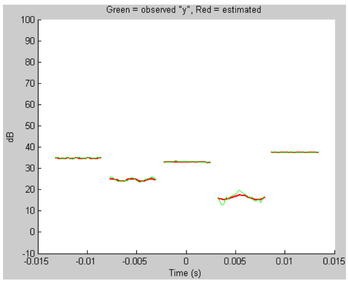

Figure 3 : input and estimated signals versus time

The "weak" target is much lower than the "strong" one (-30 dB), so that the signal is almost constant inside each burst. The variations from burst to burst correspond to target fluctuations. [Remark: 1-pulse SNR is equal to 1-burst SNR divided by the number of pulses (about -13 dB).]

In each burst, the estimated signal (red line) fits well the input one (green line).

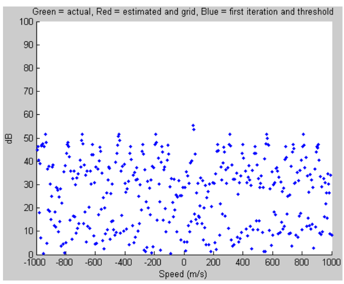

Figure 4 : matched filter output

The matched filter is an unweighted filter on each burst, followed by a non-coherent integration over the 5 bursts [Remark. This corresponds to the first iteration of equations (20) … (24)].

The matched filter output essentially reveals the "strong" target. The weak target is deeply buried into its sidelobes, it is not visible in Figure 4.

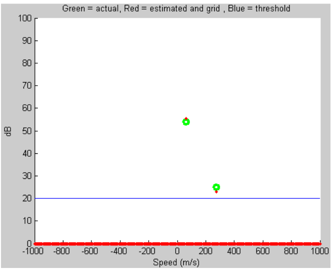

Figure 5 : input and detected targets

Green circles represent input targets, red dots represent detected targets. Both targets are detected and located at the right radial speeds. Estimated energies slightly differ from input ones.

5.2 Scenario 2, 2 targets close each other

For this scenario, the target radial speeds are set to 60 m/s and 65 m/s respectively (other parameters are kept unchanged). The speed difference between both targets is 5 m/s, that is half a speed resolution, in each burst.

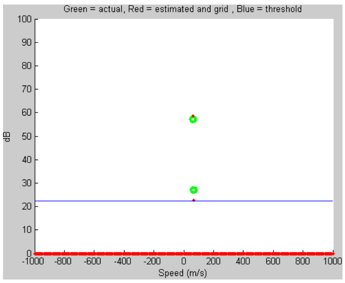

Figure 6 : input and detected SNR

Both targets are detected and located at the right radial speeds. Estimated energies again slightly differ from input ones.

5.3 Overall results analysis

Both cases show that N-CAMP Compressed Sensing algorithm is able to correctly detect and estimate the target positions, including ambiguity solving. This is achieved either when the weak target is close to the strong one, or when both targets are separated by one ambiguity speed.

Measured speeds are equal to actual speeds and measured SNR are close to actual SNR.

6. CONCLUSION

This paper describes a block-AMP algorithm that takes into account complex signals and "any" possible matrix. This algorithm has been applied to pulse to pulse radar signals. Simulation results demonstrate that the proposed strategy can achieve at the same time the functions of detection, clustering and plot measurement.

Compared to a standard radar extractor, this processing scheme gains in sensitivity:

- It detects on the basis of the full set of observed signals, in place of a burst by burst detection followed by a "K over N" process
- It does not require the use of a weighting function to decrease the sidelobe level, since the proposed strategy can naturally take into account the presence of (possibly high) sidelobes.
- Thanks to complex signals processing, small target detection is not affected by the presence of strong targets, even when ambiguity folding makes them close to each other in some bursts.

To make it fully applicable to radar processing, this processing function should be enriched by an automatic grid adaptation to target position. Applied to signals that have already been processed by a coherent filtering function (Coherent Extractor), it should also take into account the noise measurements achieved by CFAR and clutter map functions.

7. REFERENCES

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