Tracking of Fluctuating Targets Using Stroboscopic Sampling

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Abstract - This paper considers the processing of radar signals in digital radar rangefinders. Special attention is focused on the case where limited performance of digital signal processors cannot provide the input signal sampling rate corresponding to the Nyquist criterion. The considered method of stroboscopic sampling allows overcoming this limitation. A suggested pseudo random modification of the stroboscopic sampling method improves the accuracy of range measurement in the case of target amplitude fluctuations.

I. Introduction

In ultra-wideband (UWB) radar systems, limited system performance can prevent the use of optimal algorithms of real-time digital signal processing [1]. Due to wide spectral range of UWB signals, their Nyquist rates may exceed the specifications of the best analogueto-digital converters [2]. This paper considers the digital radar rangefinders in the case when the sampling frequency of the signal is lower than the Nyquist frequency. In order to increase radar system performance, different methods have been proposed:

- parallel sampling and parallel data processing [3, 4], which leads to significantly higher hardware costs;

- low-digit (including binary) signal quantisation [5, 6], which increases instrumental errors;

- under sampling, i.e. the sampling below the Nyquist frequency [2, 6].

In this paper we will analyse the use of stroboscopic signal sampling [3, 7 – 11], which allows for a reduction of range measurement error without an increase in requirements of the range finders' performance. This method allows increasing the number of samples on a signal by their small offset δ from one pulse repetition interval (PRI) T_R to another, achieving a higher accuracy range measurement with lower sampling rate. In the limiting case ADC only needs to take one sample every PRI so need for high speed ADC is avoided [10].

Stroboscopic sampling is also called Equivalent time sampling or Sequential sampling [7]. It is illustrated in Figs. 1a-d, where τ is the pulse width, *T* is the real-

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time signal sampling interval (sampling at the Nyquist rate), T_s is the stroboscopic sampling interval, offset

$$\delta = T_s / N \,. \tag{1}$$

The number of transmitted pulses needed to build the signal profile *N* equals four in the example in Fig. 1.

In the case of matched filtering of a rectangular pulse the number of samples per each signal in one PRI equals

$$n_s = 2\tau/T_s . (2)$$

Total number of samples per signal during one cycle of stroboscopic sampling is

$$n = n_s N = 2\tau/\delta \,. \tag{3}$$



Figure 1. Signal representation in the impulse radar

- a Transmitted train of pulses
- b Received pulses
- c Real-time sampling
- *d* Stroboscopic sampling
- e Series-parallel sampling
- f Pseudo random sampling

Series-parallel sampling, which is another modification of the stroboscopic method [12], is illustrated in Fig. 1e. Pseudo random sampling, which will be discussed further in the paper, is shown in Fig. 1f. The implementation of stroboscopic sampling does not require additional hardware and it is only necessary to provide sampling frequency stability (low clock drift) [8].

The remainder of the paper is organised as follows: Section II presents a radar range finder which utilises stroboscopic sampling; in Section III, a modified version of stroboscopic signal sampling, which provides better accuracy in the presence of target amplitude fluctuations, is analysed and the conclusions are formulated.

II. The influence of target fluctuations on range measurement

The radar range finder, which provides the distance to the target, represents a non-linear discrete automatic tracking system, the behaviour of which depends largely on the parameters of signal digitising. The range finder block diagram is shown in Fig. 2 and consists of a delay measuring system, a low-pass filter and a gate pulse generator producing a reference signal with the delay, which is proportional to the low-pass filter output signal. The optimum delay measuring system contains an optimum time gate and integrator, or generalised time discriminator.



Figure 2. Range finder block diagram

Range tracking is carried out in pulsed radar by the direct matching of a range gate position to the delayed echo pulse. The usual technique is a split gate range tracker, which is a form of range tracker with a pair of time gates called an early gate and a late gate, contiguous or partly overlapping in time [13, 14]. Deviation of the pair of gates from the proper tracking position increases the signal energy in one gate and decreases it in the other, producing an error signal. The range difference channel is formed by subtracting the late gate output from the early gate output and integrating the result, to form a time discriminator provides the input to an electronic tracking loop that controls the timing of the gates.

During stroboscopic sampling the samples relating to different parts of the signal are taken at different PRI. Therefore, due to fluctuations in signal amplitude (because of movement and rotation of the target, atmospheric fluctuations, etc.), the output signal of time discriminator is different from zero even at zero tracking error.

This effect is illustrated in Fig. 3. This figure shows the signal samples in the four adjacent pulse repetition periods. Since the amplitude of the signal changes, its envelope, restored at the end of the stroboscopic sampling cycle (Fig. 3e), will be distorted.

Let us consider the impact of target fluctuation on the accuracy of tracking. The signal amplitude is distributed according to Rayleigh [15], with the variance of amplitude fluctuation of input signal σ_a^2 and the correlation coefficient $r_a(t) = \exp(-|\tau|/\tau_a)$, where the correlation time of amplitude fluctuation is $\tau_a \leq NT_R$. At higher τ_a the fluctuations will have no effect on the characteristics of the time discriminator and the signal can be considered as non-fluctuating.



Figure 3. Target amplitude fluctuations

a 1st period
b 2nd period
c 3rd period
d 4th period
e Restored signal envelope

In [11] it was shown that the variance of tracking error due to target amplitude fluctuations at the output of time discriminator can be expressed as

$$\sigma_{a_{-out}}^{2} = 4\sigma_{a}^{2}\tau^{2}k_{dS}^{2}\psi(N,\mu)/n_{S}^{2} , \qquad (4)$$

where σ_a^2 is the variance of amplitude fluctuations, $k_{dS} = 0.5 \times n_S N/\tau$ is the discrimination characteristic gain, $\psi(N,\mu)$ describes the dependence of the tracking error on the number of transmitted pulses used to build the signal profile and on the relative correlation interval of amplitude fluctuations $\mu = \tau_a / T_R$.

When $\tau_a >> T_R$, which corresponds to Swerling I target fluctuation model [16], the signal can be considered non-fluctuating and $\psi(N,\mu) \approx 0$. In case of small values of μ the samples in the adjacent PRI are not correlated (Swerling II model) and $\psi(N,\mu) \approx 1/N$.

Dependence of $\sigma_{a_{out}}^2$ on n_s in (4) is due to the fact that at constant N the increase in n leads to the decrease in the stroboscopic sampling interval T_s (2), (3) thus reducing the influence of amplitude fluctuations. Maximum error occurs when $n_s = 1$. In this case stroboscopic sampling (Fig. 1d) and series-parallel sampling (Fig. 1e) are identical.

To reduce the influence of target amplitude fluctuations, N should be reduced. In this case n_s must be increased to maintain the same tracking accuracy, thus increasing the sampling frequency. In this paper we propose a different way of stroboscopic sampling, which does not lead to increase in requirements of the range finder's performance. This method is discussed in the next section.

III. Pseudo random stroboscopic sampling

As discussed in the previous section, if the samples from period to period move in one direction relative to the centre of the signal, amplitude fluctuations have significant influence on range measurement accuracy. The impact of amplitude fluctuations on the accuracy of range measurement can be reduced by a random distribution of samples on the duration of the signal. The uniform sampling is preferable to random when the tracking algorithm associates measurements to targets on scan-by-scan basis. However, if measurement is based on several PRI, then random sampling may have advantages over uniform [17]. This is especially true when the fluctuations are high.

In this paper a pseudo random sampling is proposed, which is a modified version of stroboscopic signal sampling, somewhat similar to Random equivalent-time sampling method used in digital oscilloscopes. The essence of the method is illustrated in Fig. 1f, where signal samples shifts in the adjacent periods in the opposite directions. For this purpose, two sequences of sampling pulses are used. The first samples are taken from the first sequence of sampling pulses, subsequent samples - from the second sequence.

Effect of target amplitude fluctuations on the characteristics of range finder, similar to [11], can be determined as follows. The output signal of the stroboscopic time discriminator based on early and late gate algorithms at zero error is:

$$Q_{S}(0) = -\sum_{j=0}^{N-1} \left\{ \sum_{i=0}^{(n_{S}/2)-1} u(jT_{R} + iT_{S} + j\delta) - \frac{1}{\sum_{i=n_{S}/2}^{n_{S}-1} u[jT_{R} + iT_{S} + (N/2 - j)\delta] \right\}.$$
(5)

The expression can be simplified taking into consideration that

 $u(jT_R + iT_S + j\delta) = a(jT_R) \cdot u(iT_S + j\delta) = a_j u_{ij}$ (6) where $u(iT_S + j\delta) = u_{ij}$ is the signal envelope and $a(jT_R) = a_j$ is the random amplitude fluctuation in PRI. Therefore

$$Q_{S}(0) = -\sum_{j=0}^{N-1} a_{j} \left(\sum_{i=0}^{(n_{S}/2)-1} u_{ij} - \sum_{i=n_{S}/2}^{n_{S}-1} u_{i,(N/2)-j} \right).$$
(7)

For symmetrical signal envelope $(u_{ij} = u_{i,(N/2)-j})$ and $n_s \ge 2$ the amplitude fluctuations do not affect the accuracy of tracking, i.e. $Q_s(0) = 0$. If $n_s = 1$

$$Q_{s}(0) = -\sum_{j=0}^{N-1} u_{j}(a_{j} - a_{j+1}).$$
(8)

After completing the conversions, similar to those in [11], the variance of the tracking error the case of pseudo random stroboscopic sampling can be expressed similar to (4) as

$$\sigma_{a_{-}out}^{2} = 4\sigma_{a}^{2}\tau^{2}k_{dS}^{2}\varphi(N,\mu)/n_{S}^{2} , \qquad (9)$$

where $\varphi(N, \mu)$ describes the dependence of the tracking error on *N* and on the relative correlation interval of amplitude fluctuations μ . In case of matched filtering of a pulsed signal

$$\varphi(N,\mu) = \frac{1}{N^2 \sigma_a^2} \cdot \left[\sum_{j=0}^{N/2-1} u_j (a_j - a_{j+1}) \right]^2. \quad (10)$$

The diagrams $\varphi(N, \mu)$ are shown in Fig. 4 (dashed lines). For comparison the diagrams $\psi(N, \mu)$ are shown (solid lines). It can be seen that the proposed method of pseudo random stroboscopic sampling provides significantly smaller variance of time discriminator output error than the common method. If $n_s = 1$ and $N \ge 4$, the power gain at $\mu = 0.5$ is approximately 1.2-1.4 times (0.8-1.5 dB), while at $\mu = 2$ it is 1.5-2.5 times (1.8-4 dB).



Figure 4. Influence of target amplitude fluctuation on the characteristics of time discriminator in case n=1

IV. Conclusions

In the paper a modification of the stroboscopic sampling method was suggested, which reduces the impact of amplitude fluctuations on the accuracy of range measurement by pseudo random distribution of samples on the duration of the signal.

The results of simulation, presented in Fig. 4 show that the proposed method of stroboscopic sampling allows reducing a random error of tracking, caused by the amplitude fluctuations of a signal, in comparison with the common methods. The carried out analysis has shown that this method is effective under the following conditions: small number of samples on a signal $(n_s \le 2)$ and number of PRI used for one measurement $N \ge 4$.

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