Two approaches to remote sensing via ℓ_1-minimization

Max Hügel
Hausdorff Center for Mathematics and Institute for Numerical Simulation
University of Bonn
Bonn, Germany
Email: max.huegel@hcm.uni-bonn.de

Holger Rauhut
Lehrstuhl C für Mathematik (Analysis)
RWTH Aachen University
Aachen, Germany
Email: rauhut@mathc.rwth-aachen.de

Thomas Strohmer
Department of Mathematics
University of California at Davis
Davis, CA
Email: strohmer@math.ucdavis.edu

***Abstract*—In this talk, we investigate the problem of remotely imaging a scene of targets in the far field by sending out probing signals from an array of sensors and processing the reflected echo. We will discuss and compare two possible sensor arrangements: random and uniform linear.**

To be concrete, suppose n antenna elements $a_1, \ldots, a_n \in [0, A]^2$ mounted on a square platform $[0, A]^2$ emit an isotropic electromagnetic wave of wavelength $\lambda > 0$. The spatial part of this wave emitted in $a_k = (\xi, \eta, 0)$ and recorded in a point $r_\ell = (x, y, z_0)$ at distance z_0 can be approximated by

$$G(a_k, r_j) := \frac{\exp(i\omega z_0)}{4\pi z_0} \exp\left(i\omega \frac{(x-\xi)^2 + (y-\eta)^2}{2z_0} \right),$$

where $\omega := 2\pi/\lambda$ denotes the wavenumber. Suppose now we want to image a scene of targets located in points $\{r_j\}_{j=1,\ldots,s}$ and with reflectivities $\{x_j\}_{j=1,\ldots,s} \subset \mathbb{C}^s$. If antenna element a_k transmits, then the recorded echo at antenna element a_ℓ is given by

$$y_{k,\ell} := \sum_{j=1}^{s} G(a_k, r_j) G(a_\ell, r_j) x_j.$$

If we choose the transmit-receive mode in such a way that one antenna element transmits at a time and the whole array receives the echo, we collect n^2 measurements $y \in \mathbb{C}^{n^2}$. In this talk, we will compare two possible arrangements of the antenna elements: In the first setting, we choose the a_k independently and uniformly at random from $[0, A]^2$. We then discretize the target space into a grid of N resolution cells $\{r_j\}_{j=1,\ldots,N}$. A scene of targets is represented by the vector $x = \{x_j\}_{j=1,\ldots,N} \subset \mathbb{C}^N$ of reflectivities. In many cases, only few of the typically large number of resolution cells are occupied by a target, meaning that x is sparse. See Fig. 1 for a visualization of this setup.

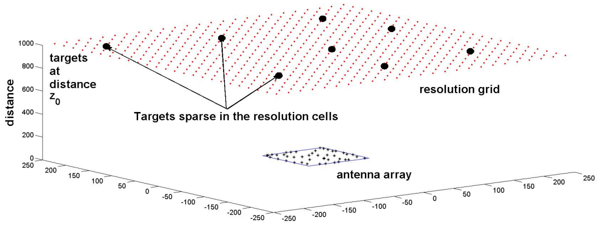

Fig. 1. The targets at distance z_0 distributed sparsely in the target domain.

Therefore, compressive sensing is applicable and one can recover the scene via ℓ_1-minimization. However, if the targets are not exactly on the grid, errors occur. Therefore, we propose a second sensor arrangement where the a_k are placed on an equidistant grid over $[0, A]^2$. Recent rigorous results on super-resolution imply that one can then recover the locations and reflectivities of a scene of targets up to infinite precision via total variation minimization, provided that a minimum distance condition between the targets is satisfied. In practice, one also discretizes the target space in the second scenario, however, one can now choose this grid arbitrarily fine and then again use ℓ_1-minimization to image the scene. In this talk, we will present some theoretical recovery guarantees in terms of the number of required antenna elements. These are essentially taken from [1] and [2]. We will moreover show some numerical results comparing the two approaches with respect to stability, robustness and sensitivity to gridding error.

References

[1] M. Hügel, H. Rauhut and T. Strohmer, *Remote sensing via ℓ_1-minimization*, to appear in FoCM.

[2] E. Candès and C. Fernandez-Granda, *Towards a mathematical theory of super-resolution*, to appear in CPAM.