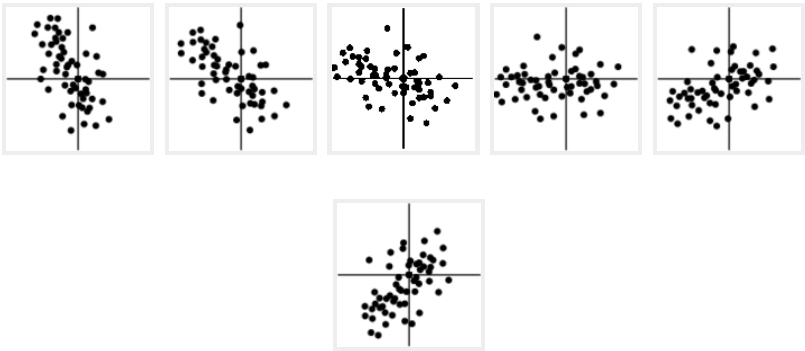

A GROUP SPARSITY IMAGING ALGORITHM FOR TRANSIENT RADIO SOURCES

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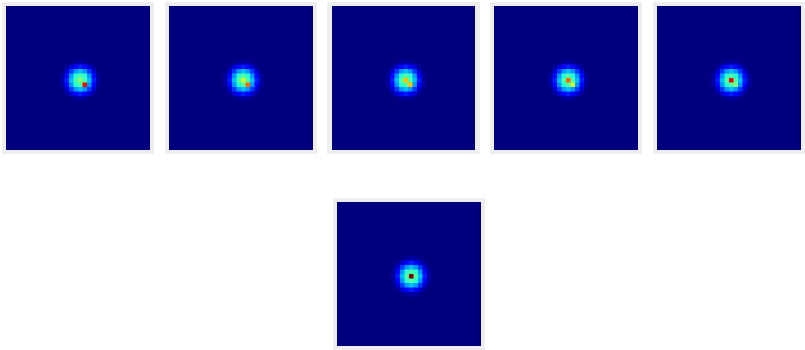
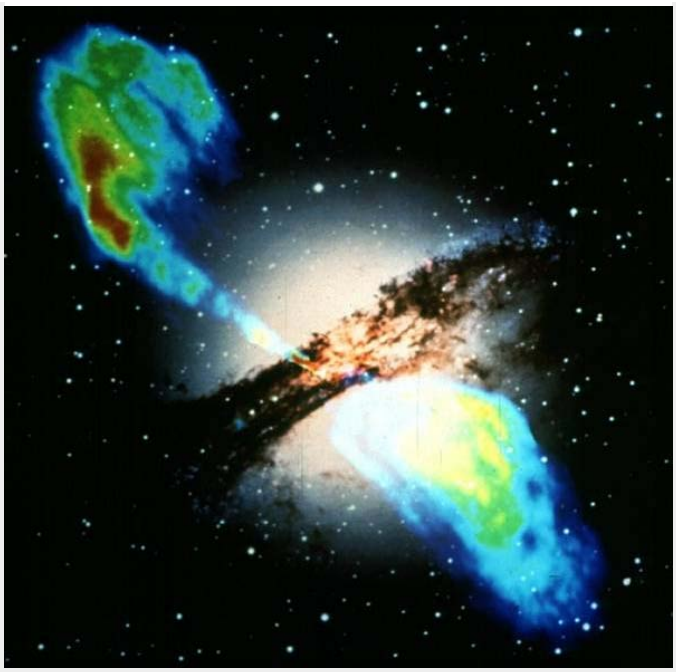
[1]Institut für Computergraphik, TU Braunschweig

[2]National Radio Astronomy Observatory, Socorro NM

[3]Physics and Astronomy Dept., University of New Mexico

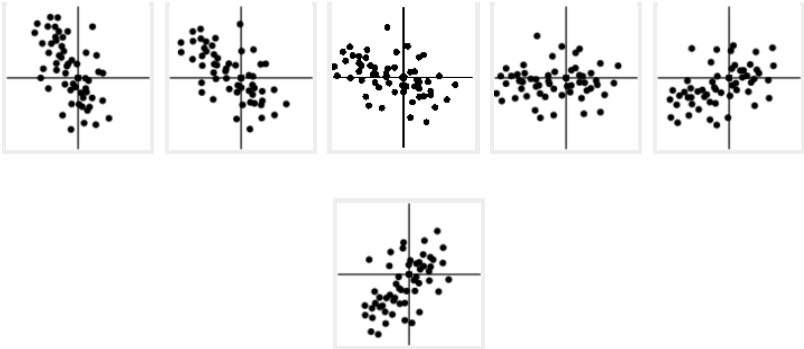


RADIO ASTRONOMY



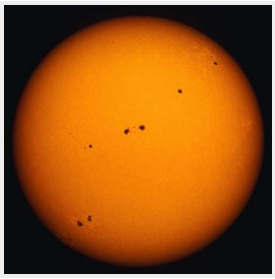
RADIO INTERFEROMETRY

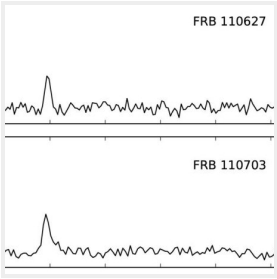
RADIO INTERFEROMETRY SAMPLING



- sparse samples in the Fourier domain
- sampling pattern rotates with Earth

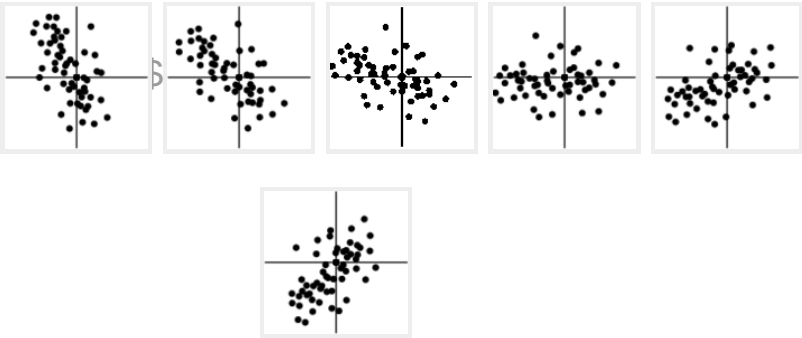
TRANSIENT SOURCES



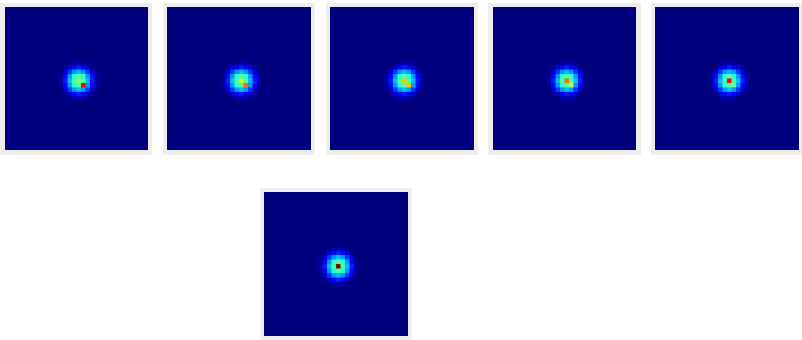


TRANSIENT TEST SOURCE

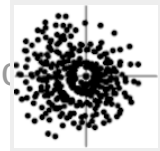
$\mathc

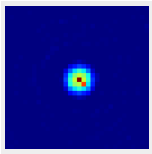


x



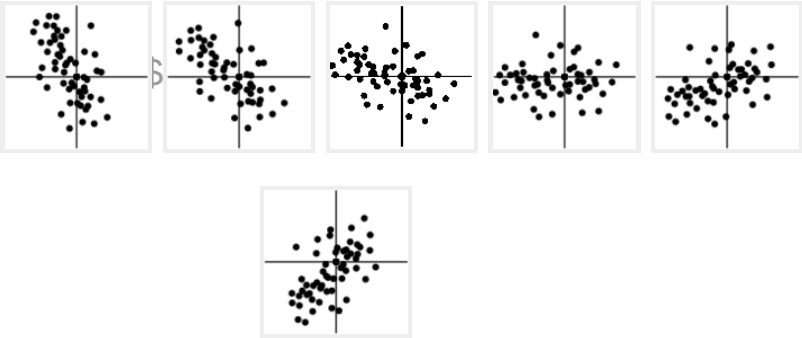
AVERAGING MANY OBSERVATIONS

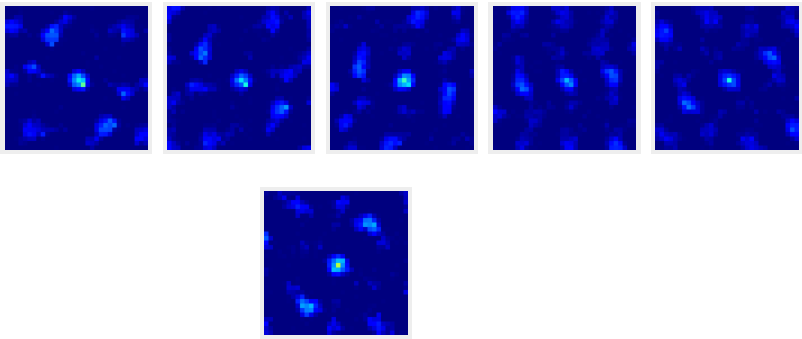
$\mathc  $

x 

+ high spatial resolution
− no temporal resolution
− artifacts due to model violation

RECONSTRUCTING INDIVIDUAL TIME FRAMES

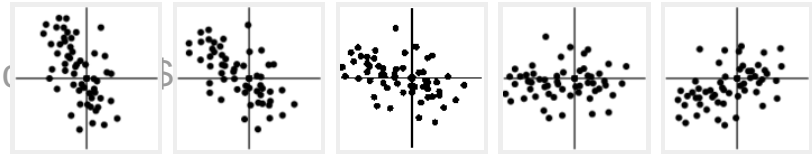
$\mathc  $

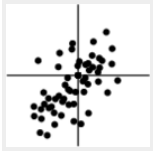
x 

+ high temporal resolution
− low spatial resolution
− artifacts due to low coverage

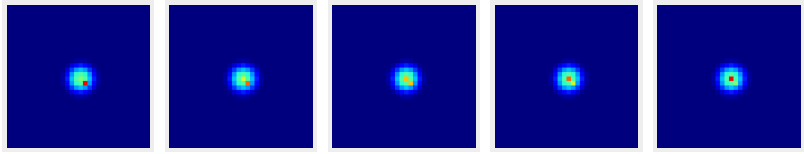
JOINT DYNAMIC RECONSTRUCTION

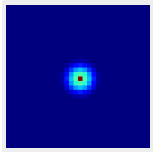
$\mathc





x





+ high spatial resolution in static parts
+ high temporal resolution in dynamic parts
+ *cross-regularization*

GROUP SPARSITY PROBLEM

$$\arg\min_{\vec{x}} \tfrac{1}{2} \norm{\mathcal{F} \vec{x} - \vec{v}}_2^2 + \tau \norm{\vec{x}(t)}_{1,\infty}$$

THE $\ell_{1,\infty}$-NORM

$$\norm{\vec{x}(t)}_{1,\infty} = \sum_i \max_t \abs{\vec{x}_i(t)}$$

EFFECTS OF $\ell_{1,\infty}$ MINIMIZATION

- locally sparse signal
- locally sparse temporal variation
- no temporal smoothing

OPTIMIZATION FRAMEWORK

$$\arg\min_{\vec{x}} \tfrac{1}{2} \norm{\mathcal{F} \vec{x} - \vec{v}}_2^2 + f(\vec{x})$$

A. Beck and M. Teboulle:
A fast iterative shrinkage-thresholding algorithm for linear inverse problems,
SIAM J. Imaging Sciences 2 (2009), pp. 183–202.

DISJOINT GROUPS

$$\sum_i \abs{g(\vec{x}_i)}$$

S. Wright, R. Nowak, and M. Figueiredo:
Sparse reconstruction by separable approximation,
IEEE Transactions on Signal Processing 57, 7 (2009), pp. 2479–2493.

PROXIMAL MAPPING FOR THE ℓ_∞-NORM

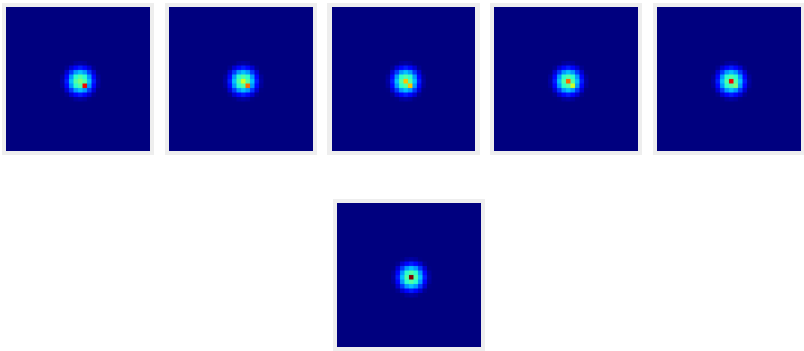
$$\max_t \abs{\vec{x}_i(t)}$$

I. Daubechies, M. Fornasier, and I. Loris:
Accelerated projected gradient method for linear inverse problems with sparsity constraints,
J. Fourier Analysis and Applications 14 (2008), pp. 764–792.

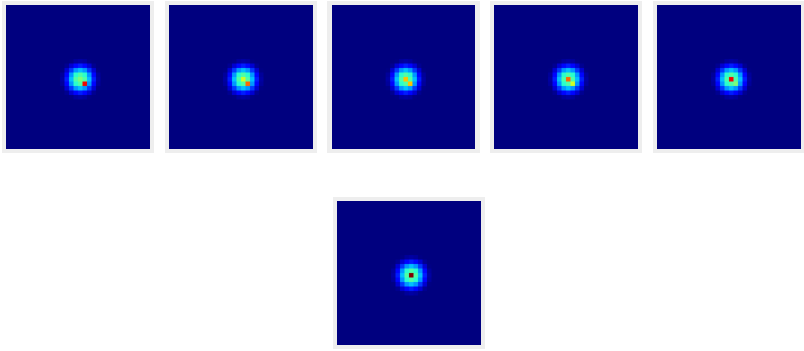
RESULTS

"BLOBS": STATIC RECONSTRUCTION

true

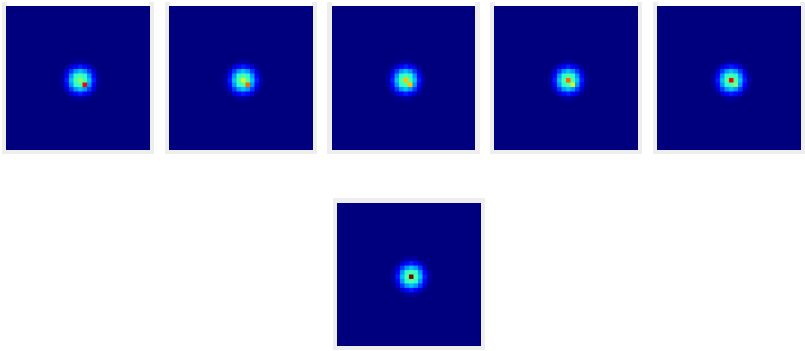


static ℓ_1

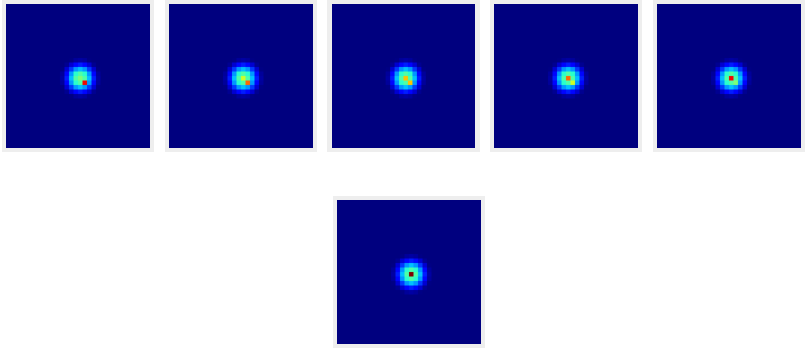


"BLOBS": INDIVIDUAL RECONSTRUCTION

true

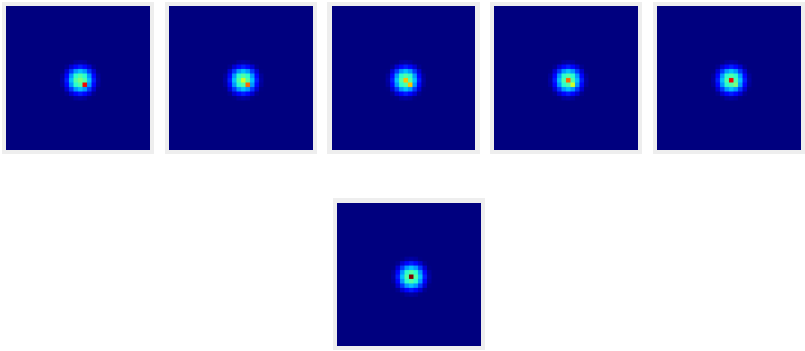


individual ℓ_1

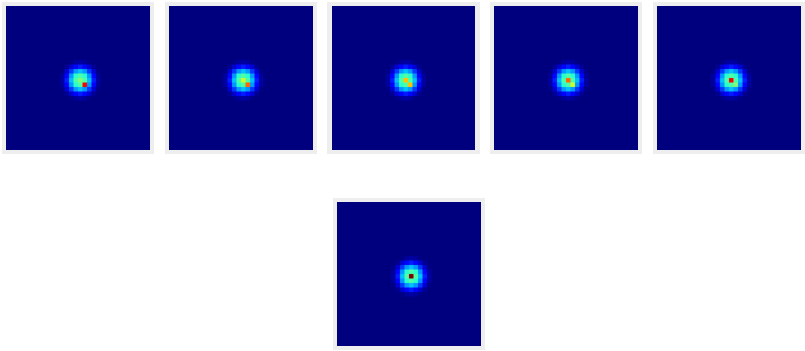


"BLOBS": $\ell_{1,\infty}$ RECONSTRUCTION

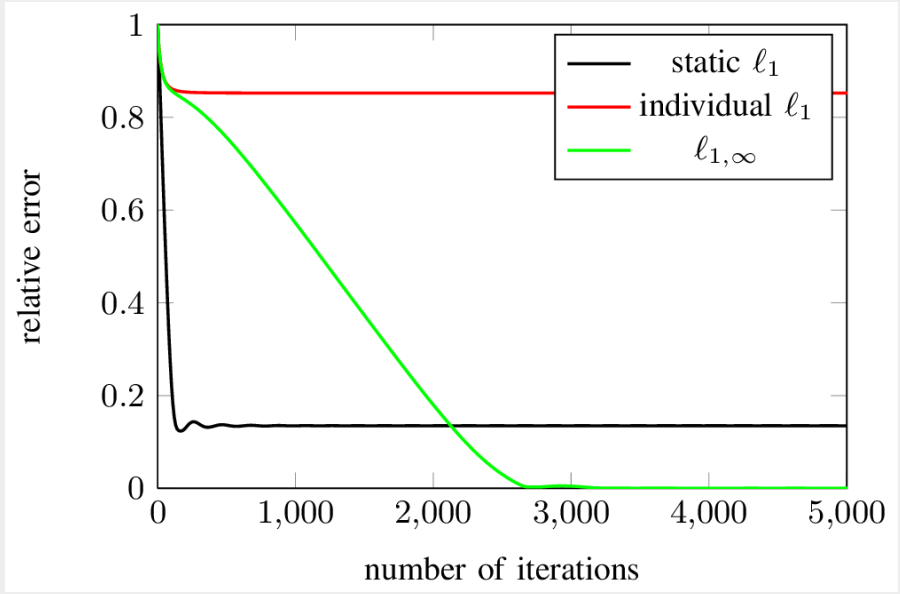
true



$\ell_{1,\infty}$

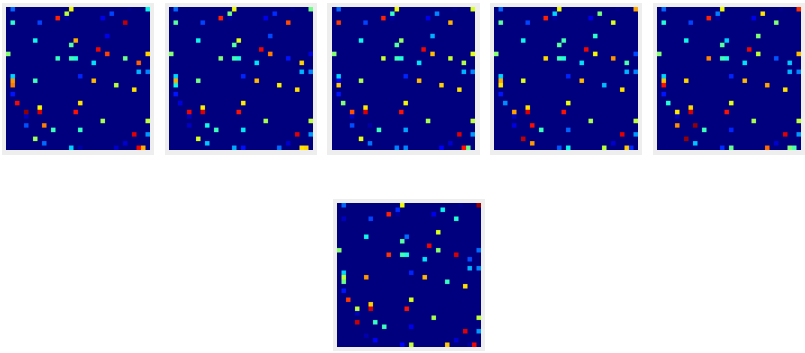


"BLOBS": NUMERICAL EVALUATION

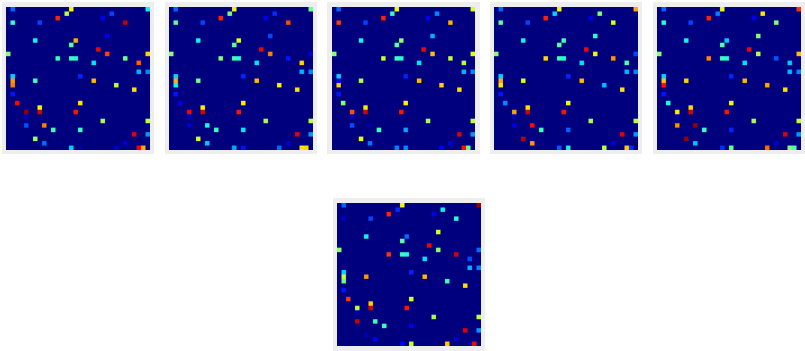


"POINT SOURCES": STATIC RECONSTRUCTION

true

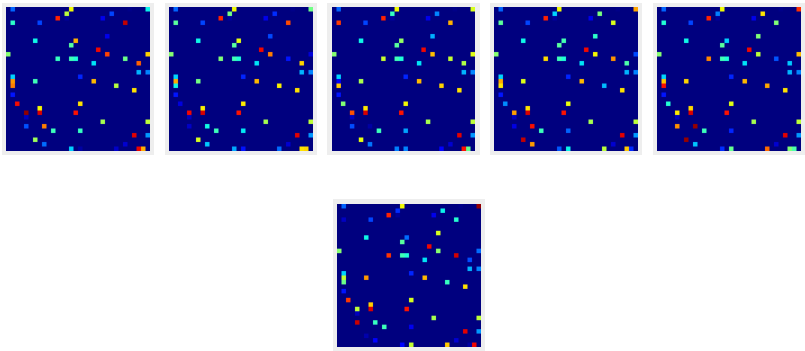


static ℓ_1

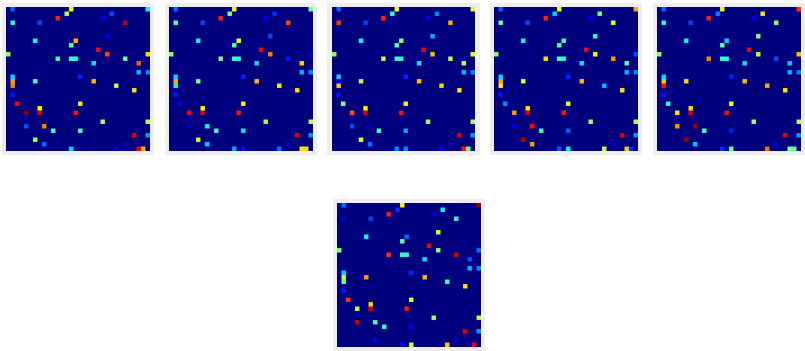


"POINT SOURCES": INDIVIDUAL RECONSTRUCTION

true

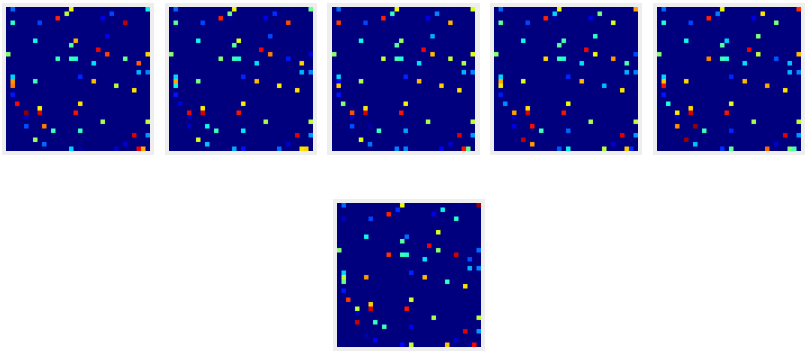


individual ℓ_1

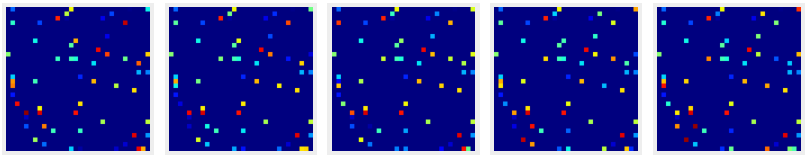


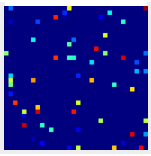
"POINT SOURCES": $\ell_{1,\infty}$ RECONSTRUCTION

true

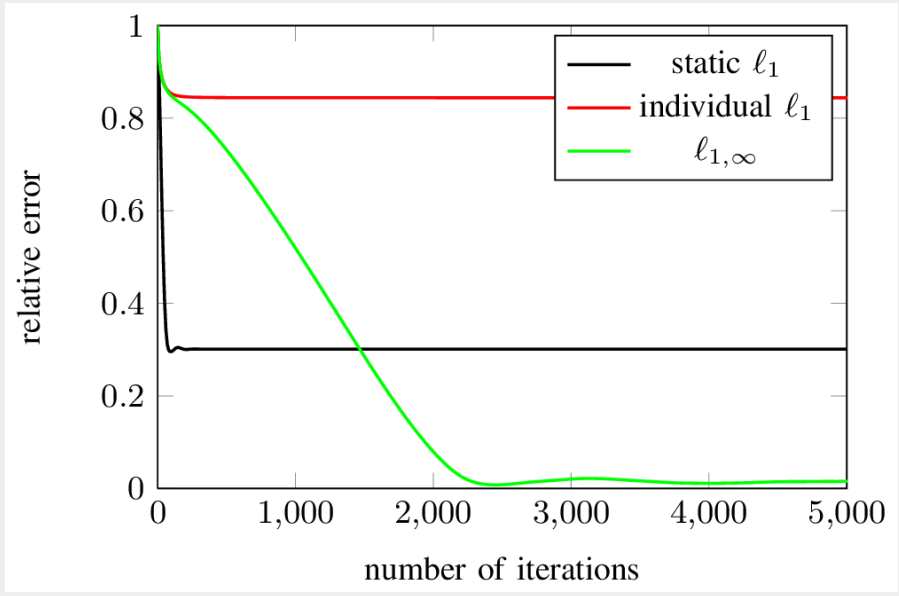


$\ell_{1,\infty}$

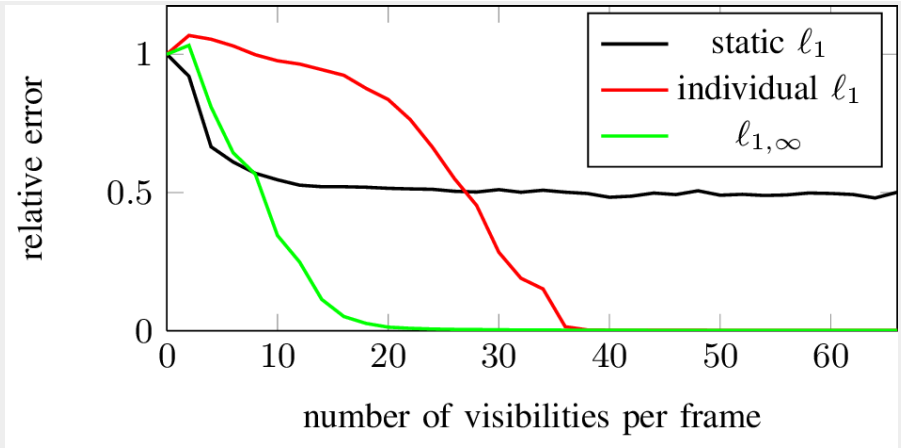




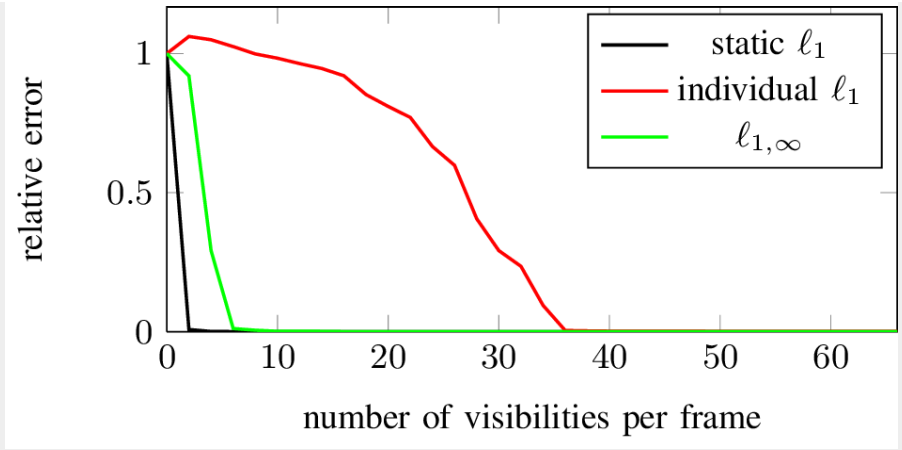
"POINT SOURCES": NUMERICAL EVALUATION



STATISTICAL EVALUATION



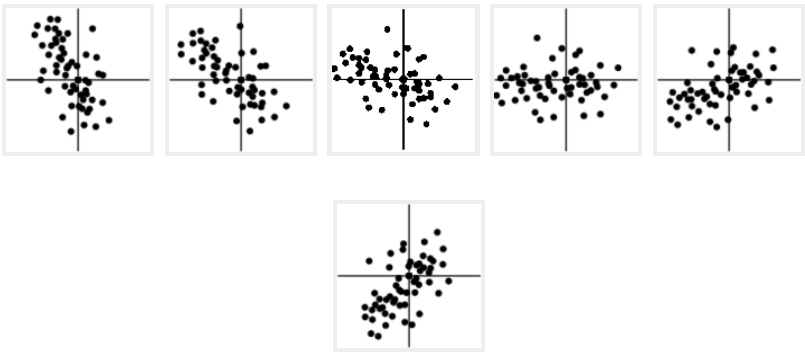
STATISTICAL EVALUATION: STATIC SCENE



CONCLUSION

+ $\ell_{1,\infty}$ regularization outperforms existing methods
  - on *data-starved* observations
  - of *sparsely varying* sources
+ otherwise, degrades gracefully
− slower convergence
− long runtime on large images (ℓ_1: $\mathcal{O}(1)$, $\ell_{1,\infty}$: $\mathcal{O}(n \log n)$)

THANK YOU!



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FOR TRANSIENT RADIO SOURCES

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