Waveform Optimization for Compressive Sensing Radar Systems

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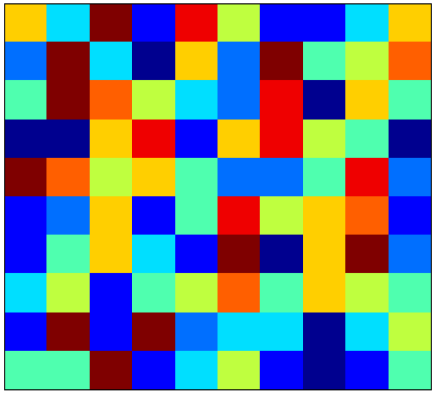
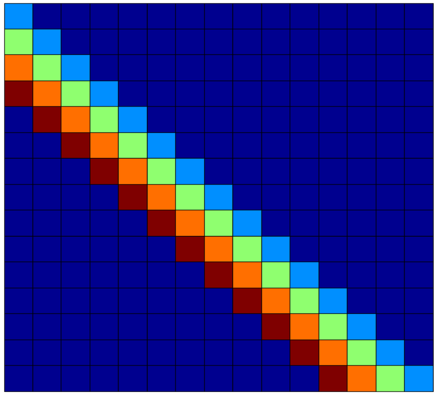
Theory Versus Practice in CS Radar

- Why do we need for optimal waveforms in CS radar? Deterministic vs. random signal acquisition
  - Lower the system complexity, less storage space
  - Clear matrix structure in radar signal model
  - Lower computational demands
  - RF system effect

Waveform design



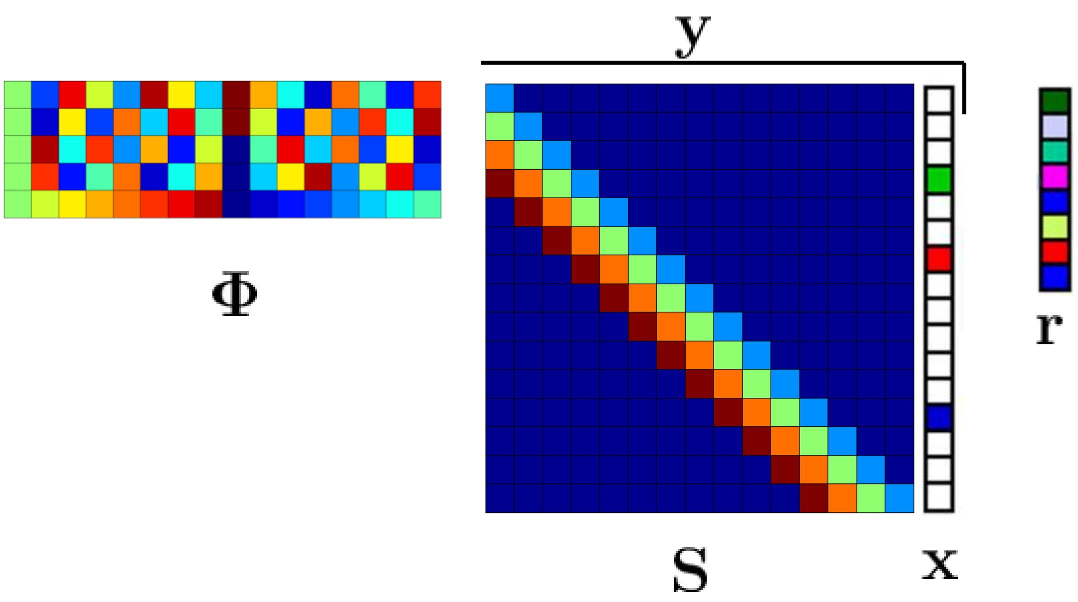


CS background

Data model

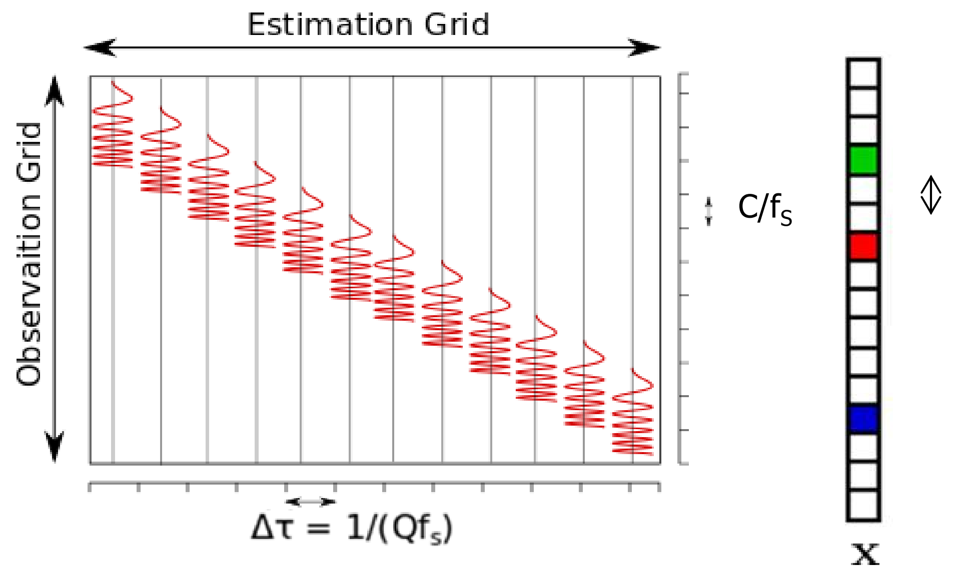
- Signal model $\mathbf{y} = \mathbf{S}\mathbf{x}, \quad \mathbf{y} \in \mathcal{C}^{P \times 1}$

- Measurements $\mathbf{r} = \mathbf{\Phi}\mathbf{S}\mathbf{x} = \mathbf{\Theta}\mathbf{x}, \quad \mathbf{r} \in \mathcal{C}^{(P/C) \times 1}$



Signal Model in CS Radar

$\mathbf{r} =$



Graphical interpretation of the measurement matrix

$$\mathbf{S} = \begin{bmatrix} s[0] & \cdots & 0 \\ \vdots & \ddots & \\ s[L-1] & & s[0] \\ & \ddots & \vdots \\ 0 & \cdots & s[L-1] \end{bmatrix}$$

$$\mu(\mathbf{S}) = \max_{i \neq k} |\langle \mathbf{s}_i, \mathbf{s}_k \rangle|$$

$$\mu(\mathbf{S}) = \max|\mathcal{A}[k]|, k \neq 0$$

$$\mathcal{A}[k] = \left| \sum_{n=0}^{L-k-1} s[n] s^*[n-k] \right|$$

Optimality of the Waveforms

- Side lobe level in the ACF, related to the mutual coherence
- Required double - sided transmission bandwidth B_f
- Ease of generation, transmission and reception
- Constant amplitude due to operation mode of amplifiers

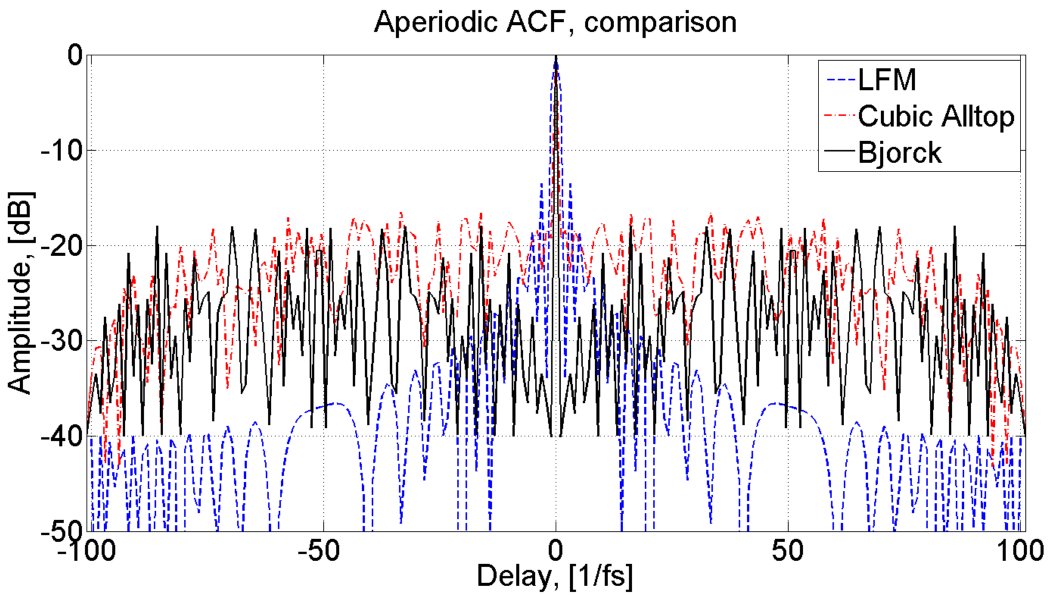
Linear Frequency Modulated (LFM) waveform
Cubic Alltop sequence
Björck sequence (CAZAC)

Optimal Waveforms

Linear Frequency Modulated waveform $s[n] = e^{\frac{j\pi n^2 B_s}{L}} / \sqrt{L}$

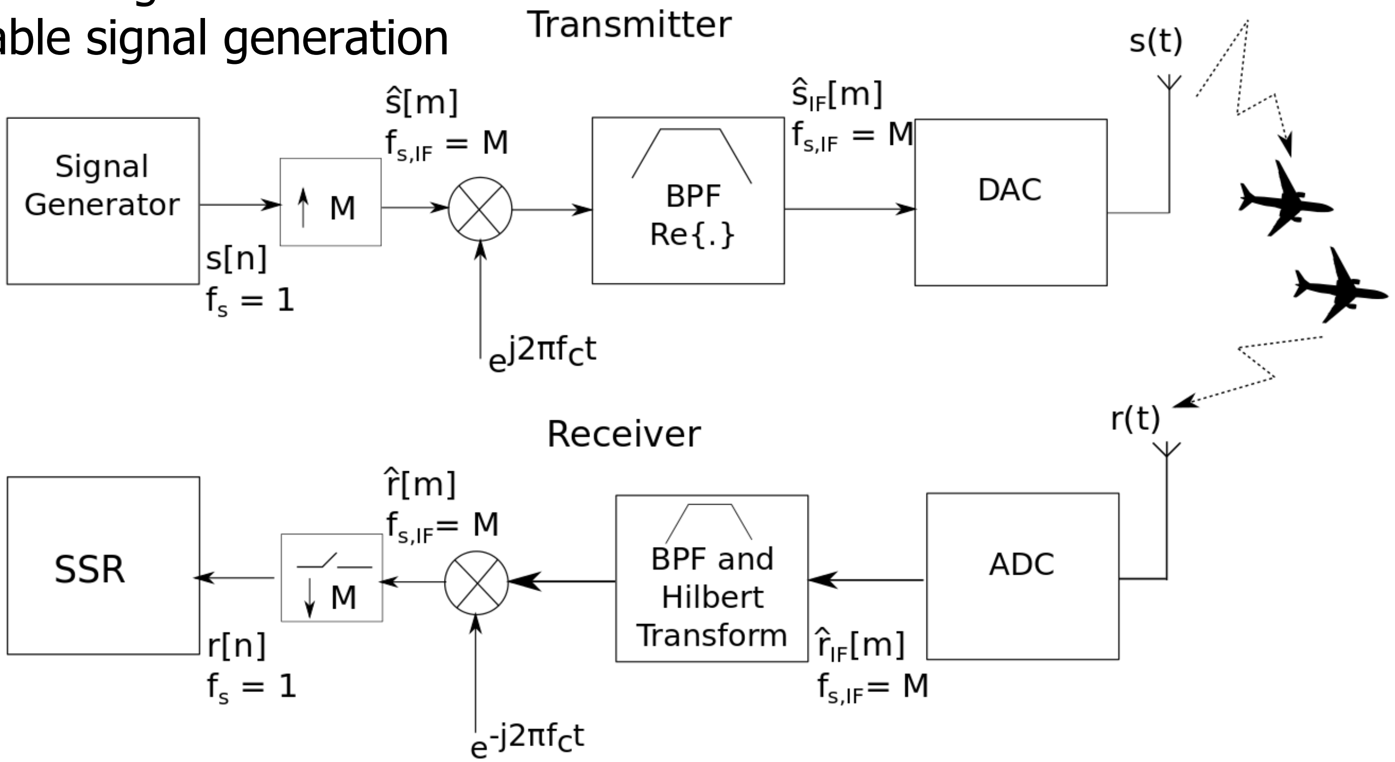
Alltop Waveform $s[n] = e^{\frac{j2\pi n^3}{L}} / \sqrt{L}$

Bjorck Sequence $s[n] = e^{j2\pi\left[\left(\frac{n}{L}\right)\right] \arccos\left(\frac{1}{1+\sqrt{L}}\right)} / \sqrt{L}$



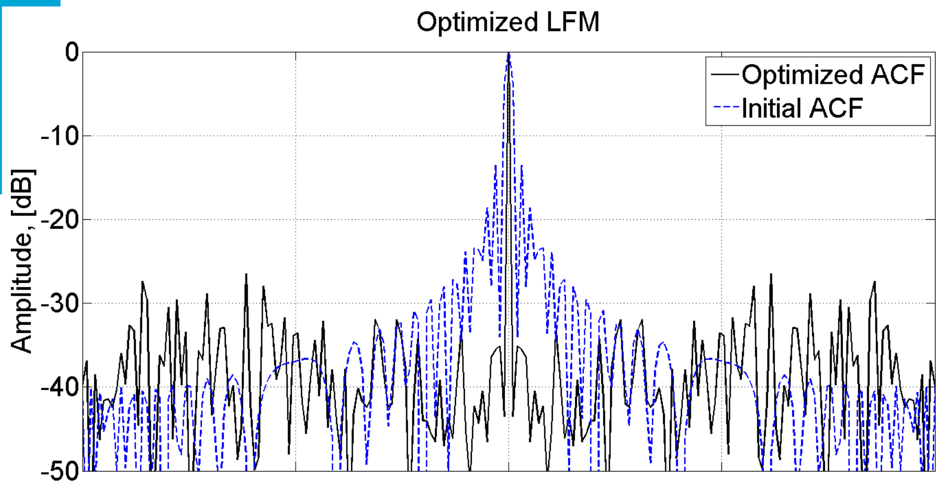
System Model

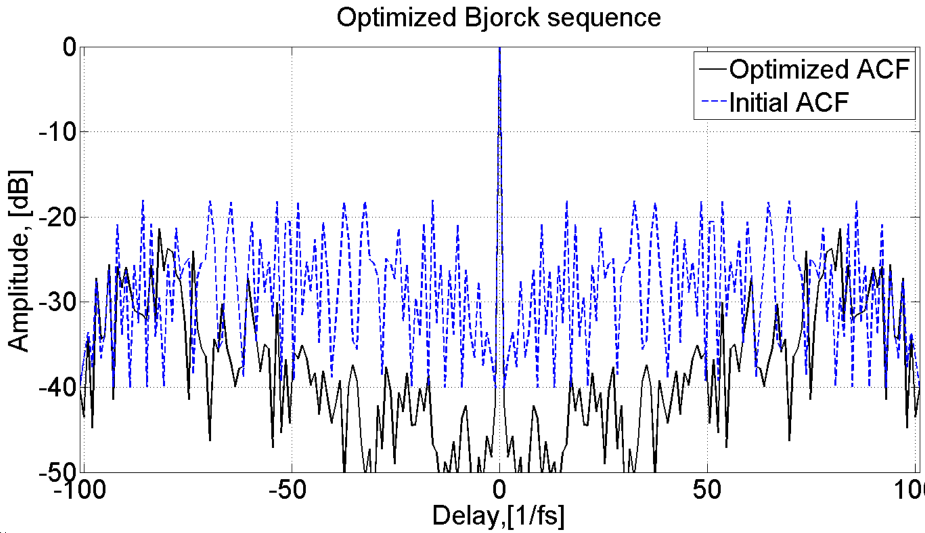
- Advantages of a digital transmit – receive system
  - Flexibility of the architecture
  - Variety of waveforms
  - Precise filtering
  - Very stable signal generation

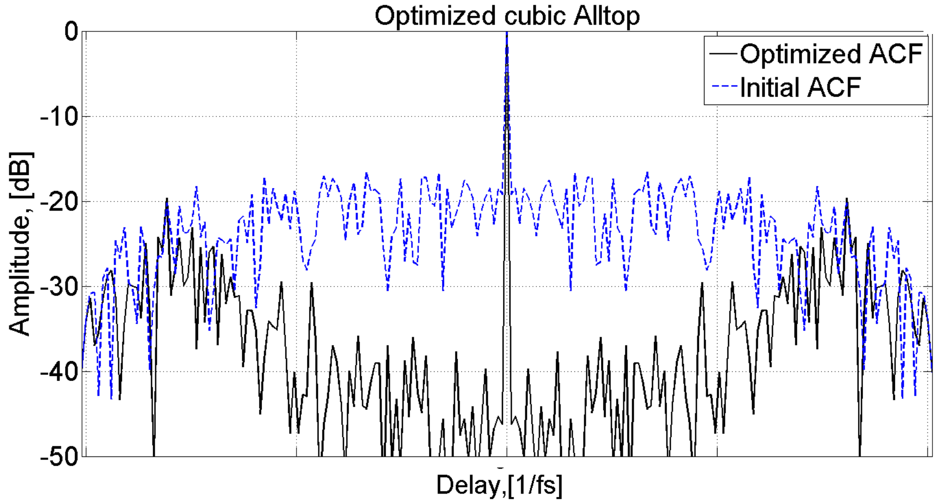


Standard digital transmit – receive system

Further Optimization of the Coherence







$$\min ||\mathbf{S}^H\mathbf{S} - \mathbf{I}||_F^2$$

- Relax the criterion,

minimize the *average* coherence

$$\mathrm{ISL} = \sum_{k=1}^{L-1} |\mathcal{A}[k]|^2$$

- Utilize the CAP algorithm

[Stoica, 2009]

Study of the Matrix Coherence

Upsampled estimation grid

Waveform	Q = 1	Q =2	Q = 4	Q = 8
LFM	-3.9dB (0.64)	-0.91dB (0.9)	-0.22dB (0.97)	-0.05dB (0.99)
Alltop	-16.6dB (0.15)	-3.18dB (0.7)	-0.55dB (0.94)	-0.13dB (0.98)
Björck	-18dB (0.13)	-3.25dB (0.68)	-0.6dB (0.93)	-0.14dB (0.98)

$\mu(\mathbf{S})$ for up-sampled estimation grid

Study of the Matrix Coherence

Compression

Waveform	C = 1	C =2	C = 4	C = 8
LFM	-3.9dB (0.64)	-3.82dB (0.64)	-3.82dB (0.64)	-4.06dB (0.67)
Alltop	-16.6dB (0.15)	-12.53dB (0.23)	-9.61dB (0.33)	-6.83dB (0.45)
Björck	-18dB (0.13)	-14.3dB (0.19)	-12.17dB (0.25)	-8.25dB (0.38)

$\mu(\mathbf{\Phi S})$ for compression with PFM

Simulation setup

- RF System effects $\mathbf{r} = \mathbf{y} + \mathbf{e} = \mathbf{A}\mathbf{x} + \mathbf{e}$, where the columns of $\mathbf{A}$ contain shifted copies of the processed (received) waveform

$$\mathbf{e} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$$

- Bandwidth dependent SNR

$$\mathrm{SNR} = \frac{\mathrm{E}\{|s[n]|^2\}}{\mathrm{E}\{|e[n]|^2\}} = \frac{1/L}{\sigma^2} = \frac{1/L}{N_0 B_f}$$

- Sparse Signal Recovery

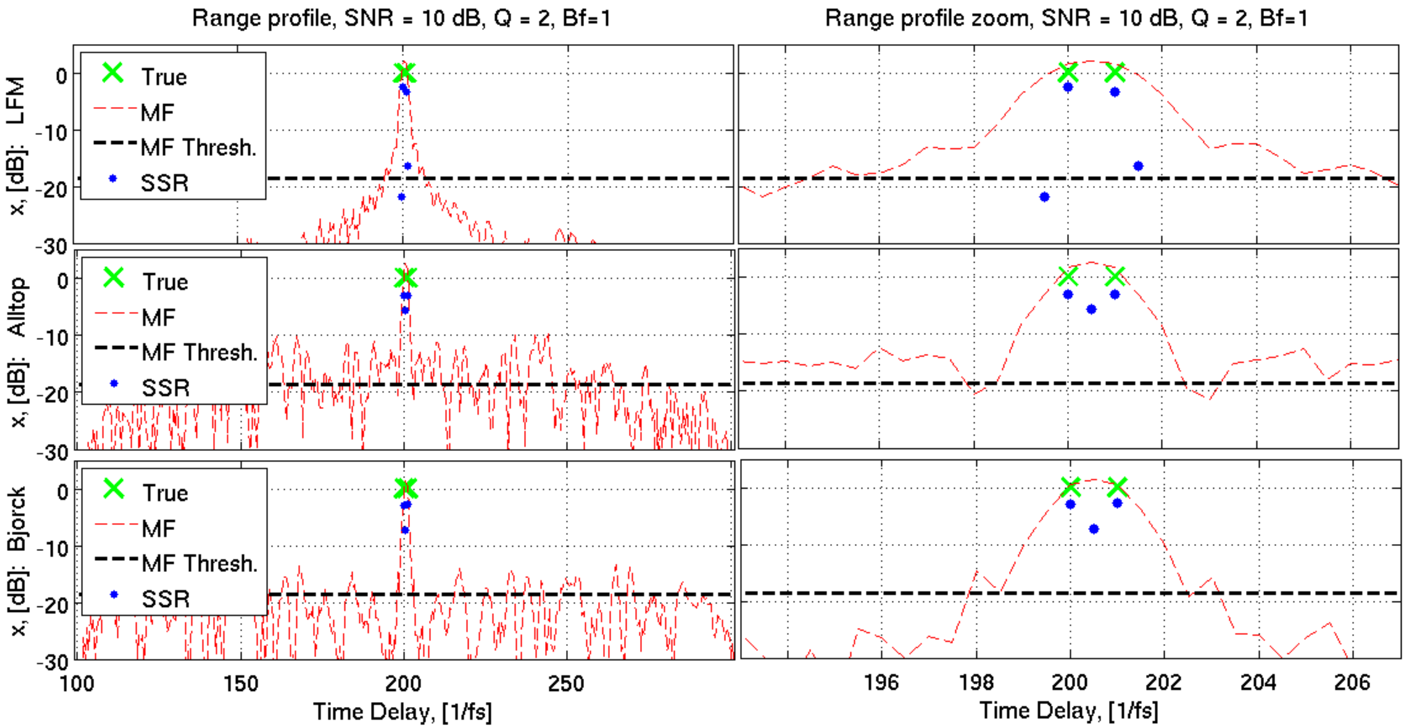
$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}}\{||\mathbf{\Theta}\mathbf{x} - \mathbf{r}||_2^2 + \alpha||\mathbf{x}||_1\}$$

Solve in the Bayesian framework with CFL, with related to P_{FA}

α

Sparse Signal Recovery

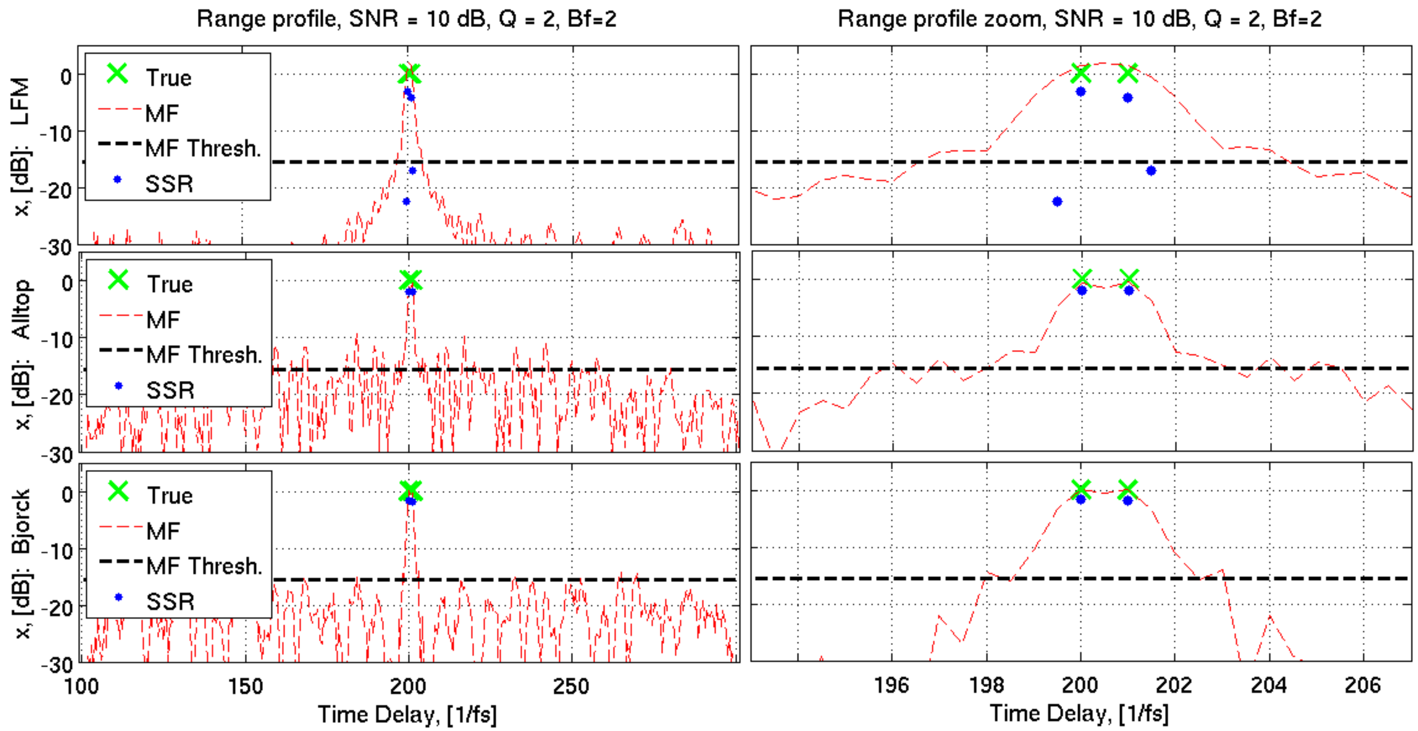
Upsampled estimation grid, Q = 2, Bf = 1



Two targets separated by one reference cell

Sparse Signal Recovery

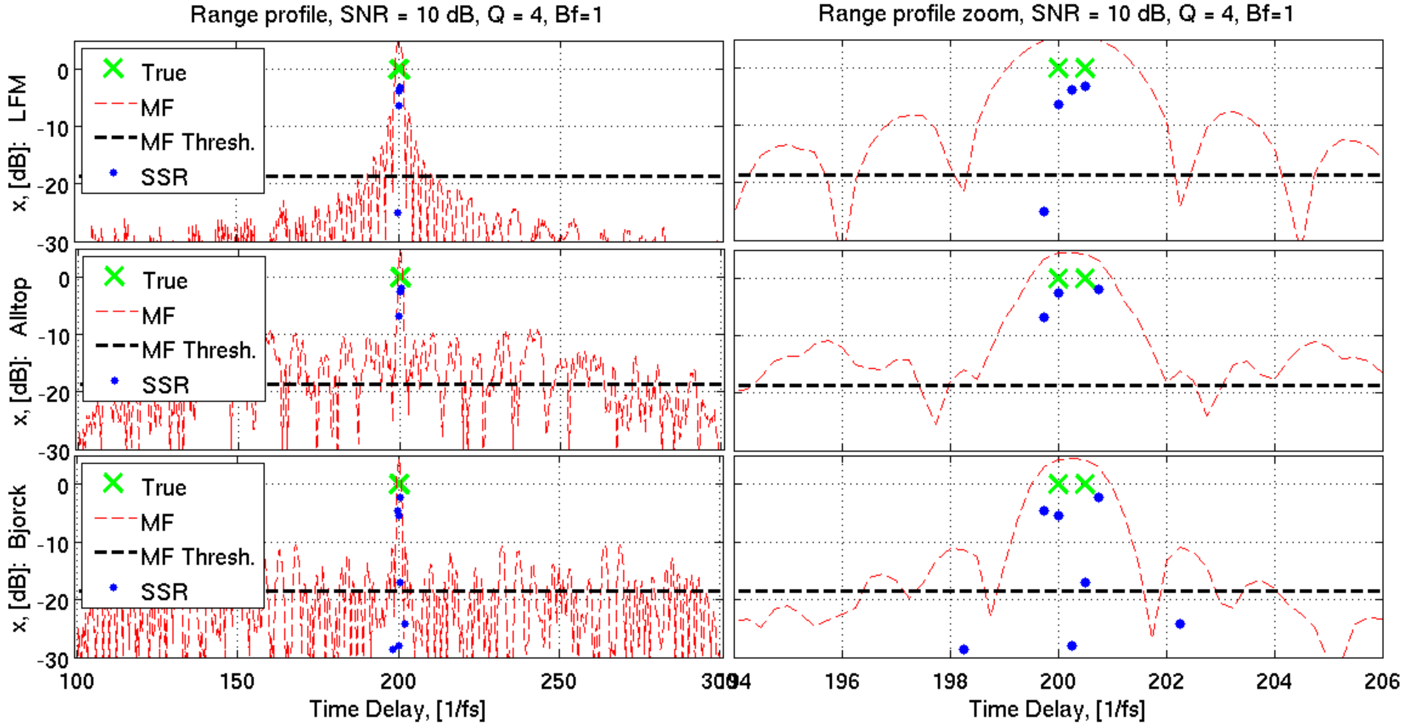
Upsampled estimation grid, Q = 2, Bf = 2



Two targets separated by one reference cell

Sparse Signal Recovery

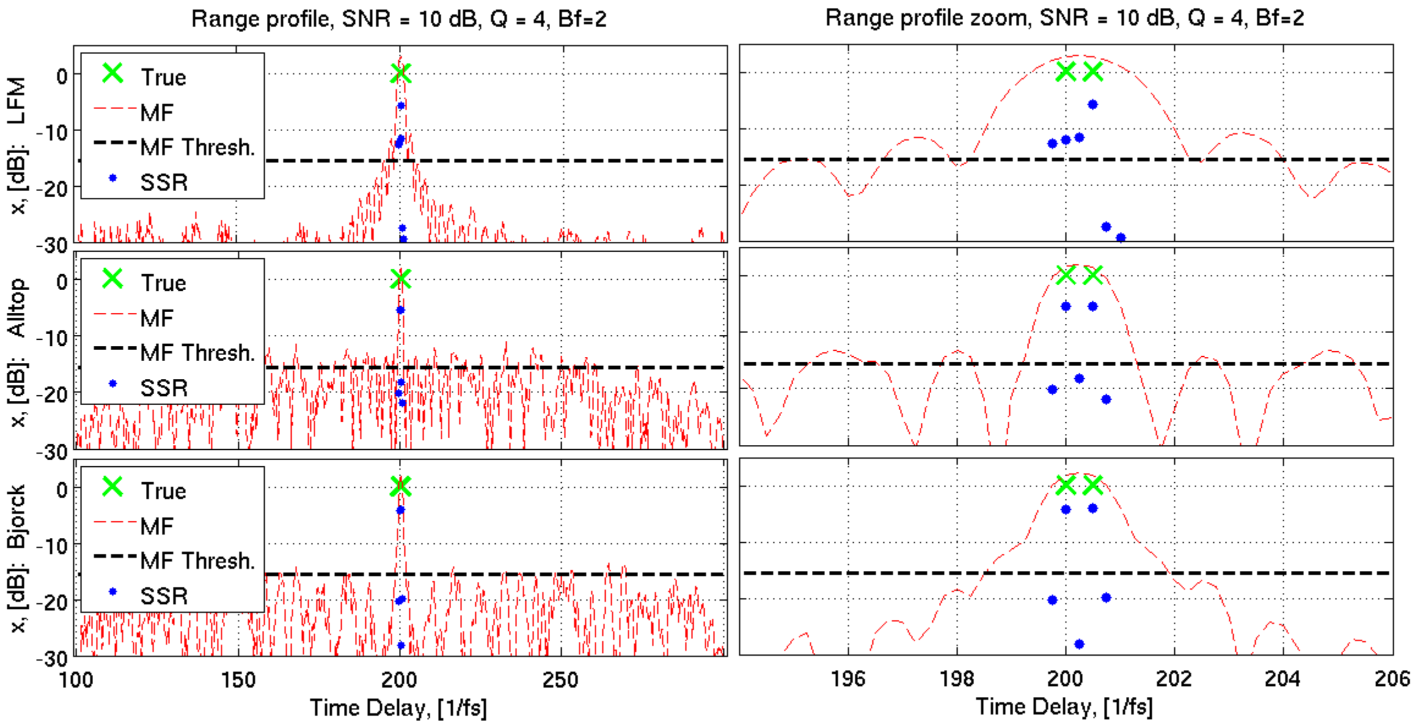
Upsampled estimation grid, Q = 4, Bf = 1



Two targets separated by half a reference cell

Sparse Signal Recovery

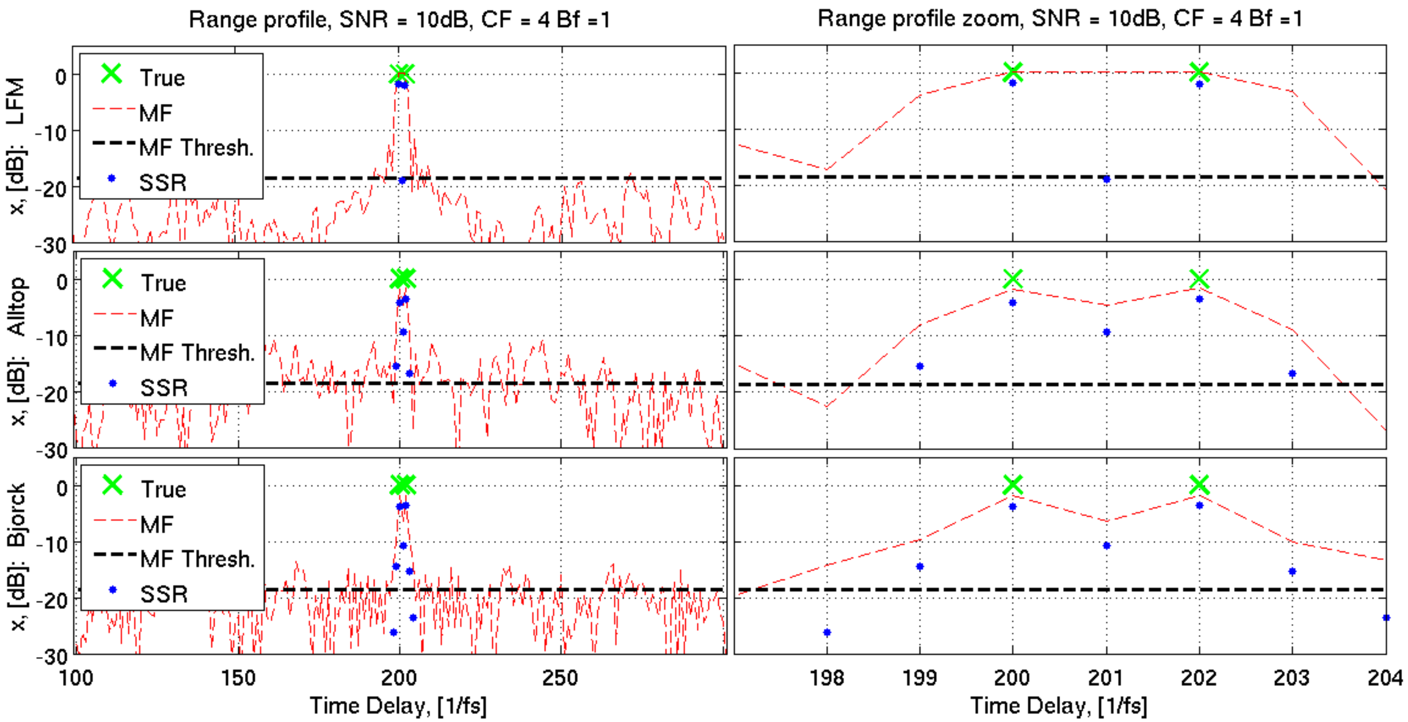
Upsampled estimation grid, Q = 4, Bf = 2



Two targets separated by half a reference cell

Sparse Signal Recovery

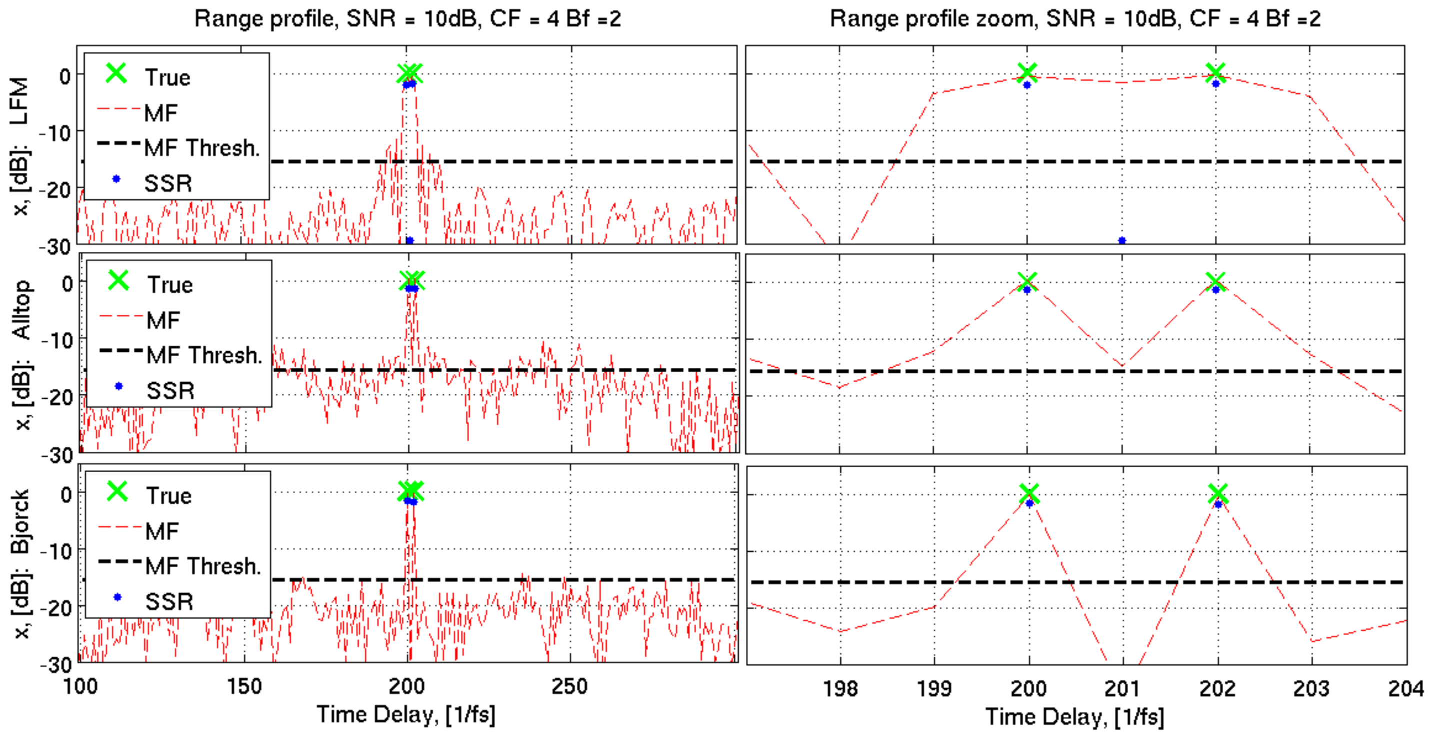
Compression, C = 4, Bf = 1



Two targets separated by two reference cells

Sparse Signal Recovery

Compression, C = 4, Bf = 2



Two targets separated by two reference cells

Conclusions

- ACF is connected to the matrix mutual coherence
- Investigated the effect of the RF system components on the ACF properties of optimal radar waveforms
- The ACF properties of the investigated waveforms can be further optimized
- The Alltop waveform is a “naturally compressed” waveform
- Proper waveform choice and SSR techniques allow closely spaced and weak targets to be resolved

QUESTIONS ?

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Thank you for the attention!