How Sparse Sampling is Useful to Radar?

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***Abstract*— The Compressive Sensing was originally developed in order to directly acquire a compressed description of a sparse scene. This can be possible, without error, provided that two conditions are met: -The scene in question must be sparse and -The sparse sampling scheme must meet certain criteria, especially the RIP criteria and a certain incoherence criteria. In Radar domain, the main advantage highlighted in the literature is the reduction of the data-flow between the R.F. front-end and processing that would otherwise intractable or uselessly costly in number of cases. But so far, most of the work in the literature has focused on the mathematical aspect of the underlying L1 inverse problem. By relying on the state of the art of Radar technology, this paper shows some examples where the use of sparse sampling is really justified. We also show that the inverse problem resolution based on the minimization of the L1 norm, even if it is elegant way to solve the problem, is not the only means to address the problem.**

Keywords-component : Compressive Sensing; Sparse scene; Inverse Problem; Radio-Regulation; Step-frequency; MPRF; Antenna Array; Analog To Information Converter.

I. INTRODUCTION

The Compressive Sensing or also Compressive Sampling (CS) is the subject of numerous studies in recent years. The basic idea of CS is to take advantage that many scenes observed by sensors, in particular by Radar, are often sparse.

The aim of CS is the direct acquisition of a compressed representation of a sparse scene, in contrast to traditional compression methods which are carried out after “normal” sampling at constant rate according to the Nyquist’s criterion. When applicable, CS consists in acquiring only a small number of samples: if there is no noise, and if the sampling meets certain criteria, the reconstruction is exact. The reconstruction process involves solving an ill-posed inverse problem by adding a sparsity constraint. This is typically carried out by a constrained L1-norm minimization.

So far, many studies have been devoted to the mathematical aspect of the inverse problem solution using this L1-norm minimization. Besides its theoretical interest, we can legitimately ask the following question, namely “In what manner CS is really useful to the art of Radar?”

In this paper, some examples where sparse sampling may solve some issues inherent to Radar are given. It will be noted we have intentionally written “sparse sampling” but neither “compressive sampling” nor “compressive sensing”.

Indeed, in most of case, the front-end of the radar system (RF, ADC) and its back-end (Digital Processing) are co-located and the main issue to be solved is not really to manage the data flow volume between both ends, which is no longer a serious problem with current digital processing systems. The main problem we are facing is that the sampling is not always ideal for a variety of reasons. As a result, some data will be unavailable but we need to ensure a correct processing.

However, in special cases (for instance, onboard very small Unmanned Air Systems, Spacecraft or in case of sensors in a tight network), the front-end and the back-end cannot be co-located. It is then necessary to have a data-link between the ends which may be a bottleneck in some applications. In these cases, the CS may be useful as data compression means.

Some “Radar” cases where we have to cope with missing data or sparse sampling are discussed in this paper:

- **Sparse sampling in Frequency– Radio-regulation issues** in broadband applications that may prevent to have a sufficient wide and continuous bandwidth.
- **Sparse sampling in Time– Range ambiguities removal**: Another interesting illustration is the CS approach for the resolution of range ambiguities in the case of Medium PRF (MPRF) Radar.
- **Sparse sampling in Space– Antenna issues**. For many reasons, it is not always feasible having large antenna arrays which are perfectly sampled (*i.e.* elements spacing is about $\lambda/2$) because of:
  - The cost of large number of RF channels in Digital Beam Forming (DBF) system;
  - The space availability.

The paper is organized as follows:

- CS principles are resumed in section II;
- Then three typical cases involving sparse sampling in Radar field are discussed in section III;
- The section IV concludes this paper.

II. Compressive Sensing

A. General Considerations on CS

As stated in introduction, CS takes advantage that many scenes are often sparse when viewed in a suitable observation basis: for instance, the Fourier basis where sinusoidal signals are represented by a set of discrete lines.

A scene containing a small number k of "targets" is said "k-sparse" if there is a representation basis in which the scene can be described as a linear combination of k signals or atoms. Such a scene is called "compressible" because it can be described using only a few multiplicative coefficients.

The original goal of CS is to perform directly a compressed acquisition (so with little redundancy) of a sparse scene. In contrast, the traditional acquisition methods start by sampling the scene at a constant rate (according to the Nyquist's criterion) regardless of any assumption about its contents. Then, a specific compression algorithm precisely takes benefit of redundancy in the data set to reduce the data volume.

Another underlying idea of CS is allowing the direct extraction of useful information contained in the scene from a few number of well-chosen samples. CS can be seen as direct "Analog to Information Conversion" (AIC) [4].

CS consists in gathering only a small number $M > k$ of well-chosen (but non-adaptively) samples or a small number of well-chosen projections of the scene onto a suitable basis. If the scene is perfectly k-sparse, and if the sampling scheme meets certain "incoherence" criteria (*cf.* section B), the reconstruction of the scene is exact [1] [2].

B. Signal Recovery

A scene of dimension N is k-sparse with respect to the representation basis $\mathbf{\Psi} = [\varphi_1,..,\varphi_N]$ if its representation in a measurement basis $\mathbf{x}$ can be written as:

$$\mathbf{x} = \mathbf{\Psi}\,\mathbf{s} \tag{1}$$

Where $\mathbf{s}$ is the k-sparse representation of the scene to be retrieved (*i.e.* $\mathbf{s}$ has up to k non-zero entries). In the measurement basis, the scene is expressed as a linear combination of up to k columns φ of $\mathbf{\Psi}$ (each column φ is an "atom" of dimension N). Each atom must be properly sampled so that any *k-sparse* scene is represented with sufficient accuracy. This notion corresponds to the accuracy of the reconstruction grid. A thin grid will provide a kind of high resolution. However, if it is too thin, the scene loses its sparse nature. This tradeoff is an important point and is currently the object of studies.

As said in introduction, CS consists in acquiring only $M < N$ compressed measurements from $\mathbf{x}$. Algebraically, this operation is expressed as:

$$\mathbf{y} = \Phi\,\mathbf{x} = \Phi\,\mathbf{\Psi}\,\mathbf{s} = \Theta\,\mathbf{s} \tag{2}$$

Where $\mathbf{\Phi}$ is the sampling matrix whose size is (M x N). It describes how getting the compressed sensing vector $\mathbf{y}$ (dimension M) from the uncompressed representation of the scene $\mathbf{x}$ (dimension N). In practical cases, the separation between $\mathbf{\Phi}$ and $\mathbf{\Psi}$ is quite arbitrary. Globally, we can tell that $\mathbf{\Theta}$ is the sensing matrix, which is a (M x N) matrix, and M/N is the compression factor.

The recovery of $\mathbf{x}$ or $\mathbf{s}$ from $\mathbf{y}$ according to (2) is an ill-posed problem. The pseudo-inverse solution, that is to say the solution which minimizes the L2-norm (the energy) within the subspace defined by (2), is not correct in the case of a k-sparse scene. However, solving the following problem constrained by the minimization of the L0-norm (3) provides an exact solution. (By definition, the L0-norm of a vector counts the number of its non-zero entries.)

$$\hat{\mathbf{s}} = \arg\min \|\mathbf{s}\|_0 \quad s.t. \quad \mathbf{y} = \mathbf{\Theta}\mathbf{s} \tag{3}$$

Such a problem involving a minimization of L0-norm is NP-hard, thus the equivalent inverse problem, which is solved, minimizes the L1-norm:

$$\begin{aligned} &\hat{\mathbf{s}} = \arg\min \|\mathbf{s}\|_1 \quad s.t. \quad \mathbf{y} = \mathbf{\Theta}\mathbf{s} && (\text{no noise}) \\ &\hat{\mathbf{s}} = \arg\min \|\mathbf{s}\|_1 \quad s.t. \quad \|\mathbf{y} - \mathbf{\Theta}\mathbf{s}\|_2 < \sigma && (\text{with noise}) \end{aligned} \tag{4}$$

The matrix $\mathbf{\Theta}$ has to fulfill the Restricted Isometry Property (RIP) [1] [2]. A more physical explanation of this condition requires that the columns φ of $\mathbf{\Theta}$ be incoherent between them, that is to say the normalized inner product[1] between two distinct columns must be as small as possible with respect to 1 [3]:

$$\mu(\mathbf{\Theta}) = \max_{j \neq k} \langle \varphi_j, \varphi_k \rangle << 1 \quad s.t. \quad \langle \varphi_j, \varphi_j \rangle_{\forall j \in [1,N]} = 1 \tag{5}$$

For example, a regular sub-sampling on a Fourier basis brings up $\langle \varphi_j, \varphi_k \rangle = 1$ for some distinct columns i and j: this is the well-known frequency folding effect. The incoherence is essential for the reconstruction. Furthermore, its value provides a lower bound on the number of measurements that need to be taken.

This incoherence property is well represented by drawing the "coherence" matrix:

$$\mathrm{M}(\mathbf{\Theta}) = \mathbf{\Theta}^H \mathbf{\Theta} \tag{6}$$

Ideally, the coherence matrix is almost equal to the identity matrix $\mathbf{I}(N)$.

C. Noise effect considerations

The presence of additive noise reduces the "sparse" nature of the observed scene. In addition, in the presence of noise, the algorithms for solving the inverse problem are more or less

[1]In case of Radar signals, the inner product is taken in the Hermitian sense (correlation between vectors).

performing. This topic is currently the subject of numerous publications within the framework of CS.

However, if the final goal of the sensing process is to decide the presence or absence of a target in the presence of noise (which is typically a “Radar” case), an issue directly related to the number of measurement arises. Furthermore, this issue appears regardless and irrespective of the method of reconstruction which is used.

Let us consider the case where we want to detect a target with a probability of detection P_D given a probability of false alarm P_{FA}. In the case of a single measurement, the required Signal to Noise ratio (*SNR*) is given by:

$$SNR = \frac{\log P_{FA}}{\log P_D} - 1 \tag{7}$$

Now suppose we perform N measurements. By taking advantage of the fact that the N measurements on the “useful” signal are redundant while the N noise measurements have no coherence, the lower bound of the required *SNR* is now $SNR_{(N)} = SNR / N$. If we use a compressed acquisition method using only $M < N$ samples, the lower bound of the *SNR* will be only $SNR_{(M)} = SNR / M$. Whatever the reconstruction algorithm, the loss of SNR cannot be less than $N/M > 1$ (*i.e.* the compression factor). A compressed acquisition is inherently less sensitive than a fully redundant acquisition according to the Nyquist’s criterion.

This intrinsic loss of sensitivity on receive (due to the compressed acquisition) must be compensated as much as possible on transmit. All things being equal, the Radar detection theory [5] states that the best detection range R_{MAX} obeys to the following proportionality relationship:

$$R_{MAX}^4 \propto P_T \, T \, A_R = E_T \, A_R \tag{8}$$

Where P_T is the transmitted power, T is the total illumination time (taking in account the duty cycle, *i.e.* the sum of the transmitted pulses durations during the observation), E_T is the “utilized” radiated energy toward the target and A_R is the “electrical” surface of the antenna array system on receive (taking in account the possible sparcity of the array). According to the dimension where applies incomplete sampling (space, time, frequency, etc.), it will be possible or impossible to compensate for the loss of receiver sensitivity (due to under-sampling) by using an “*ad-hoc*” transmission scheme. This is an extremely important point in Radar art, because the transmitted power is generally an expensive resource.

III. Examples of Sparse Sampling Uses in Radar

A. *High Resolution SAR with incomplete Band*

There exist mainly two reasons for using such a scheme:

- Radio Regulations and saving of spectrum resources: In some Radar bands, it is not possible to find enough continuous bandwidth. For instance a SAR image with 30 cm resolution requires at least 500 MHz of bandwidth.
- Waveform issues related to Range-Doppler ambiguities in particular when using a step-frequency waveform [6]. An illustration of such an issue is given Figure 1. Indeed, these waveforms are an elegant and versatile means for generating broadband signals with low instantaneous bandwidth hardware. Reducing the number of steps while preserving the total bandwidth (resolution) leads to deal with an incomplete bandwidth.

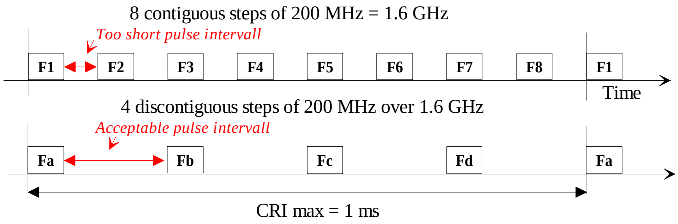

Figure 1. Range-Doppler issue with step-frequency waveform

1) *SAR mage properties*

A useful SAR image is not generally a sparse scene. Indeed, such an image consists of two parts: - the background and, - strong but localized echoes such as point-like targets or ridge targets.

The background spreads on very large parts of the scene. The background has absolutely no “sparse” features but its good reproduction is essential to the operational use of images (roads, runways, water areas, shadows produced by elevated targets, etc.). In contrast, the second category of echoes (strong point-like echoes) has the characteristics of “sparse” targets and these strong echoes are also essential to the operational use of images.

While the background (not sparse) has a moderate dynamics, the dynamics of point-like echoes (sparse) may be very high (over 40 dB above the background and even much more in the case of high-resolution images). The standard matched filter processing is no longer sufficient: we must correct the effect of the missing band so that the background is not corrupted by the side-lobes responses such as in Figure 2.

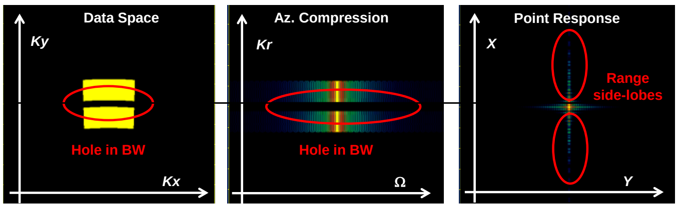

Figure 2. Range sidelobes induced by incomplete band.

Thus, the practical need is absolutely not to reduce the flow of data using a compressed acquisition, but it is to interpolate the missing data between sub-bands that induce side lobes with an incomplete bandwidth. Note that each sub-band is properly sampled (at Nyquist’s rate).

Among all applicable methods, we can mention two specific ones:

- Auto Regressive (AR) interpolation;
- L1-modified regularization [7] [8].

Whatever the interpolation method that is used, it is always implemented after the azimuth compression so that the number of strong echoes per azimuth bin is as small as possible (the scene corresponding to the strong echoes must be the as “sparsest” as possible).

2) AR interpolation

In frequency domain, the point-like echoes appear as complex sine signals while the background appears more or less as a Gaussian white noise. Such a scene is convenient for being represented by AR model of low order. The AR modeling allows extrapolating the missing signals around each sub-band (Figure 3.).

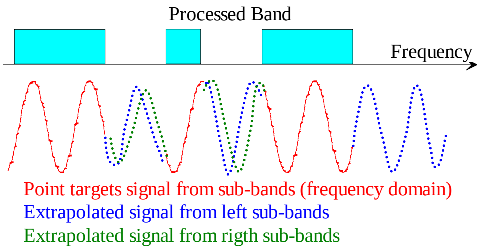

Figure 3. Principle of A.R. Extrapolation

Each missing part is retrieved by a weighted average of:

- The extrapolation toward high frequencies from the lower sub-band;
- The extrapolation toward low frequencies from the upper sub-band.

Such a reconstruction on real data is presented in Figure 4.

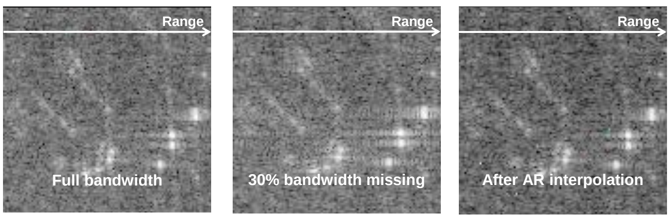

Figure 4. AR reconstruction

As the number of “hole” is usually small, this interpolation is easy to mechanize. The advantage of the AR interpolation method is its low computational need. On the other hand, the main issue is to automatically find out the required order of the model.

3) Regularization method

Regularization allows solving an ill-posed problem by adding *a priori* information on the “right” solution to be found [8].

Let $\mathbf{S}$ be the reconstructed range profile (N-vector), $\mathbf{Y}$ be the input signal (M-vector), that is to say the received spectrum vector from the usable sub-bands only. $\mathbf{S}$ must fit with the input data, that is to say must fulfill the following condition:

$$\left\|\mathbf{Y} - \mathbf{\Phi}\ \mathrm{FFT}(\mathbf{S})\right\|_2^2 < \sigma_n^2 \tag{9}$$

where σ_n^2 denote the measurement noise and $\mathbf{\Phi}$ is the concatenation (M, N) matrix. $\mathbf{\Phi}$ is constructed from the identity matrix where the lines corresponding to missing data have been removed.

The added *a priori* knowledge states both the background is not sparse but the strongest echoes are sparse (*cf.* section 1)). This assumption is expressed by the fact that the right solution should minimize the L2-norm on small signals (not sparse) and should minimize the L1-norm on strong echoes (sparse) [9]. A convenient way to express this condition consists in minimizing the following “L1-modified norm”:

$$\left\|\mathbf{S}\right\|_{1\#} = \sum \sqrt{\left|s_i\right|^2 + \varepsilon^2} \tag{10}$$

A representation of this norm in a 2D space for $\varepsilon = 1$ is given in Figure 5. The transition from spherical constant norm surfaces to polyhedral constant norms surfaces with their spikes along basis axes is clearly shown.

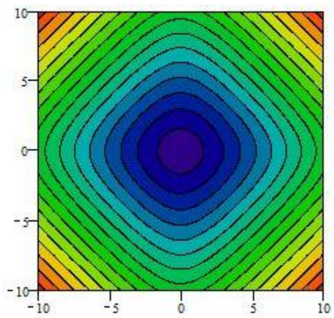

Figure 5. L1-modified norm in 2D

The constant ε determines the transition between an L2 behavior and an L1 behavior of the norm. Its suitable value depends on the background level.

The regularization allows retrieving the right solution which satisfies (9) subject to minimize (10). The problem is usually solved using a convex relaxation:

$$\hat{\mathbf{S}} = \arg\min \left\|\mathbf{Y} - \mathbf{\Phi}\ \mathrm{FFT}(\mathbf{S})\right\|_2^2 + \lambda \left\|\mathbf{S}\right\|_{1\#} \tag{11}$$

By noticing that $\mathbf{X} = \mathrm{FFT}(\mathbf{S})$ may be written algebraically $\mathbf{X} = \mathbf{\Psi S}$ (where $\mathbf{\Psi}$ is the Fourier basis), equation (11) becomes:

$$\begin{aligned} \hat{\mathbf{S}} &= \arg\min \left\|\mathbf{Y} - \mathbf{\Phi}\ \mathbf{\Psi}\ \mathbf{S}\right\|_2^2 + \lambda \left\|\mathbf{S}\right\|_{1\#} \\ \hat{\mathbf{S}} &= \arg\min \left\|\mathbf{Y} - \mathbf{\Theta}\ \mathbf{S}\right\|_2^2 + \lambda \left\|\mathbf{S}\right\|_{1\#} \end{aligned} \tag{12}$$

This writing is mathematically one possible formulation for CS reconstruction (LASSO).

Some results obtained on real data are presented below. The comparison between the matched filter solution (missing data are assumed null in matched filter solution) and the regularized reconstruction by solving (11) is presented in Figure 7.

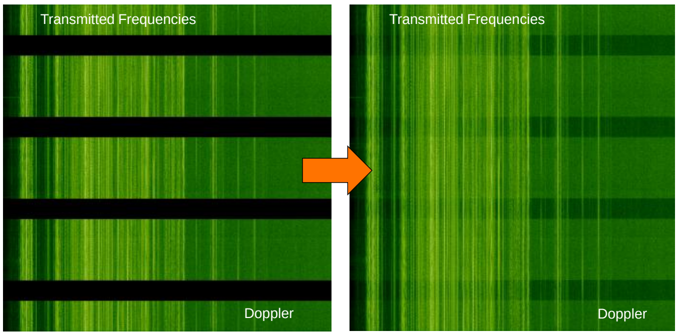

Figure 6. Interpolation of missing data

Furthermore, the interpolation effect provided by the regularization is illustrated in Figure 6. The left image corresponds to the raw data (frequency / azimuth) while the right image displays the Fourier transform along distance of the reconstructed scene $\mathbf{S}$. The strongest echoes are right extrapolated while a partial but sufficient interpolation is provided on small echoes from background.

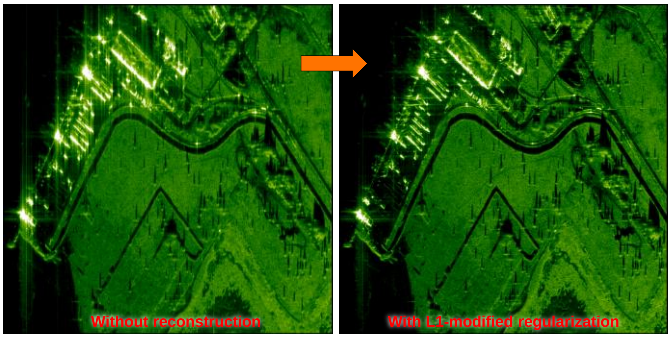

Figure 7. Left: MF without interpolation. Right: regularized reconstruction.

As discussed in section II.C, the sparse sampling leads to a loss of processing gain on receive, thus requires a higher *SNR* at sampling level. However, this loss can be fully compensated by transmitting signal only in the processed sub-bands on receive. In the case of a step waveform frequency, this means increasing the pulse duration of each step (the compression ratio). There is no wasting of power and the average radiated power (thus the cost) remains exactly the same as for a full bandwidth.

The coherence matrix (6), corresponding to the sensing scheme of Figure 6. , is displayed Figure 8. The coherence according to (5) is about 0.39.

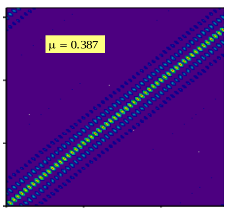

Figure 8. Coherence matrix

In this example, mathematical tools, similar to those used in the CS framework, were used. However, the acquisition process is not really incoherent and we do not make really CS, that is to say a compressed acquisition.

B. *Range ambiguities removal for MPRF Radar*

1) *MFRF waveforms*

Such a waveform does not provide direct measurements, neither in range nor in Doppler. It is usually utilized with a staggered PRF scheme. Several (*N*) PRI are sent according to: $PRI_i = n_i\,\tau$ where n_i is a set of *Q* co-prime integers.

2) *Classical range unfolding methods:*

Let *R* the unambiguous quantized delay (range) of the target. For each set of $PRI_i = n_i\,\tau \quad Q \in [1, Q]$, we can only measure the remainders of the division of *R* by n_i . That is to say, the set of measurements is $r_i = R \bmod n_i$.

By adopting the framework of representation of CS:

- The output $\mathbf{S}$ is a "long" and sparse *N*-vector corresponding to the unfolded range domain. It contains either binary digits or values corresponding to the non-ambiguous locations of the targets.
- The representation basis $\mathbf{\Psi}$ is the identity matrix $\mathbf{I}(N)$: the sampling basis and the representation basis are the same. The matrix $\mathbf{\Psi}$ enumerates all the possible locations of a target from 1 to *N*.
- The sampling matrix $\mathbf{\Phi}$ is obtained as indicated in Figure 9.

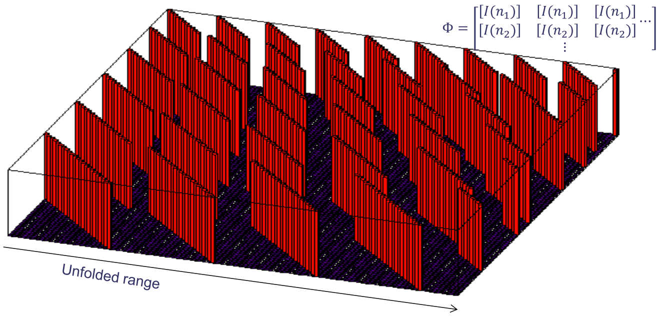

Figure 9. Sampling matrix for MPRF radar

  - By concatenating horizontally, for each PRI_i , $\mathbf{I}(n_i)$ until reaching a columns number greater or equal to *N*. The resulting matrix is then truncated to the size (N, n_i).
  - By stacking vertically the matrix obtained at last step for all *PRI*. We get a (*M*, *N*) matrix with $M = \sum_{i=1,Q} n_i$.

- The measurement *M*-vector $\mathbf{y}$ is obtained by stacking *Q* vectors containing either binary digits or values corresponding to the <u>ambiguous</u> locations of the targets:

(13) $$\mathbf{y} = \mathbf{\Phi}\,\mathbf{\Psi}\mathbf{S} = \mathbf{\Phi}\,\mathbf{S}$$

The basic unfolding algorithm consists in calculating (14) then to apply a threshold.

(14) $$\hat{\mathbf{S}} = \mathbf{\Psi}^{-1}\mathbf{\Phi}^{T}\mathbf{y} = \mathbf{\Phi}^{T}\mathbf{y}$$

This basic algorithm works correctly as long as there is only one target. In the case where there are at least two targets or noise, ghost targets appear. This is because ambiguous responses from distinct targets, or due to noise, create spurious or wrong summations in (14). This effect is quite similar to the occurrence of side lobes responses with an incomplete sampling (Figure 10.).

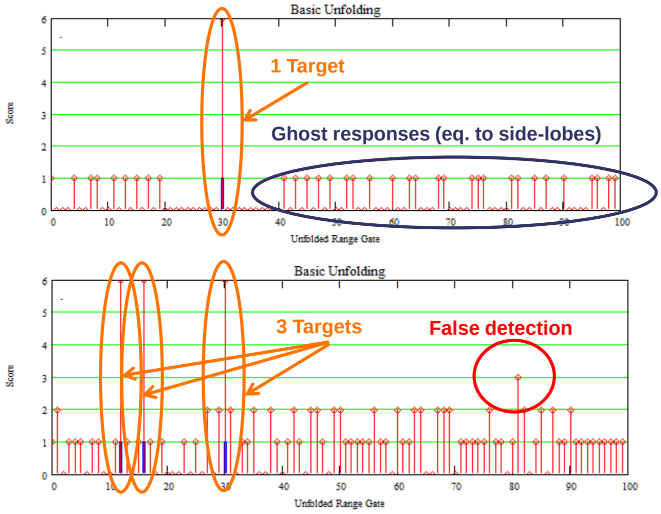

Figure 10. Ghost targets due to wrong summations. $Q = 6$ differents PRI based on co-prime numbers. Detection threshold is 3. Top - one target: sidelobes occurs but they are too low to create wrong detection. Bottom – 3 targets: false detection appears.

3) *Ambiguities unfolding based on a CLEAN approach*

The issue of ghost targets in the case of basic unfolding comes mainly from the fact that one target before unfolding can contribute in the occurrence of more than one target after unfolding.

Nowadays, algorithms more efficient than the basic unfolding algorithm are employed. One of them is based on a similar approach to the CLEAN method. Its principle is to iterate the following steps:

a) Set the index of validated output $i = 1$.

b) Calculate $\mathbf{S} = \mathbf{\Phi}^{T}\mathbf{y}$.

c) Rank the non-null outputs within $\mathbf{S}$ according to a relevant ranking criterion *(e.g.* the value of each element of $\mathbf{S}$).

d) Retain as valid output $s(i)$ the element of $\mathbf{S}$ having the highest rank, then set to zero all elements in the measurement vector $\mathbf{y}$ that have contributed to the value $s(i)$.

e) Increment $i = i+1$

f) If all remaining elements of $\mathbf{S}$ have nil value then exit.

g) Otherwise do loop to b).

As the scene is sparse, *i.e.* the number of targets to be detected is low; this algorithm requires only a small number of iterations. It is therefore very efficient in terms of computing resources provided there is sufficient memory to store the matrix $\mathbf{S}$ and $\mathbf{\Phi}$.

It is worth to notice that this method can be applied either in the classical "unfold after detect" approach ($\mathbf{y}$ elements are binary detections) or in "detect after unfold" approach ($\mathbf{y}$ elements values depends on the received power; the threshold is then applied at $\mathbf{S}$ level).

4) *CS solution to range unfolding*

The coherence matrix is shown in Figure 11. If the set of *PRI* is obtained from a set of co-prime number, the coherence matrix exhibits good properties for reconstruction using the L1 minimization (diagonal structure and coherence = 1/N).

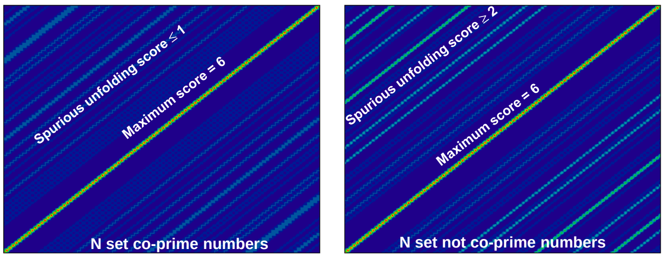

Figure 11. Coherence matrix

The use of CS solves the previous issue. Indeed, it search for the sparsest solution, that is to say the one that fit (13) without extra spurious summations. CS approach allows finding the best solution, but it requires much more computing resources that the method described at section 3). Moreover, some "special" extra information related to the operational Radar behavior is difficult to express with matrix algebra.

Nevertheless, the use of L1 minimization for the range unfolding is really a good example of application of CS to radar. CS is here really used in its founding spirit, and not as a method of regularization of ill-posed problem as it is most often the case in Radar issues.

C. *Beam-forming with sparse array*

An interesting case study is constituted by the very large radio telescopes such as the "SKA" (Square Kilometer Array) radio telescope [10], Figure 12.

Figure 12. The SKA radio telescope – credit: SKA Organisation/Swinburne Astronomy Productions.

Such a telescope is intended for imaging of galactic radio sources. The scene which is imaged is made of a few point-like sources surrounded by nothing. That is exactly a sparse scene.

The first characteristic that is sought after is the angular resolution, which is why these systems extend over several kilometers in domain of radio waves. Then, the best sensitivity is looked for: the goal is to maximize the total surface of the receiving part of the array.

It is obvious that with such overall dimensions, it is absolutely impossible using a non-lacunar array, therefore the maximum sensitivity of the system has to be sacrificed to enable its feasibility. This is why sparse sensing techniques are essential, hence the irregular appearance of the network.

We do not know precisely what is the reconstruction method used on the SKA telescope. However, regardless of the reconstruction method, the irregularity of the array is necessary.

Following the CS framework:

- The scene to retrieve $\mathbf{S}$ is expressed in basis of cosine coordinates (Fourier basis).
- The measurement vector is $\mathbf{y} = \mathbf{\Phi}\,\mathbf{\Psi}\,\mathbf{S}$ where:
  - $\mathbf{\Psi}$ is a kind of Fourier matrix
  - $\mathbf{\Phi}$ describes how the sparse sampling is done. It is the identity matrix where lines corresponding to missing receivers have been deleted.

1) Basic Beam-forming solution (M.L. sense)

The basic beam-forming consists in calculating:

$$\widehat{S_{BF}} = \underbrace{\mathbf{\Psi}^{-1}}_{\substack{\text{Matrix (NxN) of} \\ \text{steering vectors}}} \underbrace{\mathbf{\Phi}^{H}\boldsymbol{y}}_{\substack{\text{Fill with zero entries} \\ \text{to get size N}}} \tag{15}$$

This operation does not provide a single solution, if sources are well disposed that is to say if the sampling scheme meets the incoherence criterion, we get a peak response and side-lobe responses.

In fact this problem is well known for a long time in radio astronomy [11]. The raw images are corrected using de-convolution methods based on the point spread function (the “dirty beam”) of the system.

2) Compressive Sensing solution:

By using the CS concept, the reconstruction is given by:

$$\widehat{S_{CS}} = \mathbf{argmin}(\,\|\boldsymbol{S}\|_1)\ s.t.\,\|\boldsymbol{y} - \mathbf{\Phi}.\mathbf{\Psi}.\boldsymbol{S}\|_2 < \sigma \tag{16}$$

This method provides a single (no side-lobes) and exact solution in noiseless conditions, if sources are well disposed. In fact, that is equivalent to the basic beam-forming with a sparsity constraint.

IV. Conclusion

The problems of the reconstruction of missing data and subsequent reduction of side-lobes are not a novelty. They are used in radio astronomy since 70’s and the regularization methods have been effectively used since 90’s, especially in medical imagery. The use of random sampling for reconstructing sparse scenes is also not new. The unfolding of MPRF ambiguities is a good example.

In practice, the concept of CS is not a novelty of the 2010's. However around it, many researches are being conducted that have theorized the concept and are producing efficient L1-nom minimization algorithms. It would be a pity not to take advantage of this to efficiently solve ill-posed problems by the means of conventional regularization.

Except the cases where the front-end and processing are distant or distributed, the main interest of CS in radar is not to reduce the incoming data flow (which is no longer really an issue with current digital technologies) but to reconstruct observed scenes when sampling cannot be ideal for practical questions.

The current “fashion” about CS finally deserves to ask again the question of what is really needed, not for reconstructing a signal, but for directly extracting the useful information it contains.

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