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How Sparse Sampling is Useful to Radar.

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Summary

1. ***Compressive Sensing, Sparse Sampling, their utilities in radar applications.***

2. ***Radar "cases"***

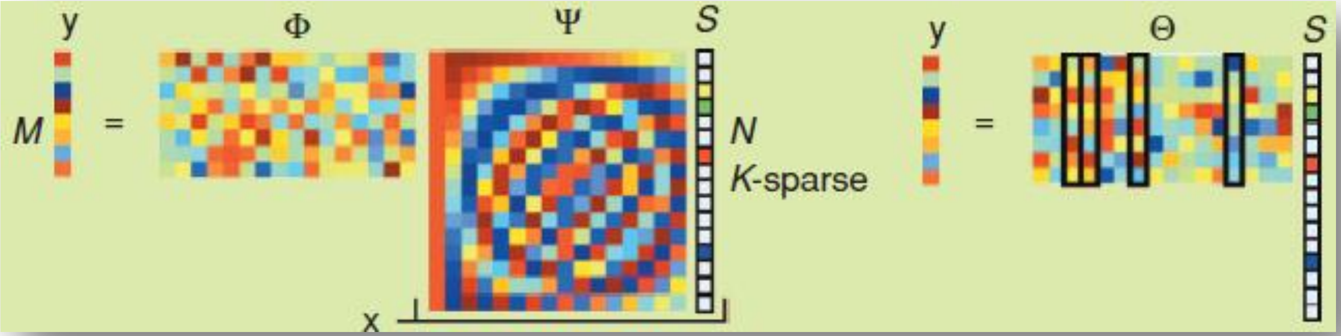
   - **Broadband Applications with incomplete BW;**
   - **Example for fun - MPRF unfolding;**
   - **Sparse Arrays Issues.**

3. ***Conclusion***

What is C.S.?

- Originally developed to directly acquire a compressed description of a sparse scene.
- The sparse scheme ≠ Nyquist's criterion
- Sampling constrained to other criteria such as the RIP condition.
- Leads to a reconstruction process involving the solve of an ill-posed problem.

☞ Lot of mathematical studies on minimization algorithms (minimization of L1-norm especially).



How Sparse Sampling may be useful in Radar.

Out of its theoretical interest, what does C.S. bring really to the Radar?

The usual case in radar = not to deal with compression/acquisition but with the sampling issues.

Three Radar cases dealing with missing data or sparse sampling :

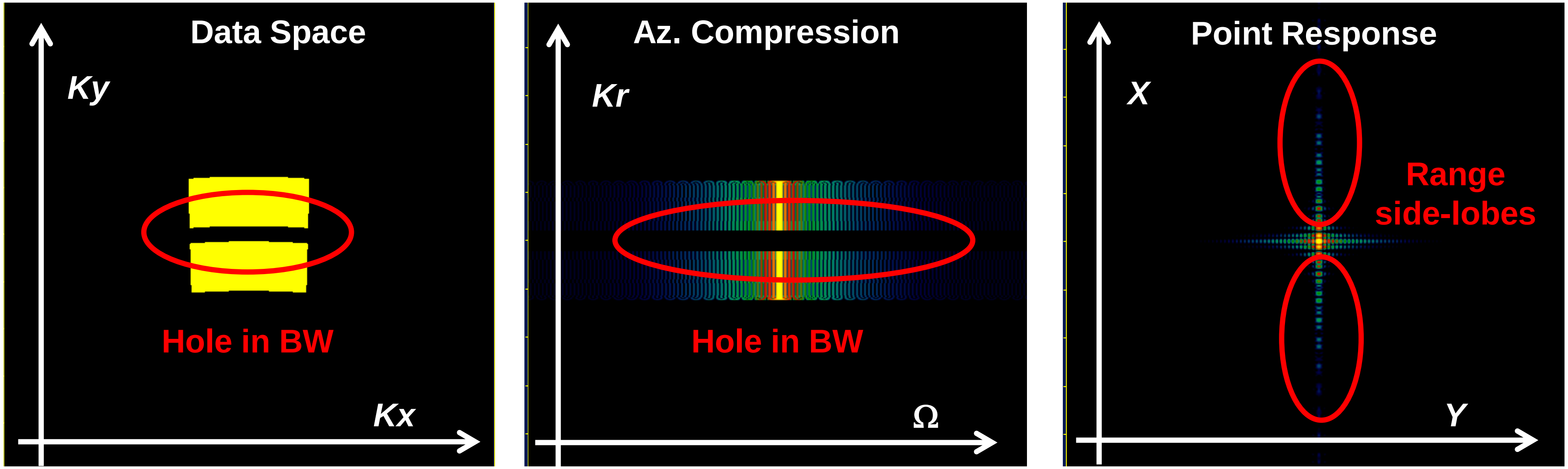
- **Sparse Sampling in Frequency in broadband applications**
- **Sparse sampling in Time-Range ambiguities**
  - Waveforms issues: Example MPRF unfolding
- **Sparse sampling in Space - Antenna arrays issues:**
  - Cost of large number of RF channels in DBF systems.
  - Space availability on platforms

Sparse Sampling in Frequency (1/2)

1st reason for using incomplete frequency BW:

- **Radio Regulations and saving of spectrum resources:**
- **Issue: In some bands (especially low bands for FOPEN applications), it is not possible to find enough continuous bandwitdh.**
- **Ex.: SAR image with 30cm Doppler resolution ➔ BW > 500 MHz**

➔ Solution: "Incomplete BW"



The problem here is not to reduce the input data flow.
The problem is to cope with unavoidably missing data.

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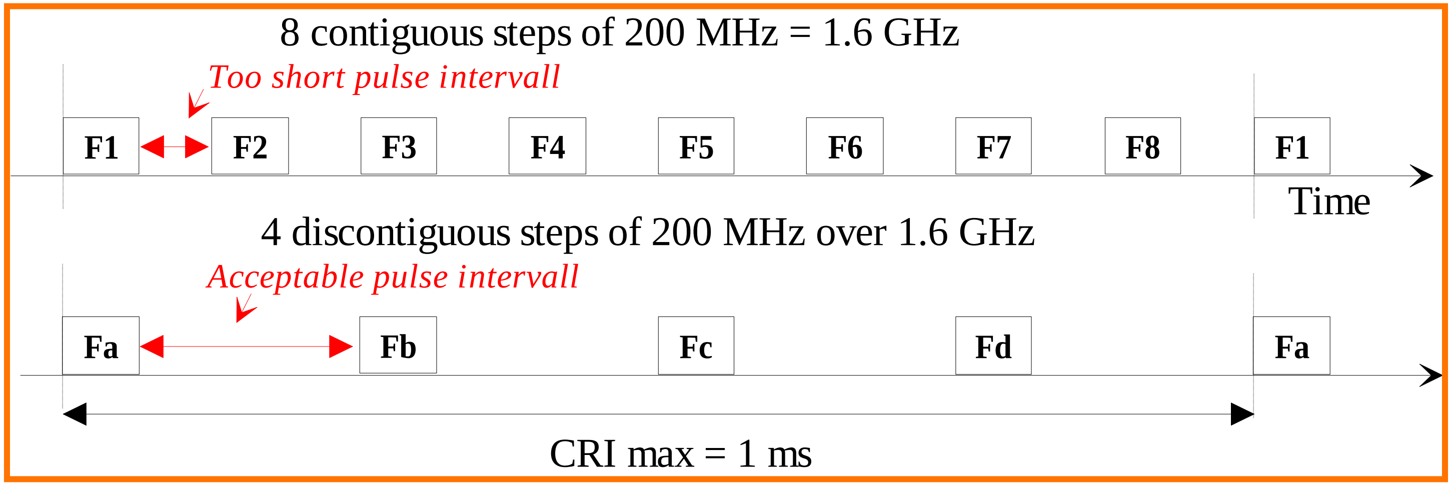
Sparse Sampling in Frequency (2/2)

2nd reason for using incomplete BW:

- **Range ambiguities when using stepped frequencies WF**
  - Limited instantaneous BW ➔ More affordable technologies, more flexible and scalable



- **Issue:**
  - Number of steps is $N_{step} = \frac{Total\ BW}{Instant\ BW}$
  - Doppler processing requires that Cycle Repetition Interval (CRI) < Max. value.
  - N_{step} too large ➔ range ambiguities occur.

Solution: using fewer number of steps➔ Sparse spectrum
Radio Regulation requirements can be easily incorporated

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Sparse Sampling in Frequency : SAR imaging application (1/6)

What is really needed?

- **From a practical point of view, the need <u>is not</u> to reconstruct a sparse scene with a small number of random projections.**

➔ A useful SAR image is not a sparse scene!

- **The real need = to interpolate missing data corresponding to strong echoes that have side lobes because of an incomplete transmitted bandwidth.**
  - But, each sub-band is properly sampled (Nyquist's rate).

➔ So, the goal is to restitute both strong echoes and background without being corrupted by side-lobes from strong echoes.

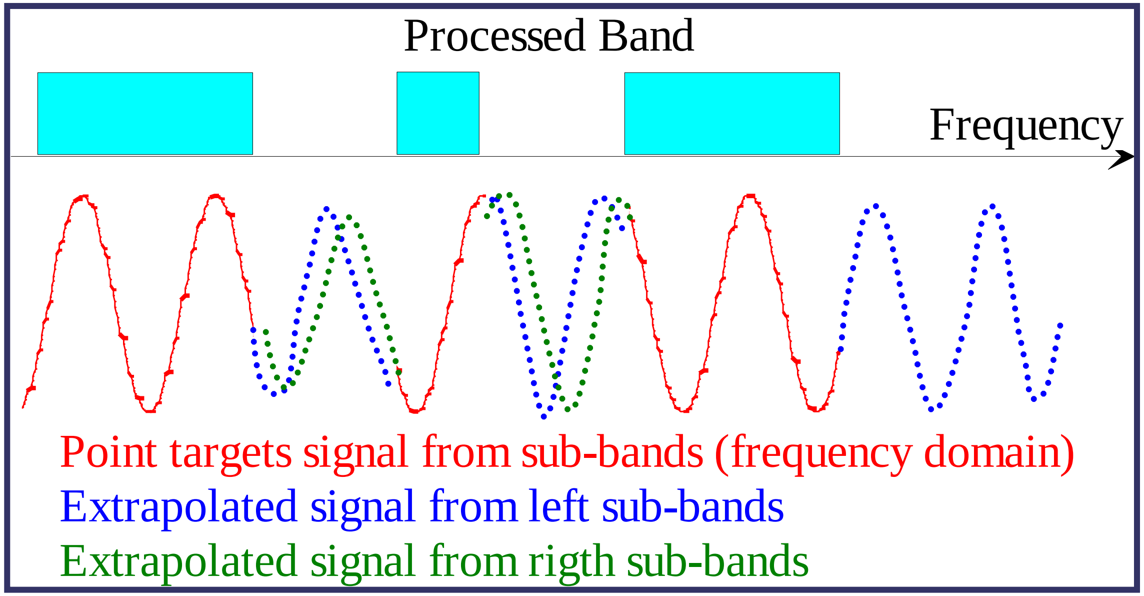
- **Some candidate methods are:**
  - Auto Regressive (AR) interpolation;
  - L1-norm regularization reconstruction methods;
  - Clean-Relax, etc.

Goal = to interpolate, in frequency domain, the missing data corresponding to strong point-like targets equivalent to a sum of sinusoidal signals.

Sparse Sampling in Frequency : SAR imaging application (2/6)

A.R. interpolation of missing data from available sub-bands :

- **Applied after azimuth compression:**
- **A.R. models provide extrapolations from left and right sub-bands.**
- **Small number of “holes” ➔ easy to mechanize.**
- **Low computing req.**



- **Final interpolation of missing data = weighted sum of left and right extrapolations.**

Only the signal corresponding to strong & discrete targets is reconstructed.

Sparse Sampling in Frequency : SAR imaging application (3/6)

A.R. interpolation – real data

- Full bandwidth
- 30% bandwidth missing
- After A.R. interpolation

Sparse Sampling in Frequency : SAR imaging application (4/6)

Another possible method: L1-modified Regularization

- Range reconstruction applied after Doppler compression;
- Input signal Y: received signal spectrum only in usable sub-bands;
- Reconstructed range profile S:

$$\boldsymbol{S} = \textbf{argmin}(\|Y - \Phi.\boldsymbol{FFT}(\boldsymbol{S})\|_2{}^2 + \boldsymbol{\lambda}\,\|\boldsymbol{S}\|_{1\#})$$

To preserve image's background: $\|\boldsymbol{S}\|_{1\#} = \sum \sqrt{(\boldsymbol{s_i})^2 + \boldsymbol{\varepsilon}^2} \Rightarrow \begin{cases} = \text{L1 norm on strong signals} \\ \sim\text{L2 norm on small signals} \end{cases}$

- Φ is the "concatenation matrix". It removes from $FFT(\boldsymbol{S})$, some parts of the bandwidth where no data was gathered.
- Φ = $\boldsymbol{I}(N)$ where lines corresponding to missing data have been deleted.

- "*X = FFT(S)*" may be written algebraically "*X* = Ψ*.S*":
  - Ψ = Fourier basis matrix ➔ $\boldsymbol{S} = \textbf{argmin}(\|\boldsymbol{X} - \Phi.\boldsymbol{\Psi}.\boldsymbol{S}\|_2{}^2 + \boldsymbol{\lambda}\,\|\boldsymbol{S}\|_1)$
  - That is mathematically one possible formulation for C.S. reconstruction.

L1 regularization: not used as in CS approach (sparse random projections)
Goal: interpolate missing data ➔ lower side-lobes on point-like targets

Patented algoritms. THALES, 2005, 2007.

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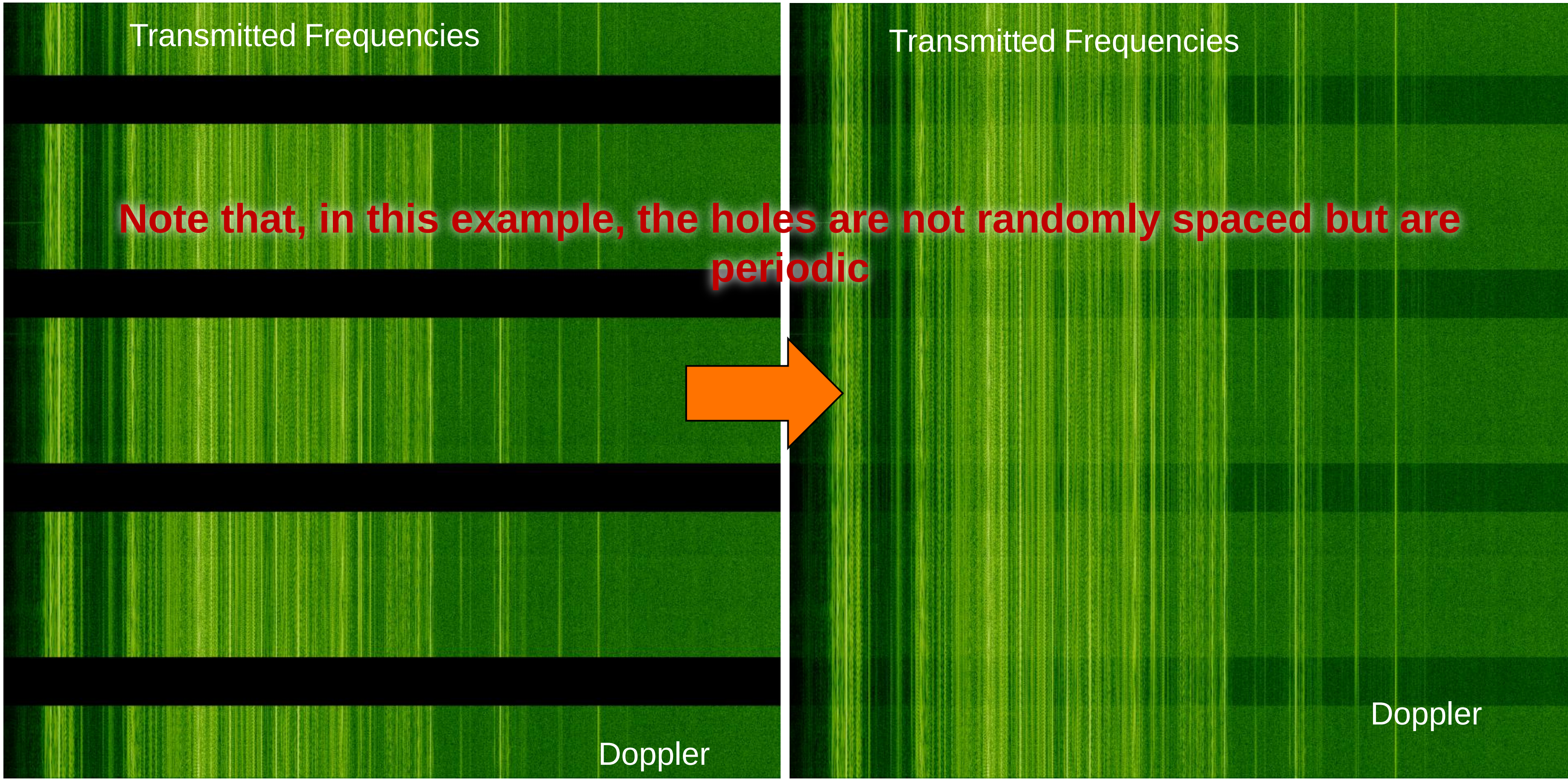
Sparse Sampling in Frequency : SAR imaging application (5/6)

L1 Regularization – Real data:



Note that, in this example, the holes are not randomly spaced but are periodic

As for the A.R. method, the signal corresponding to strong & discrete targets in Range is well reconstructed

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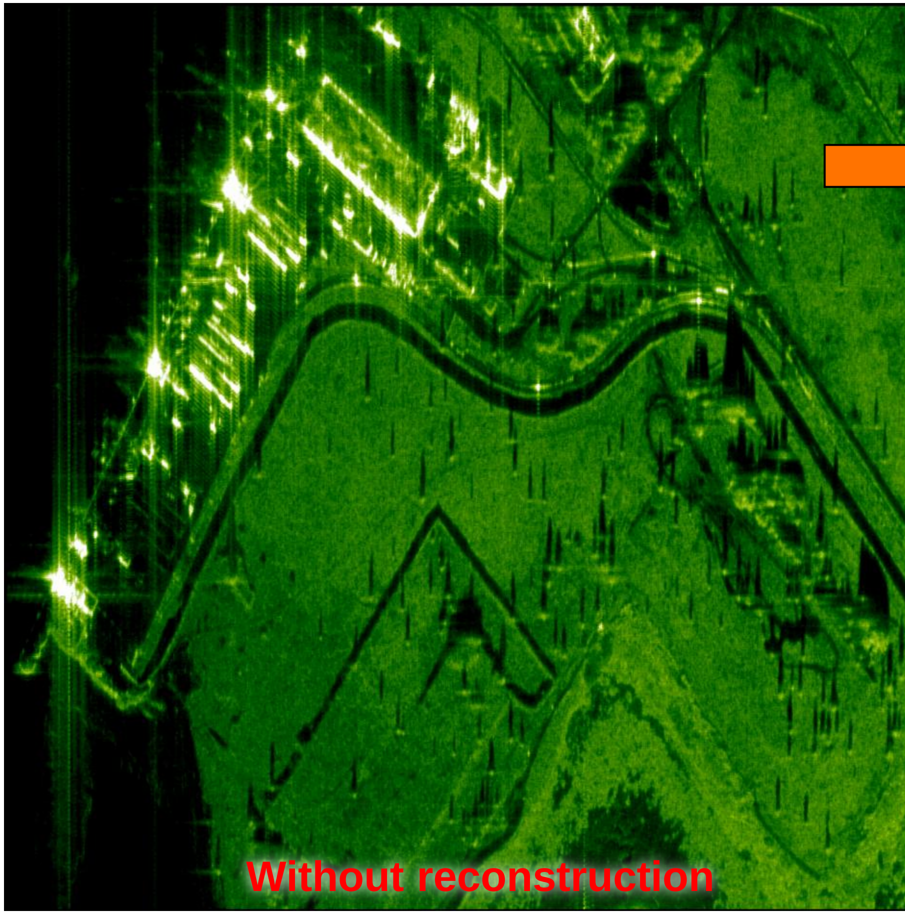
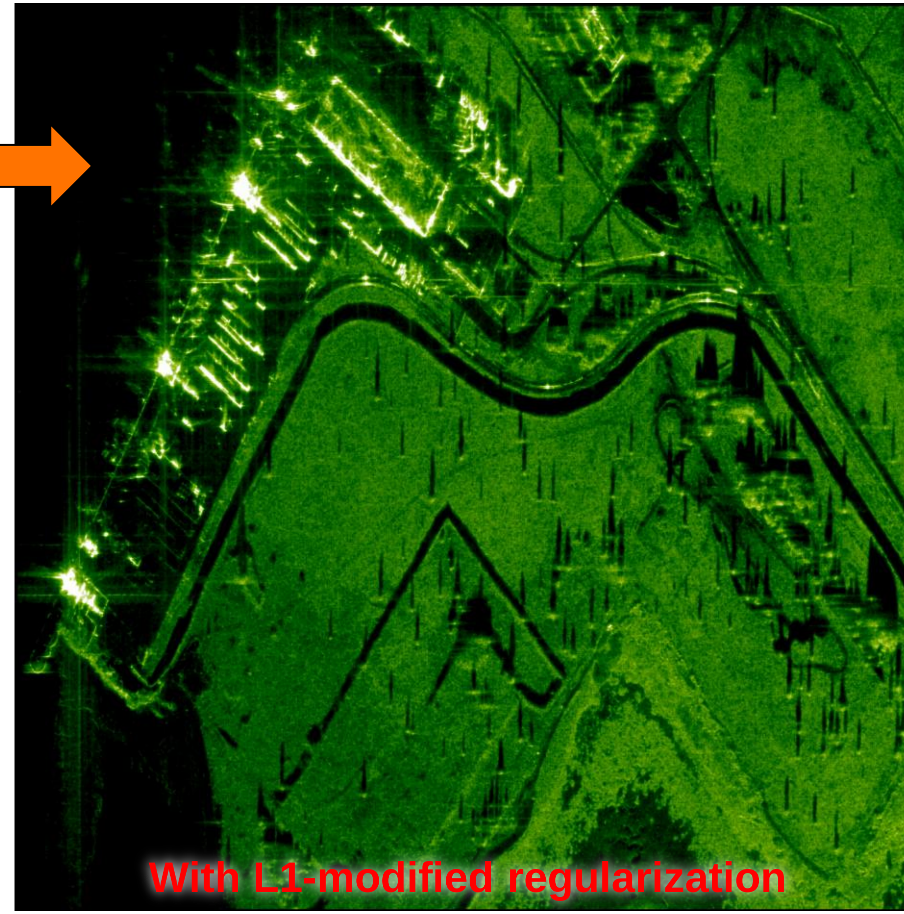
Sparse Sampling in Frequency : SAR imaging application (6/6)

L1 Regularization – Real data:





Due to the modified L1 norm, point-like strong echoes are well reconstructed while the background details are preserved.

Second case : MPRF unfolding (1/6)

Question: Is the compressive sensing approach useful for ambiguities solving ?

Measurement of ambiguous ranges, example:

- **N_{PRI} = 6 PRI corresponding to: n = 11, 13, 15, 17, 19, 23 range gates**
  - Six co-prime numbers.
  - Range unfolding up to N = 100 unfolded range gates.
- **Measurement vector X:**
  - Vertical stacking of NPRI = **6 binary detections** vectors (0 or 1) in folded range domain

Dictionary matrix: Ψ

- **Sampling and reconstruction are using the same basis. Ψ enumerates all the possible locations of a target from 1 to N $\Rightarrow \; \Psi = I(N)$**

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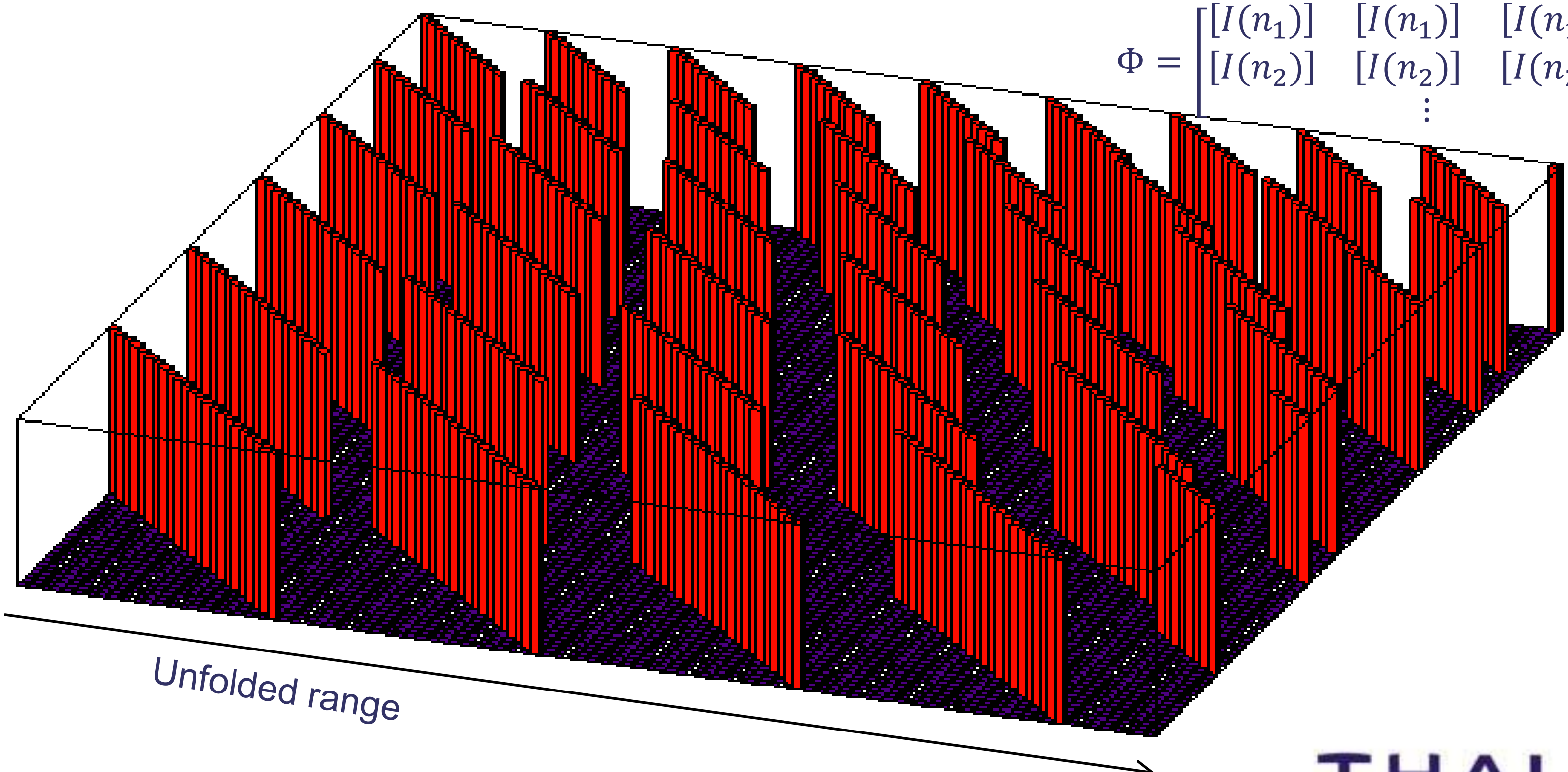
Second case : MPRF unfolding (2/6)

Measurement matrix Φ building:

- **For each PRI:**
  - Horizontal stacking of identity matrix (range ambiguities for this PRI): $[[I(n)] \quad [I(n)] \quad [I(n)] \ldots]$.
  - Truncation to get a rectangular matrix (N x n)
- **Vertical stacking of N_{PRI} = 6 rectangular matrix (N x n)**

$$\Phi = \begin{bmatrix} [I(n_1)] & [I(n_1)] & [I(n_1)] & \cdots \\ [I(n_2)] & [I(n_2)] & [I(n_2)] & \\ & \vdots & & \end{bmatrix}$$



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Second case : MPRF unfolding (3/6)

Basic unfolding consists in:

- **For each PRI:**
  - To initialize a detection vector whose size is the unfolded domain;
  - To enumerates all possible locations for a given ambiguous pre-detection (0 or 1);
  - To write “1” in all possible locations.
- **For the set of N PRI:**
  - To sum all of the N_{PRI} detections’ vector
  - To validate an unfolded detection if its score is greater than K (K out of N criterion)
- **Algebraically, this is equivalent to calculate:** $\boldsymbol{S} = \boldsymbol{\Psi}^{-1}\boldsymbol{\Phi}^{T}\boldsymbol{X} = \boldsymbol{\Phi}^{T}\boldsymbol{X} \underset{<}{\geq} K$

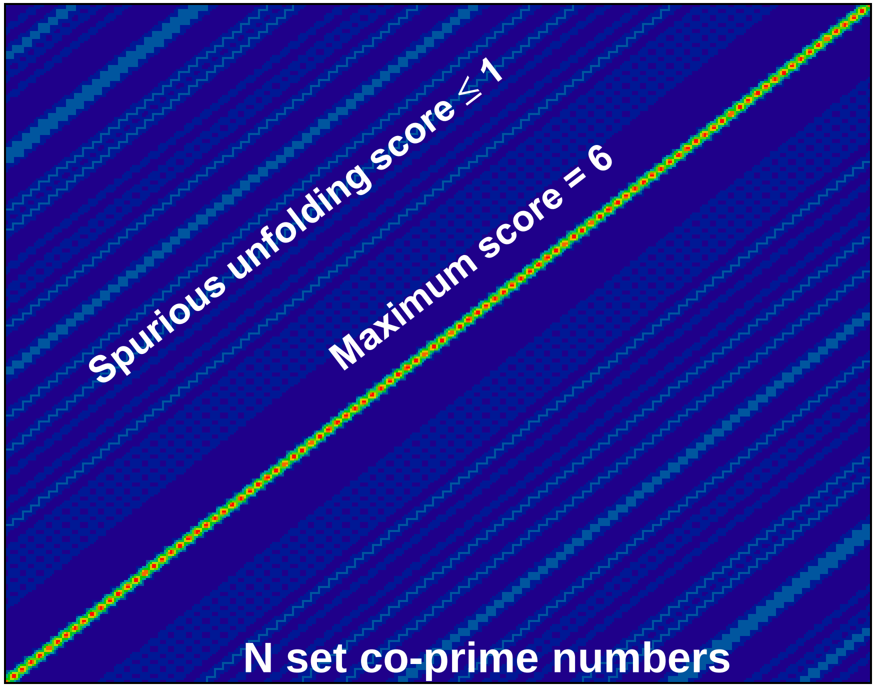
Main issue occurs in multi-targets condition:

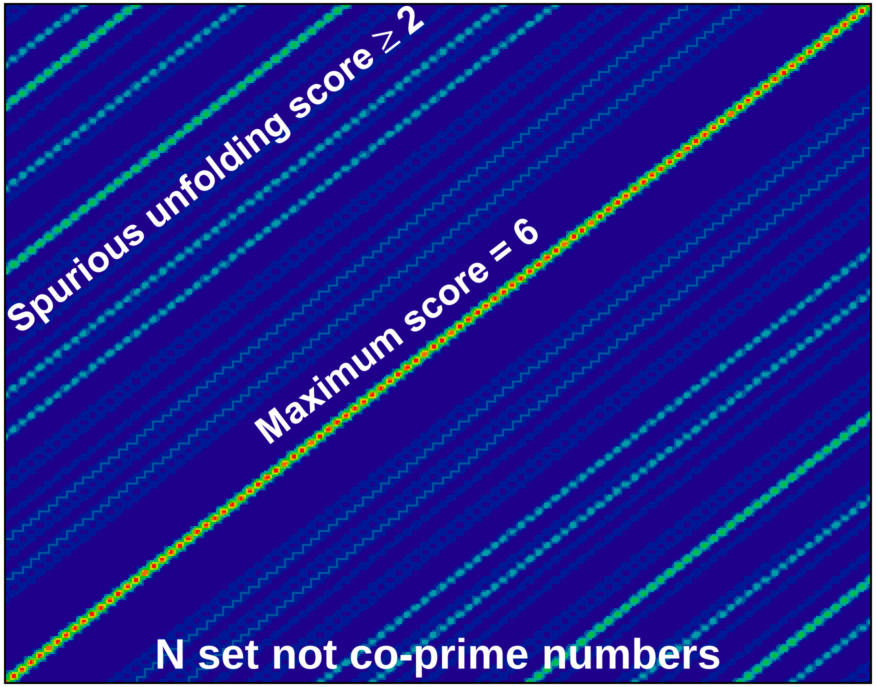
- **Indeed, one pre-detection may contribute to more than one unfolded detections.**

Basic unfolding may create ghost targets in multi-targets conditions.

Second case : MPRF unfolding (4/6)

Matrix $\Theta^T\Theta$ acts as "ambiguity function" of unfolding:





X : true location,Y : possible location, Z: association score [0, 6]

- **The basic unfolding is to calculate the score of association of pre-detections from N_{PRI} ambiguous domains.**
- **Then a threshold (k out of N_{PRI}) is applied.**

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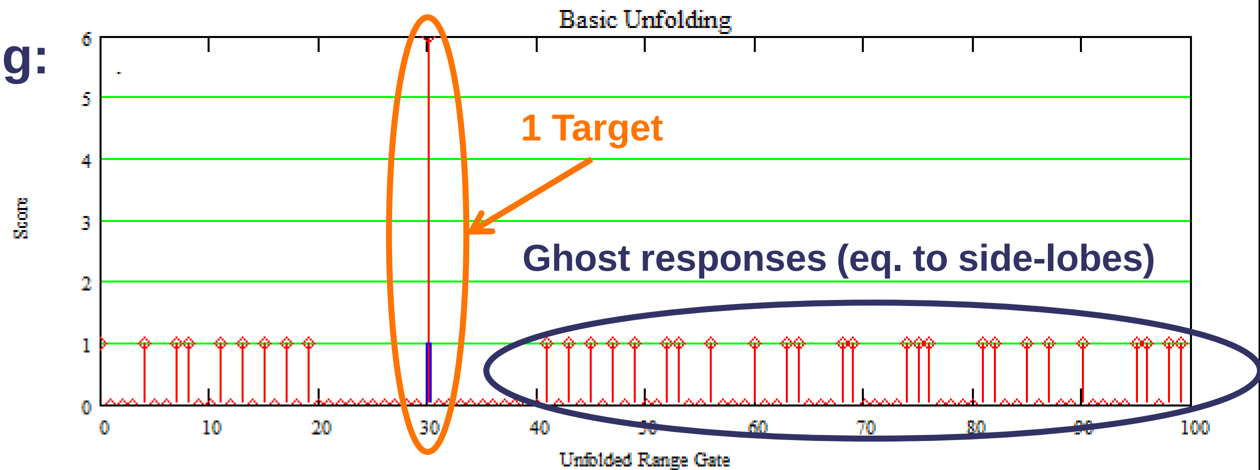
Second case : MPRF unfolding (5/6)

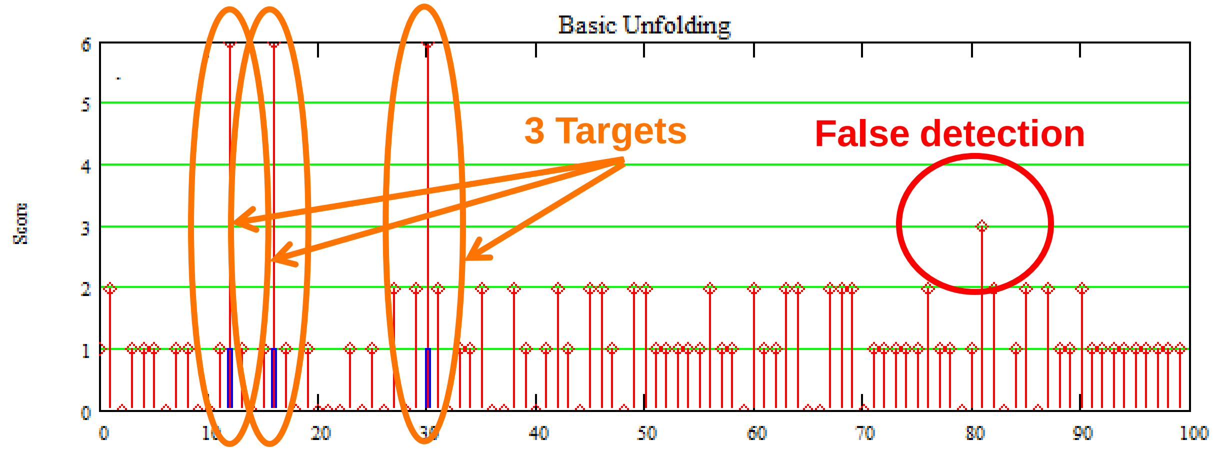
What can bring C.S. concept to MPRF unfolding?

- With basic unfolding:

➔ Single target case:



➔ Multi-target case:



The "sparsity" constraint provided by the L0 or L1 minimization avoids false recombination (only the sparsest solution is retained).

Second case : MPRF unfolding (6/6)

Is CS concept really useful in this case?

- **In practice, the unfolding is performed both in Range and Doppler.**
  - Mathematics of CS thus lead to manipulate very large matrix.
- **Modern algorithms may operate as follows:**

Initialize $\boldsymbol{S}^0 = \boldsymbol{\Phi}^T \boldsymbol{X}$
Loop:
(Apply ranking criteria on elements of *S*)
Retain as unfolded detection the highest rank with S(i) ≥ K
Then, set S(i) = 0
If all elements are < K then exit
Return in folded domain $\boldsymbol{X} = \boldsymbol{\Phi}.\boldsymbol{S}$
(Apply ranking criteria on elements of *X*)
Set all non-null entries to 1
Return in unfolded domain $\boldsymbol{S} = \boldsymbol{\Phi}^T \boldsymbol{X}$
End Loop

- **This method proceeds in a similar idea as CLEAN algorithm.**

L1 minimization leads to similar results but with higher complexity. Moreover, some "special" criteria are difficult to express with matrix algebra.

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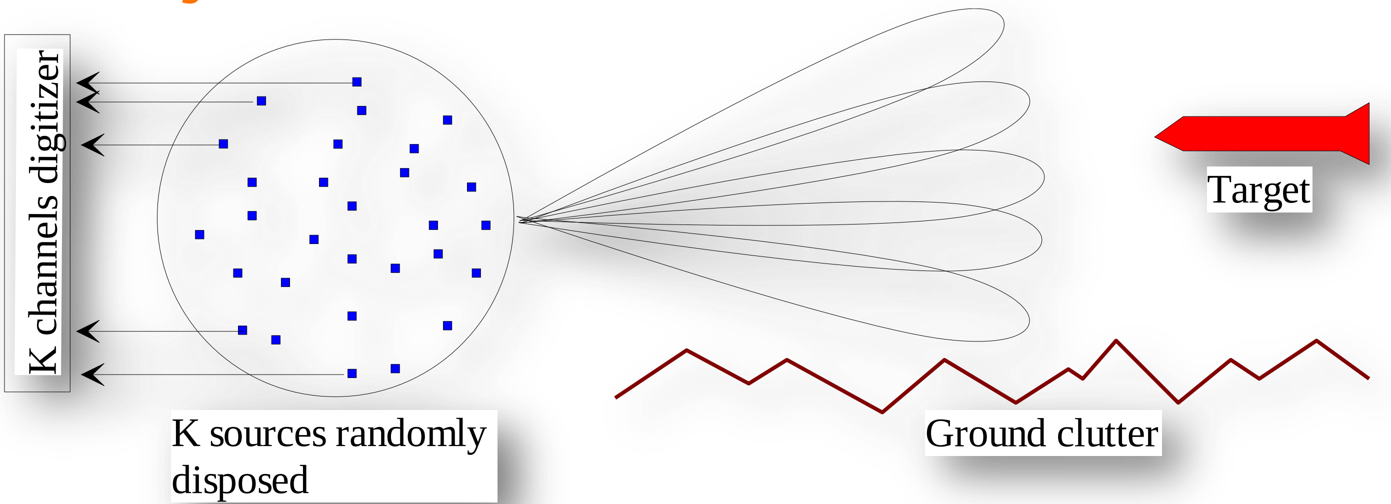
Sparse Sampling in Space– Accurate DOA measurement (1/4)

Ex. Air Detection System:



- **Need to steer devices➔ High angular accuracy ➔ Large D/λ array size**
- **Doppler processing:**
  - Only a few targets within a Doppler bin;
  - Fast close-in targets ➔ Targets on thermal noise Doppler region;
  - No critical requirement on antenna due to clutter rejection ➔ Side-lobes are not an issue.
- **We face to a sparse scene, affordable cost & complexity ➔ sparse array**

But "sparse array" means "less antenna surface", so less received power. Concept is viable only if the sensitivity is not an issue for short range systems

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Sparse Sampling in Space– SKA Radio-telescope (2/4)

Another similar study case: the SKA radio-telescope

- **What it is first searched for? : Angle accuracy and resolution.**
  - Need for a very large array;
- **How are the radio sources? :**
  - Spatially isolated from nothing;
  - Frequency separated : The observed wavelengths depend on phenomena involved in.
- **Is a fully filled array possible ? No, impossible to cover hundreds of km²!**

Sparse and random array = the only affordable solution.

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Sparse Sampling in Space– Accurate DOA measurement (3/4)

Beam-forming vs. C.S. approach :

- The scene of sources is *S* expressed in basis of angle coordinates
- The measurement vector is $X = \Phi.\Psi.S$ where:
  - Ψ is a kind of Fourier matrix
  - Φ describes how the sparse sampling is done. $\Phi = I(N)$ where lines corresponding to missing receivers have been deleted.

→ Basic Beam-forming solution (M.L. sense):

$$\widehat{S_{BF}} = \underbrace{\Psi^{-1}}_{\text{Matrix (NxN) of steering vectors}} \underbrace{\Phi^{H} X}_{\text{Fill with zero entries to get size N}}$$

Not a single solution, if sources are well disposed, peak response and side-lobes.

→ Compressive Sensing solution:

$$\widehat{S_{CS}} = \mathbf{argmin}(\ \|S\|_1)\ s.t.\ \|X - \Phi.\Psi.S\|_2 < \sigma$$

Single (no side-lobes) and exact solution in noiseless conditions, if sources are well disposed. In fact, that is equivalent to bacic beam-forming with sparcity constraint

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Sparse Sampling in Space– Accurate DOA measurement (4/4)

Beam-forming vs. C.S. approach :

- **After Doppler filtering and clutter removing, the probability for having more than one single echo in a range-Doppler cell is low.**

→ **In most of cases, the “basic” beam-forming at M.L. sense followed by a conventional detection process is sufficient:**

  - Threshold and selection of the peak response.

→ **Accurate DOA can be obtained from a kind of “mono-pulse”.**

However, in some cases, the exact reconstruction using the L1-norm minimization can help.

Concept of sparse and random sampling on one hand, and L1-norm minimization on the other hand are two separate things: L1-norm minimization is only an elegant method to solve a problem, not a new concept itself.

Conclusions

- Reconstruction of missing data and reduction of side-lobes are not a novelty. They are used in radio astronomy since 70’s and regularization methods have been effectively used since 90’s.
- The use of random sampling to reconstruct sparse scenes is also not new. Unfolding of MPRF ambiguities is a good example.
- Finally, the concept of CS is not a novelty of the 2000's. However around it, many research are being conducted that have theorized the concept and are producing efficient L1-norm minimization algorithms.
- Except cases where front-end and processing are distant or distributed systems, the main interest of CS in radar is not to reduce the incoming data flow (which is no longer really an issue with current digital technologies) but to reconstruct observed scenes when sampling cannot be ideal for practical questions.
- The current “fashion” about CS deserves asking again the question of what is really needed, not for reconstructing a signal, but to directly extract the useful information it contains.

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Thank you for your attention

Questions?