On the Choice of Mixing Sequences for SNR Improvement in Modulated Wideband Convertor

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I. Problem Description

The Modulated Wideband Convertor (MWC) is one of the architectures for sub-Nyquist sampling of wideband analog signals containing few narrowband transmissions [1], [2]. In the MWC, the received signal $s(t)$ is first multiplied in M parallel channels with a different random periodic waveform, then low-pass filtered, and finally critically sampled. From the digital data it is possible to reconstruct $s(t)$ by exploiting the sparsity of the signal in frequency domain via the framework of compressed sensing.

The behavior of the MWC can be better understood in frequency domain. Let W and B denote the bandwidths of the input signal $s(t)$ and of the low-pass filter, respectively. The spectrum of the wideband sparse input signal $S(\omega)$ can then be divided in $K = {}^{W}/_{B}$ subbands, denoted by $S_i(\omega)$, where $k = 1, 2, \ldots K$. The output $y_m(t)$ of the m-th channel is a signal with bandwidth B whose spectrum $Y_m(\omega)$ is obtained by a weighted sum of the subbands $S_i(\omega)$ as follows

$$Y_m(\omega) = \sum_{k \in \mathcal{K}_m} \gamma_{m,k} \cdot S_k(\omega), \qquad (1)$$

where the index set $\mathcal{K}_m$ defines the $\tilde{K} \leq K$ subbands which are to be included in the sum. The weights $\gamma_{m,k}$ are determined from the mixing sequences chosen in time domain. When choosing these sequences randomly as proposed in [1], [2], we have that $\tilde{K} = K$ as $\mathrm{E}\left[\gamma_{\mathrm{m,k}}\right] = {}^{1}/\sqrt{\mathrm{K}}$. As a consequence, all subbands contribute to the sum in (1) although only a very few will contain useful signal (due to the sparsity of $S(\omega)$).

II. Contribution

In this paper we propose to design the mixing sequences, i.e., the weights $\gamma_{m,k}$ such that $\tilde{K} < K$ subbands are considered in the sum in (1). Compared to the traditional approach, i.e., $\tilde{K} = K$, this leads to an improvement of the SNR at the m-th branch when the narrowband signals fall within $\mathcal{K}_m$, as fewer subbands containing only measurement noise are added in (1). On the contrary, when $\mathcal{K}_m$ does not include the subbands with useful signal, the SNR will degrade. Notwithstanding, on average, taking fewer subabands in (1) leads to an improvement of the SNR. This can be seen from the Figure 1 representing average SNR of one branch for varying $\tilde{K}$. The plot is obtained from a Monte-Carlo simulation where $s(t)$ consists of a single narrowband emission of bandwidth B at randomly chosen center frequency and random $\tilde{K}$ weights $\gamma_{m,k}$. The SNR within the band B of the desired signal was 20 dB. Unfortunately, taking lower $\tilde{K}$ does not bring only benefits. In fact, as Figure 1 shows, the average mutual coherence of the equivalent measurement kernel increases for lower $\tilde{K}$, implying that while the SNR increases for each branch, we need to take more measurements, i.e., higher number of branches M, to be able to apply the reconstruction successfully.

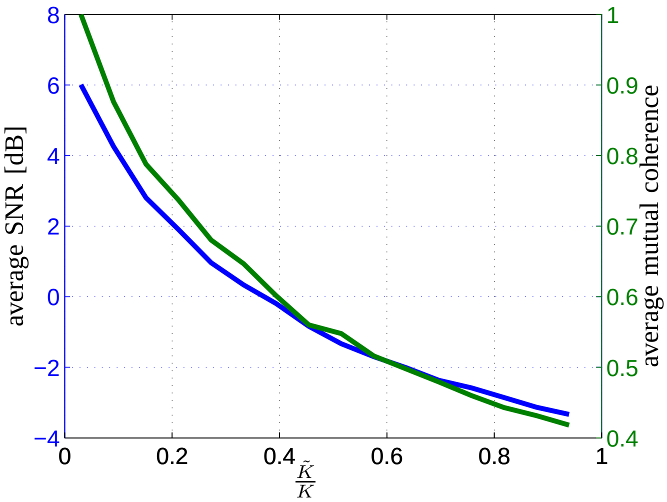

Fig. 1. Average SNR for one branch for a varying number $\tilde{K}$ of subbands included in the sum in (1) and the corresponding average mutual coherence of the equivalent measurement kernel.

In the full paper we investigate the tradeoff between taking lower $\tilde{K}$ to increase the average SNR at the cost of making the measurement less efficient. For a given number of branches M and sparsity of the problem there exists an optimal $\tilde{K}$ which maximizes the SNR while still allowing for perfect reconstruction.

References

[1] M. Mishali and Y. C. Eldar, *From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals*, IEEE Journal of Selected Topics in Signal Processing, vol. 4, no. 2, April 2010, pp.375-391.

[2] M. Mishali and Y. C. Eldar, *Xampling: Signal acquisition and processing in union of subspaces*, IEEE Transactions on Signal Processing, vol.59, no.10, 2011, pp. 4719-4734.