

Some remarks on the Herman-Strohmer high resolution CS-radar

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Main publication

Herman-Strohmer publication

- M.A. Herman, T. Strohmer, *High-Resolution Radar via Compressed Sensing*,
- proposed the compressed sensing approach to high resolution radar
- we discuss Herman-Strohmer approach
- we relax their assumptions and present simulations

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Herman-Strohmer framework

Assumptions

- $N \times N$ - time-frequency grid
- $s_{kl} \in \mathbb{C}$ - reflection coefficient
- $f(n) = \frac{1}{\sqrt{N}} e^{2\pi i n^3/N}$ - continuously emitted signal
- echo $\sum_{k,l} s_{kl} f(n-k) e^{2\pi i nl/N}$
- compressed sensing algorithm \rightsquigarrow exact recovery s_{kl}

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Herman-Strohmer Assumptions

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- **N - prime number**
- time and frequency grids the same
- Herman-Strohmer measure $\{0, 1, \dots, N - 1\}$ echo
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Beyond Herman-Strohmer framework

Relaxing the time-frequency grid assumption

- $N \times (2M + 1)$ - time frequency grid
- N, M - arbitrary natural numbers
- cutoffs and grids \rightsquigarrow natural units t_N, μ_M

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- periodic T -signal
- probing coefficient P
- Fourier decomposition of the signal:
$$f(n) = \sum_{d=0}^{P-1} a_d e^{2\pi i d \cdot n / P}$$
- Echo $s_{kl} f(n - k\Lambda) \omega^{nl}$, $\Lambda \equiv Pt_N/T$, $\omega = e^{-2\pi i \mu_M T / P}$
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$1/r^2$ - decay of the signal

- $1/r$ decay: $\vec{E} = \vec{E}_+ \frac{f(r-ct)}{r}$
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The simulation results

Non-Alltop signal

- emitted signal \rightsquigarrow Bernoulli noise periodically repeated
- K - number of objects
- square time - frequency grid
- on the y - axis we show the maximal K - number for exact recovery

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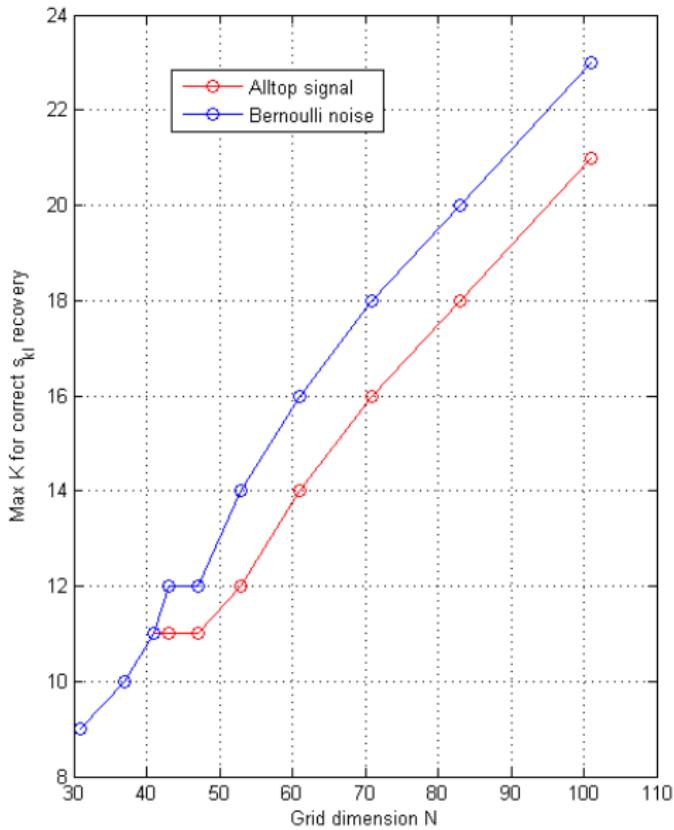
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Shift coefficient Λ and period ω

- $\Lambda \neq 1 \rightsquigarrow$ CS performs well only when Λ and P relatively prime
- CS performs well only when $P \geq N$
- recovery is successful only when $|\{\omega^{kl} : k, l \in \mathbb{Z}\}| \geq 2M + 1$

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Decreasing the number of measurement

- removing a number of randomly distributed measurements by a factor of 2 we got exact recovery
- degree of possible measurement size reduction depends on the size of frequency plane ($2M + 1$)
- $N = 100$ with $K = 10$ objects on the square grid
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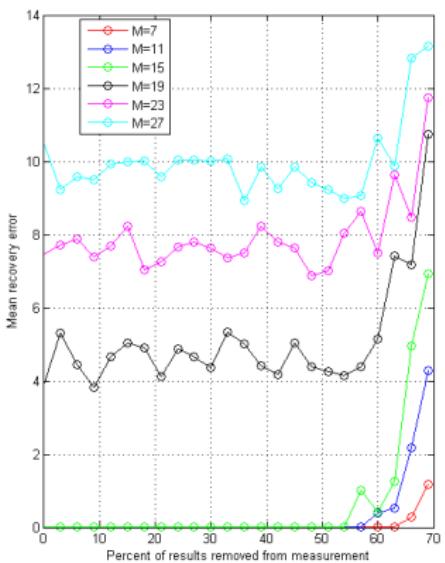


Figure : Recovery error when limiting number of measurements

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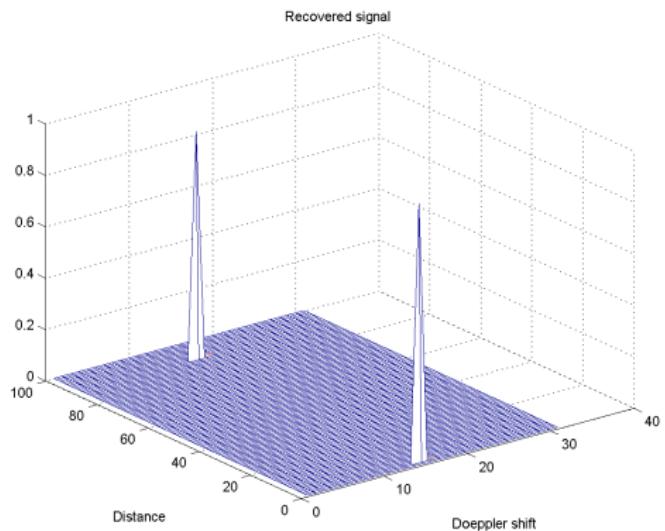


Figure : $1/r^2$ - decay of the signal