Sub-Nyquist Sampling for TDR Sensors: Finite Rate of Innovation with Dithering

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***Abstract*—In this paper, Finite Rate of Innovation (FRI) is applied to time domain reflectometry and it is aimed at significantly reducing the data acquisition requirements. The sensitivity of FRI to quantisation noise is addressed given the stringent practical constraints on the resolution of the deployed analogue to digital converters in miniature reflectometry sensors. Dithering with averaging is proposed to combat the effects of quantisation noise whilst maintaining remarkably low operational sampling rates. The substantial benefits of the adopted FRI-based reflectometry is demonstrated in the presented simulations. The trade-off between the resolution of the quantiser, time averaging and sampling rate is also depicted in terms of the quality of the signal recovery attained from the sub-Nyquist FRI samples.**

I. Introduction

Majority of the DSP in modern electronic systems is governed by the celebrated Shannon sampling theorem. It stipulates that the data acquisition rate should exceed the Nyquist rate, which corresponds to the bandwidth of the signal. In several application areas, including Time Domain Reflectometry (TDR) [1], Nyquist rates can be prohibitively high imposing stringent requirements on the data acquisition and processing module(s). This results in high Size, Weight, Power and Cost (SWPaC) solutions. In this paper, we propose utilising Finite Rate of Innovation [2] in TDR level sensors (otherwise known as guided wave radar level sensors) to facilitate operating at significantly low sub-Nyquist sampling rates. This leads to substantial SWPaC reductions without compromising the sensor performance.

Fig. 1 depicts a TDR level sensor for determining the liquid level in an industrial tank/container by measuring the Time of Flight (ToF) of transmitted electromagnetic pulses. An example of the operational specifications of a guided wave radar level sensor is listed in Table I. With classical Nyquist DSP, the analogue signal should be sampled at a rate of several giga samples per second to establish the liquid level. Such rates pose formidable design challenges given the miniature size of a TDR sensor. Therefore, notably reducing the data acquisition rate via sub-Nyquist sampling techniques is highly desirable to produce low SWPaC guided wave radar level sensors. Since the processed signal is composed of a sum of finite number of pulses that can appear anywhere along the time axis, FRI is adopted in lieu of Compressed Sensing (CS) [3]. With the latter, achieving very high resolution along the time domain to capture the location of the present pulses is a cumbersome task with the discretisation typically involved in CS to tackle the sparse problem in a finite union of subspaces. Additionally, FRI is easily implementable and/or integrable into existing TDR designs, see for example Fig. 2, unlike CS and Equivalent Time Sampling (ETS) [4]. With equivalent time sampling, a complex bulky circuitry is necessary to generate the equivalent sampling rates introducing hardware design challenges such as reliable phase locked loops, etc. Besides, ETS demands notably longer analysis time windows compared with FRI with implications on the sensor response time and the incurred latency.

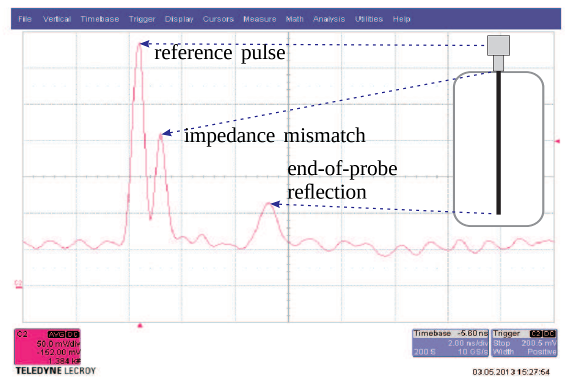

Fig. 1. Measured TDR signal depicting the sent pulse, its first reflection due to impedance mismatch and the pulse reflection at the end of the probe (no medium is present).

In FRI, the sampling rate is typically increased to well above twice the signal information rate (theoretical minimum rate) and denoising algorithms are deployed to combat the effects of any present noise [2], [5], [6]. Such remedies degrade the FRI gains in terms of the furnished savings on the data acquisition compared with classical Nyquist DSP and limits its practical applications. In the considered TDR sensor and due to practical SWPaC constraints, the resolution of the on-board Analogue to Digital Converter (ADC) is limited, e.g. to a maximum resolution of 8 bits. This introduces quantisation noise that the subsequent processing, such as signal reconstruction from the captured sub-Nyquist samples, is ought to handle or tolerate [7]. Conventionally, either the sampling rate is increased and/or an ADC with higher resolution (whenever possible) is used to deliver the sought sensing performance.

In this paper, we show that FRI is very sensitive to the presence of quantisation noise even with various denoising algorithms such as Cadzow and total least squares; or a combination of both (see [2], [5], [6]). Consequently, sampling rates remarkably higher than the set theoretical minimum in FRI [2] are required to enable reliable FRI-based TDR. Here, we

TABLE I. TECHNICAL SPECIFICATIONS OF AN OPERATIONAL TDR SENSOR.

Requirement	Value
Measuring Range	5 cm ... 10 m
Inaccuracy	$<$ 5 mm
Resolution	$<$ 0.5 mm
Response Time	$<$ 100 ms

propose introducing dithering to the signal prior to sampling followed by an averaging to suppress/eliminate the effects of the present quantisation noise without significantly increasing the operational sub-Nyquist sampling rates and/or resorting to a higher resolution ADC. The trade-off between the sampling rate, ADC resolution (i.e. level of present quantisation noise), averaging and accuracy of ToF measurements are evaluated. In [8] various FRI sampling kernels are examined with the presence of additive noise. Unlike the latter, the aim here is to combat the quantisation noise by dithering and ensemble averaging.

II. TDR AND PROBLEM FORMULATION

A. TDR and Adopted Signal Model

A TDR measurement system locates the discontinuities of the waveguide impedance along the propagation path of an electromagnetic wave. It exploits the fact that at every discontinuity a wave reflection occurs and the amount of the reflected energy depends on the impedance change described by the reflection factor:

$$R = \frac{Z_0 - Z_1}{Z_0 + Z_1}. \tag{1}$$

when the waves travel from a space with impedance Z_0 into a space with impedance Z_1.
A TDR sensor sends a pulse $p(t)$ and analyses its reflections to establish the locations of the present discontinuities, e.g. a medium-change, by determining their associated ToF. The measured TDR signal can be expressed by:

$$x(t) = \sum_{i=0}^{K-1} a_i p(t - t_i) = g(t) \star p(t) \tag{2}$$

where K is the number of reflected pulses centred at the delay time instants $\{t_i\}$, $|a_i| \leq 1$, $\delta(t)$ is the Dirac delta and

$$g(t) = \sum_{i=0}^{K-1} a_i \delta(t - t_i). \tag{3}$$

Thus the processed signal $x(t)$ has $2K$ degrees of freedom and FRI-sampling with the annihilating filter can be applied.
Since this work is motivated by suppressing/eliminating the impact of the quantisation noise on FRI-based TDR, a number of simplification to/in the model in (2) are made. The pulses in $x(t)$ are assumed to not overlap and a minimum distance between any two adjacent pulses is maintained as in [5] and [8]. Here the latter is set to 10σ where σ is the standard deviation of the Gaussian shaped pulse $p(t)$. Electromagnetic interference (EMI) and analogue noise are discarded. Whereas, the magnitude attenuation factor a_i in (2) can vary for distinct time delays $\{t_i\}$ unlike in [5] and [8]. In the considered TDR application, reflections with magnitudes in the range of $|a_i| \in \{0.05 \ldots 1\}$ are of an interest, leading to a magnitudes dynamic range of 26 dB.

B. Problem Formulation and Proposed Approach

Whilst FRI offers a means to remarkably reduce the prohibitively high data acquisition rates of TDR sensors that abide with Shannon sampling theorem, it is shown to be very sensitive to the presence of quantisation noise. It is particularly severe given the low resolution ADC(s) typically used in TDR sensors due to size and power limitations. In lieu of increasing the averaging sampling rate to combat the effects of noise, dithering with averaging is proposed to facilitate effective FRI-based reflectometry. In addition to the sampling rate and ADC resolution, the level of introduced dithering and time averaging affect the quality of attained sensing results. Accordingly, the trade-off between these parameters is examined using extensive simulations. It is noted that reliably estimating the pulse location(s) in a TDR sensor is crucial to fulfil the accuracy requirement of the system. For example, in Table I the specified maximum relative ToF measurement error between two pulses imposes a pulse-location estimation error of approximately less than $t_{\text{error}} = {}^{s}/_{c} = 33\,\text{ps}$. The proposed FRI-based TDR with dithering and averaging is shown to substantially improve the pursued estimation accuracy.

III. FRI-BASED REFLECTOMETRY

A. FRI Sampling

The basic principle of FRI is to translate the highly non-linear dependency between t_i and $x(t)$ in (3) in a linear system. This is done by obtaining the Fourier transform of the samples of $x(t)$. Applying methods from spectral estimation the signal $x(t)$ can be reconstructed from the few collected sub-Nyquist samples $\{x(nT_s), n = 1 \ldots N\}$. One well investigated method is the annihilating filter [2]. Prior to sampling at remarkably low rates compared to the Nyquist counterpart, a sampling kernel $s^*(-t)$ is used to filter the analogue signal (see Fig. 2). The Sum of Sincs (SoS) kernel is adopted in the sequel due to its suitability for data contaminated with additive noise [8] and its compact support in time domain allowing to sample finite as well as infinite length FRI signals [9]. SoS kernel can be described in the frequency domain by:

$$G(\omega) = \frac{\tau}{\sqrt{2\pi}} \sum_{k \in \mathcal{K}} b_k \operatorname{sinc}\left(\frac{\omega}{2\pi/\tau} - k\right) \tag{4}$$

where $b_k \neq 0$ are the coefficients and τ is the period of the present pulses. For the signal in (2) with $2K$ degrees of freedom, FRI enables the full recovery of $x(t)$ as long as the number of collected samples N exceeds the signal rate of innovation, i.e. $N \geq 2K + 1$ albeit Nyquist, and $2K$ is commonly referred to as the critical sampling or minimal rate. The sampling frequency is given by N/τ and the processed signal is typically sampled at rates exceeding that of the critical sampling such that $N = 2\beta K + 1$ and $\beta > 1$ is the oversampling factor.
In the presence of noise, various denoising algorithms, e.g. Cadzow, are used with FRI. However, such denoising algorithms are shown to be ineffective for achieving notable improvements in the quality of the signal reconstruction in the presence of quantisation noise; a noise exasperated by practical

limitations on the resolution of the used ADC. Alternatively, the sampling rate can be substantially increased to well above the critical minimum to enhance the quality of results; an option that undermines the benefits of FRI in terms of easing data acquisition requirements. A top-level block diagram of the proposed TDR system is shown in Fig. 2. After triggering the pulse generator, the Gaussian shaped pulse propagates through the electronics towards the probe. Both sent pulses and their reflections are guided to the sampling kernel, and then dithering is added to the signal prior to the low rate quantiser. This produces a discrete sequence with a sampling period of T_s. Dithering coupled with averaging prior to the signal reconstruction is shown to significantly improve the accuracy of estimating the TDR pulse locations circumventing the need to choose high FRI oversampling factor.

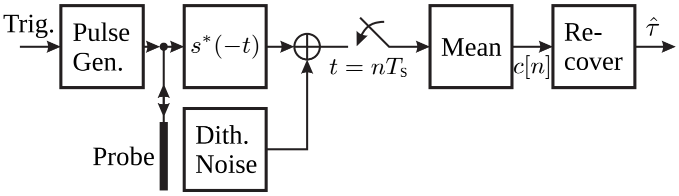

Fig. 2. TDR with FRI; $s^*(-t)$ is the sampling kernel such as sums of sincs filter.

B. Dithering with Averaging

Ensemble averaging can be modelled like an oversampling process. The quantisation noise energy $E[\cdot]$ is [10]

$$E[e_{noise}^2] = \frac{Q^2}{12L}, \tag{5}$$

where L is the oversampling factor and Q is the quantisation step. This oversampling factor L must not be confused with β pertaining to the FRI sampling. It can be derived that the gain G of the averaging process in bits can be expressed with:

$$G = 1/2 \cdot \log_2 L. \tag{6}$$

However, this only holds for linear systems. As quantisation is a highly non-linear process information is lost. This is because for each quantisation level all corresponding amplitude levels are subsumed to a single value. Therefore, averaging an already quantised data will not necessary lead to the average of the data. To overcome this problem, a linearisation of the system is applied using additional dithering noise [11]. When for example a uniformly distributed noise with an amplitude range of $\pm Q/2$ is added prior to quantisation, the probability of an amplitude level being quantised to the next higher or lower quantisation level depends linearly on the level, i.e. on average the system is linear again. Averaging the dithered and quantised levels therefore significantly reduces the noise level, although some dithering noise was added.

In the TDR application, each contribution to the ensemble averaging is taken after a pulse is transmitted. Assuming a total signal length or pulses period of $\tau = 500\,\mathrm{ns}$ the overall signal acquisition time for the averaging process with 250 averages is $125\,\mu\mathrm{s}$. As the measurement condition can be considered to be stationary over a time of several milliseconds, the resulting time delay can be tolerated. Additionally, the sensor requirement for the maximum permitted latency, e.g. set in Table I at $100\,\mathrm{ms}$, clearly indicates that there is sufficient time to perform the computational tasks involved (latency refers to the time between a change in the measurement condition and the response of the TDR sensor to the aforementioned change).

IV. Simulations

In this section, extensive Monte Carlo simulations are conducted to quantify the advantages of the proposed approach, i.e. dithering with averaging, in FRI-based reflectometry. Here, we adopt the model described in (2) where Gaussian shaped pulses with $\sigma = 200\,\mathrm{ps}$ are used by the TDR sensor as in typical practical scenarios. Five sent/reflected pulses, $K = 5$, are assumed to be present and the period of the observed signal is $\tau = 50\,\mathrm{ns}$. This leads to a minimum permissible sampling rate of $1/T_s = N/\tau = (2\beta K + 1)/\tau = 220\,\mathrm{MHz}$ with $\beta = 1$ as per the signal rate of innovation; it is remarkably lower than the Nyquist sampling counterpart. The sums-of-sincs filter is deployed as the sampling kernel with a bandwidth of $B = 1/T_s$ and the recovery is carried out using the Cadzow algorithm. In a practical TDR system, the pulse absolute amplitude value can notably vary, e.g. in Table I the processed signal has a dynamic range of $26\,\mathrm{dB}$. Accordingly, we assess the impact of treating pulses with varying amplitudes on the accuracy of the attained results compared with the scenario where all the pulses take amplitudes of ± 1. In all the presented plots, 2,000 independent experiments are averaged to obtain the displayed Root Mean Squared (RMS) and absolute maximum errors of estimating the exact locations of the present pulses. The pulse locations are chosen arbitrarily in each of the aforementioned experiments; however the restriction that the distance between any two adjacent pulses is never less than 10σ is maintained. Dither that is uniformly distributed in the region of $\pm Q/2$ is introduced (whenever applicable) and it is accompanied by averaging.

In Fig. 3 and Fig. 4, we show the maximum and RMS errors of the FRI pulse locations estimation for a varying ADC resolution with and without dithering plus averaging. The oversampling ratio is set at $\beta = 2$, i.e. the sampling rate is $440\,\mathrm{MHz}$, and 250 averages are made, i.e. the signal aquisition time (latency) is $12.5\,\mu\mathrm{s} \ll 100\,\mathrm{ms}$. Whilst in

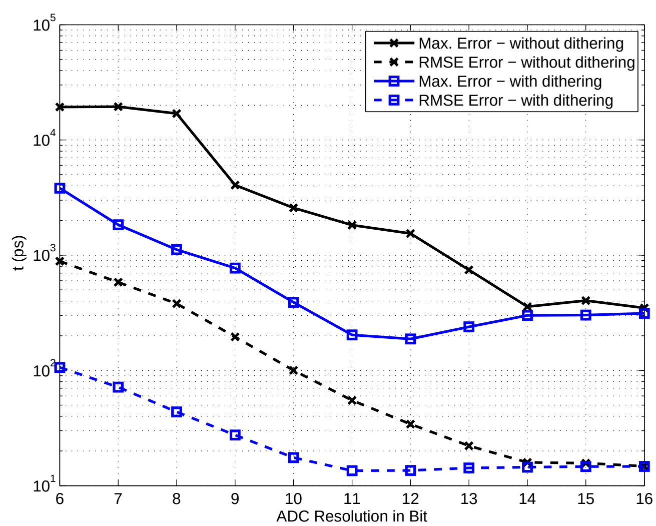

Fig. 3. Maximum and RMS errors of the FRI pulse locations estimation for a varying ADC resolution; pulses amplitudes take a value of ± 1.

Fig. 3 the pulses take an amplitude of ± 1, in Fig. 4 we have $|a_i| \in \{0.05 \ldots 1\}$. It is clear from both figures, the proposed FRI-based TDR with dithering plus averaging leads to notable reductions in the delay estimation errors. Besides, increasing the ADC resolution without dithering plus averaging does not necessarily lead to lower maximum estimation accuracy due to the random nature of the quantisation noise. Whereas, introducing dithering plus averaging suppresses such erratic behaviour of the obtained error. It is noted here that in practice restraining the maximum absolute error is crucial to ensuring robust TDR sensor operation and fulfil the pre-set system specifications. As seen in Fig. 3 and Fig. 4, the achieved gain from using dithering plus 250 averages is approximately $3\,\mathrm{bits}$, which is consistent with (6). For example, in Fig. 3 FRI without dithering requires 12 bits to achieve the RMS error of FRI with dithering of a resolution of 8 bits. Additionally, Fig. 3 and Fig. 4 depicts that applying FRI to TDR for pulses with a relatively large dynamic leads to substantially larger estimation errors compared to when the present pulses have equal magnitudes, e.g. $a_i \in \{-1, +1\}$.

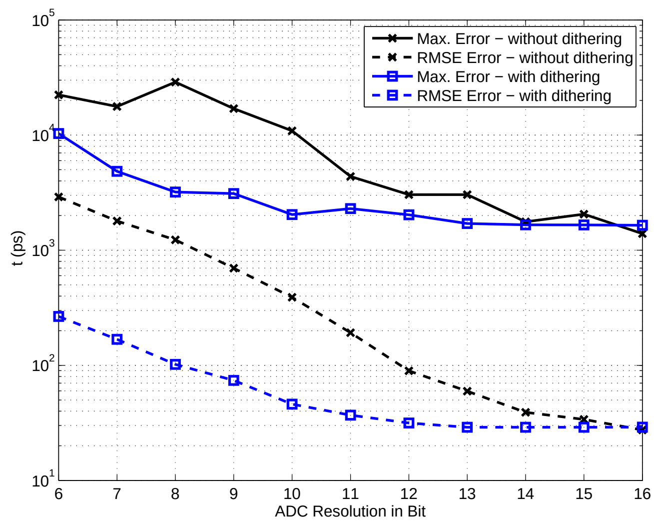

Fig. 4. Maximum and RMS errors of the FRI pulse locations estimation for a varying ADC resolution; pulses amplitudes have a dynamic range of $26\,\mathrm{dB}$.

In Fig. 5, the effects of FRI oversampling ratios and number of averages on the accuracy are examined. The five present pulses amplitudes vary with a dynamic range of $26\,\mathrm{dB}$ and an ADC resolution of 6 bits is assumed. It can be noticed from the figure that the estimation accuracy improves as the oversampling ratio increases. The slope of this improvement is significant till $\beta = 8$ and then the benefits of increasing the sampling rate becomes marginal. In terms of the number of averaged signal acquisitions, it is clear from Fig. 5 that increasing the number of averages leads to better estimates with lower RMS and maximum errors. It is noted here that in practice a trade-off is present between the ADC operational sampling rates and the achieved ENOB. This implies that high resolutions at high sampling rates can be infeasible.

V. Conclusion

Whilst FRI technique enables TDR with substantially low sub-Nyquist rates, it is very sensitive to the presence of quantisation noise which is severe in TDR systems due to

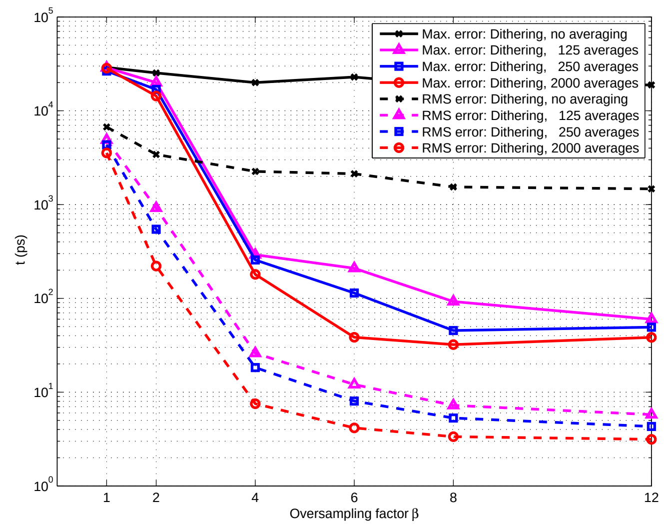

Fig. 5. Time resolution as a function of the oversampling factor and the number of averaged sequences (the absolute time mismatch is displayed using solid lines, while dashed lines are used to indicate the RMSE-values).

practical hardware limitations. Introducing dithering prior to sampling and then averaging directly after sampling leads to significant performance improvements of the FRI approach. However, such improvements are not sufficient to meet stringent sensing requirements in practical reflectometry sensors, e.g. a maximum error of $33\,\mathrm{ps}$. Given FRI amenability to implementation in hardware and ease of integration into existing sensor architectures, this paper serves as an impetus to further research into FRI-based TDR, especially in terms of further reductions on the maximum ToF estimation error via novel reconstruction methods.

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