Initial Analysis of SNR / Sampling Rate Constraints in Compressive Sensing based Imaging Radar

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***Abstract*—The compressive sensing based imaging radar, as a combination of compressive sensing (CS) and microwave imaging, is attracting people's interest for its several advantages, but also facing application difficulties including the lack of performance evaluating tools. People have developed some tools e.g. RIP, MC and phase transit to analyze and evaluate the performance of a sparse reconstruction system, but large computation complexity restricts their usefulness. Here we suggest that an analytic expression of SNR, down-sampling ratio and sparsity constraints in CS based imaging radar system could be achieved by adopting the sharp bound analysis in noisy sparse measurement problem.**

As the representative of modern microwave imaging technology, synthetic aperture radar (SAR) has been widely used in various fields of remote sensing. According to radar resolving theory and Nyquist sampling theorem, with the rapid improvement of SAR resolution, the data amount would increase dramatically. The theory of compressive sensing (CS) and sparse reconstruction provides a scheme to overcome the problem of increasing sampling rate, and has also been introduced to the microwave imaging. Plenty of works about the CS based imaging radar have been published [1]–[4]. Both theoretical analysis and experiments show that, comparing with traditional SAR, CS based imaging radar has potential advantages in not only reducing the system complexity, but also improving the system imaging performance.

The performance evaluation is very important in the radar system design and performance analysis. Evaluating tools in traditional radar theory e.g. the radar equation is difficult to be applied under the CS scheme due to totally different signal processing strategies. This makes us in lack of tools to calculate the constraints of several critical radar system parameters e.g. signal-to-noise ratio (SNR), sparsity and sampling rate, and brings difficulties to system designing and applications. In the compressive sensing theory, we have several classical criteria to investigate the performance of sparse recovery system, such as restricted isometry property (RIP) and mutual coherence (MC). While, they are either too inaccurate, or too computational-complex. Phase transit has also been proposed as a candidate of evaluating tool. It is an imperial tool and basically a Monte-Carlo, while still costs too much computation.

In this paper, we will introduce a sharp bound analysis in noisy sparse measurement problem to CS based radar imaging, and try to achieve an initial analytic expression of the constraints among SNR, sampling date and sparsity of a radar system. The theory of such sharp bound was developed by Dossal et al in [5], where bounds of minimal SNR and down-sampling rate of an sparse reconstruction problem is achieved. We treat the radar imaging problem as a support recovery problem. We recall the bounds achieve by [5], and adopt it in the radar imaging problem as follow,

$$\mathrm{SNR} \geq \frac{5.5}{\rho\sqrt{1-\alpha}}\sqrt{\frac{2\log(p)}{p}}, \tag{1}$$

$$\rho \geq \frac{2\log(p)k}{\alpha\beta p}, \tag{2}$$

where SNR is defined as the ratio of backscattering coefficient of a point target to the noise energy in echo. ρ is the down-sampling ratio, p is the length of target, k is the number of non-zero elements in target (i.e. k/p is the sparsity), and α, β are two balance constants. This is a sufficient condition, provides a non-asymptotic expression to the constraints of SNR and sparsity.

we investigate the performance of a CS based imaging radar system with different sampling rate, sparsiy and SNR using Monte Carlo simulations, and compare it with the theoretical results obtained from (1) and (2). The result shows that the our theoretical estimations by (1) and (2) does approximately matches the empirical results. We need to mention that the result of [5] is obtained for Gaussian measurement matrices, simulation results confirm that it is still accurate for a chirp convolution matrix which is used in radar imaging.

References

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