Initial Analysis of SNR / Sampling Rate Constraints in Compressive Sensing based Imaging Radar

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Imaging Radar

Imaging radar is a technology that uses radar to do imaging and remote sensing. The main technology of imaging radar is the synthetic aperture radar (SAR).

Synthetic Aperture Radar

As one of the two main modern remote sensing methods, SAR has been widely used in various fields such as agriculture, forestry monitoring, oceanic observation, topography mapping and military reconnaissance.

Problem Arise

Difficulties of Future Development of Imaging Radar

Modern SAR systems usually come with high resolution and wide swath with various additional functions. Increasing system complexity brings difficulties to the future development of SAR systems.

To solve this problem, there exist many articles that introduce the sparse signal processing theory esp. the compressive sensing (CS) to the radar imaging.

Advantages of CS based Imaging Radar

The CS based imaging radar shows its advantages in several aspects.

- To current radar systems, we can adopt the signal processing method of CS based imaging radar to improve the performance of existing SAR systems, e.g. lower sidelobes and ambiguity, better distinguish ability, etc.
- We can also design optimized CS based imaging radar system to reduce the system complexity and improve the imaging performance.
  In this paper, we mainly study the system parameter constraints during the system design period.

Performance Evaluation

For a sparse reconstruction system, the choice and constraints of several critical parameters, such as *system SNR*, *scene sparsity* and the *down-sampling rate*, is very important. This is the guidance for us to design and implement the system.

For a radar system, the analysis of these parameters is the task of performance evaluation. Performance evaluation is also very important for system design and performance analysis.

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Traditional Performance Evaluation Tools

Radar Equation

$$\mathrm{SNR} = \frac{PG^2\lambda^3\sigma c}{256\pi^3 R^3 KTB_T F_n L_s V \sin\theta} \tag{1}$$

Radar equation is the main evaluating tools in traditional radar theory. It is difficult to be applied under the CS scheme due to totally different signal processing strategies. This makes us in lack of evaluation tools and brings difficulties to system designing and applications in CS based imaging radar.

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Analytic Evaluation Tools

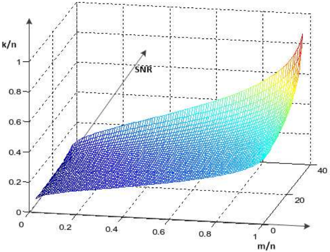
In the compressive sensing theory, we have several tools to evaluate the performance of system measurement matrix.

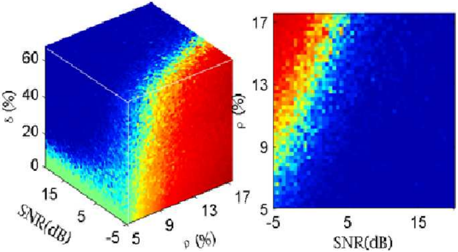
- Restricted isometry property (RIP)
- Exact recovery coefficient (ERC)
- Mutual coherence (MC)
- ...

They are either too inaccurate, or too computational-complex.

Experimental System Performance Evaluation Tools

Phase transit has also been proposed as a candidate of evaluating tool. It is an imperial tool and still costs too much computation (esp. in 3D cases).





B. Zhang, W. Hong, and Y. Wu, "Sparse microwave imaging: Principles and applications," Science China Information Sciences (Series F), vol. 55, no. 8, pp. 1722–1754, 2012.

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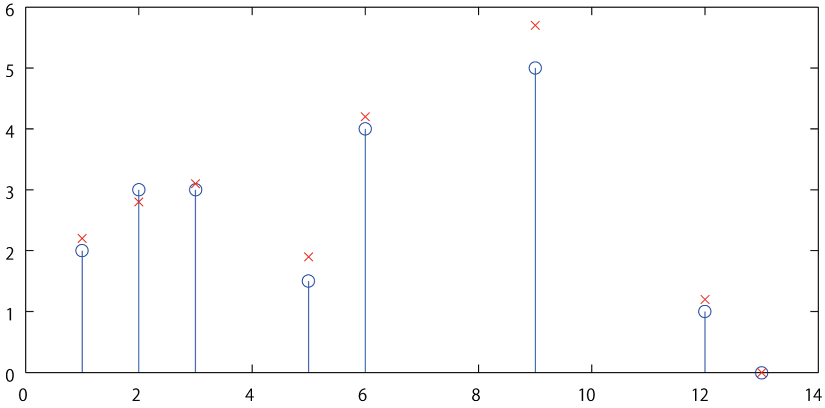
Analytic Expression Based on Sharp Bound

We will introduce a sharp bound analysis in noisy sparse measurement problem by C. Dossal et.al to CS based radar imaging. In the theory of sharp bound, the bounds of minimal SNR and down-sampling rate of an sparse reconstruction problem is achieved.

C. Dossal, M. Chabanol, G. Peyré, and J. Fadili, "Sharp support recovery from noisy random measurements by ℓ_1-minimization," Applied and Computational Harmonic Analysis, no. 10, 2011.

In CS based radar imaging, we treat it as a support recovery problem i.e. the accurate support recovery as the successful reconstruction because of the existence of noise.



Analytic Expression Based on Sharp Bound

Based on the sharp bound theory, the analytic expression of the constraints among SNR, sampling date and sparsity of a radar system is,

$$\mathrm{SNR} \geq \frac{5.5}{\rho\sqrt{1-\alpha}}\sqrt{\frac{2\log(p)}{p}}, \tag{2}$$

$$\rho \geq \frac{2\log(p)k}{\alpha\beta p}, \tag{3}$$

ρ: the down-sampling ratio
p: the size of scene
k: the number of non-zero elements
k/p: the sparsity
α, β: two balance constants

This is a sufficient condition, provides a non-asymptotic expression.

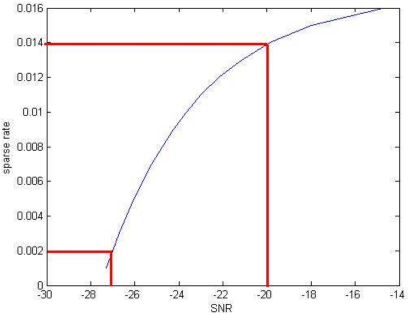
Explanation

From this expression, we can conclude

- For a specific CS based imaging radar system, the required sampling rate is mainly determined by the target scene sparsity.
- The number of sampling $n = \rho p \geq 2k\log(p)/\alpha\beta$ i.e. $n \sim O(k\log(p))$.
- For a specific CS based imaging radar system, the required system SNR is determined by the target scene and the sampling rate.
- SNR is defined as the ratio of backscattering coefficient of a point target to the noise energy in echo.

Explanation

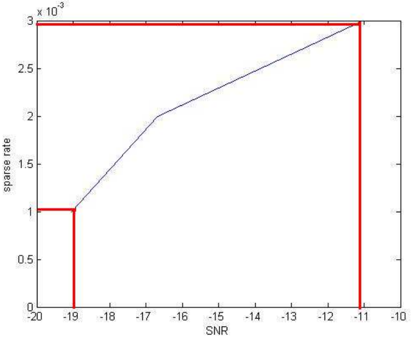
We analysis the SNR-sparsity curve.



- Under-sampling ratio $\rho = 50\%$.
- As SNR increase from -28dB to -20dB, the allowed maximal sparsity increases from 0.1% to 1.4%.

Explanation

We analysis the SNR-sparsity curve.



- Under-sampling ratio $\rho = 10\%$.
- As SNR increase from -19dB to -11dB, the allowed maximal sparsity increases from 0.1% to 0.3%.

Explanation

- The SNR here is quite low because it is defeined in the echo, without any extra gain of signal processing. In fact, we can get extra SNR increment during the signal processing period.
- Under a specific sampling rate, the relationship between SNR and sparsity in non-linear.
- As the sampling rate increases, the same SNR increment allows more sparsity increment.

Explanation

Hence, we can derive some design principles of CS based imaging radar.

1. According to the sparsity of target scene, calculate the required sampling rate based on (3).
2. According to the target scene and required sampling rate, calculate the required system SNR based on (2).
3. According to the sampling rate and SNR, calculate other radar parameters such as PRF, transmitting power and bandwidth based on radar theories.

Explanation

We must mention that the result is obtained for Gaussian measurement matrices. Our simulation results show that it still works for a chirp convolution matrix which is used in radar imaging.

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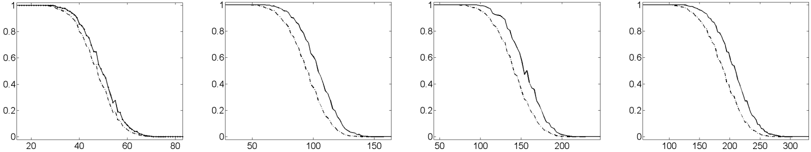
Simulations of Gaussian and Chirp Matrices

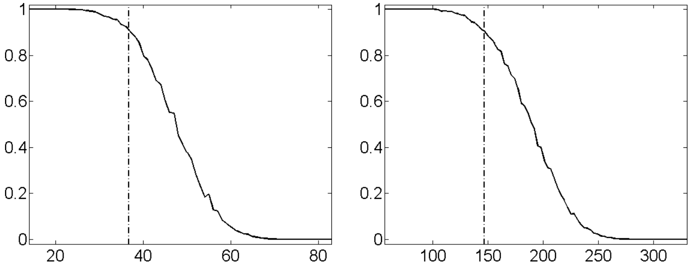


$\rho = 20\%$ $\rho = 40\%$ $\rho = 60\%$ $\rho = 80\%$

Solid line is Gaussian and dash line is chirp, $p = 4000$, $\alpha = 0.8$, $\beta = 0.8$. The simulation shows

- The performance of Gaussian matrix is close to, and better than that of radar convolution matrix.
- When the down-sampling rate (ρ) is low, performance of Gaussian matrix and radar matrix are quite close, and the difference becomes larger when sampling rate is higher.

Simulations



$\rho = 20\%$ $\rho = 80\%$

horizontal axis is the number of non-zero points, and vertical axis is the probability of successful reconstruction.

The simulation shows that the theoretical bound is close to the simulation. The theory is effective.

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Conclusion

- Based on the theory on sharp bound in sparse reconstruction, we investigate a performance evaluation tool of CS based imaging radar. The result shows that the our theoretical estimations does approximately matches the empirical results.
- We need to mention that the result is obtained for Gaussian measurement matrices, simulation results show that it is still works for a chirp convolution matrix which is used in radar imaging.

Conclusion

- To implement the concept of CS based imaging radar, it is necessary to find an performance evaluation tool to analysis the relationship between system SNR, sampling rate, sparsity and reconstruction performance.
- To achieve the best evaluation, we can use both analytic expression and the phase transit diagram, and combine them.
- The analysis of SNR, sparsity and sampling of CS based imaging radar is an important problem, but we are still facing many difficulties.

Thanks!

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