## Azimuth Ambiguity Suppression for SAR Imaging based on Group Sparse Reconstruction

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Abstract—The classical methods for synthetic aperture radar (SAR) azimuth ambiguity suppression are based on deterministic methodologies. However, the reconstruction with azimuth ambiguity problem is underdetermined. In this paper, we present a group sparse modeling based approach in which the reflectivity of target is extended as group sparse signal and its components are jointly recovered by  $\ell_q$  regularization method. The proposed method is verified on airborne SAR data, which shows effectiveness in reducing the ambiguous power while details of the reconstruction are preserved.

The azimuth ambiguity [1] in SAR imaging is an underdetermined problem in mathematical view. The sparse signal processing provides SAR imaging with potentials of achieving better imagery quality [2]. The methodology of group sparse allows the efficient reconstruction of signals whose support is contained in the union of a small number of disjoint groups [3]. We address that the azimuth ambiguity problem can be suppressed by group sparse modeling based approach [4].

In the modeling, we focus on analyzing the properties of the azimuth signal. By proper discretion of the scene, the echoes of the observed scene can be expressed as:

$$y(\eta) = \sum_{n} x[n] \cdot p(\eta; \eta_c(n)), \tag{1}$$

where x[n] is the reflectivity function of the target at the *n*-th discrete grid.  $p(\eta; \eta_c(n))$  is the azimuth signal of point target at azimuth time  $\eta_c(n)$ . In order to represent the ambiguity part,  $p(\eta; \eta_c(n))$  is divided by azimuth time  $f_a/k_a$ , where  $f_a$  is the pulse repetition frequency,  $k_a$  is the azimuth frequency rate. After certain sampling on  $y(\eta)$ , we have

$$y[m] = \sum_{k} \sum_{n} x[n] \exp(-j2\pi k f_a \eta_c(n)) \cdot p_k[m;n], \quad (2)$$

where  $p_k[m;n]$  refers to the *m*-th signal sample of the *n*-th target in its *k*-th time interval, e.g., the signal in  $k = \pm 1$  generate the first left and right azimuth component. Finally, we obtain the group sparse observation model:

$$\mathbf{y} = \sum_{k} \mathbf{\Phi}_{k} \mathbf{x}_{k} + \mathbf{n}, \tag{3}$$

where k-th component of  $\mathbf{x}_k = x[n] \exp(-j2\pi k f_a \eta_c(n))$ , **n** is the additive noise. It can be indicated that the all of the scene  $\mathbf{x}_k$  share the same support. The group sparsity of ambiguous SAR image and the group sparse based  $\ell_q$  regularization reconstruction model:

$$\rho(\mathbf{x}) = \|\sum_{k} \operatorname{diag}(\mathbf{x}_{k}^{t} \mathbf{x}_{k})\|_{q}^{q}, \tag{4}$$

$$\min_{\mathbf{x}} \{ \|\mathbf{y} - \sum_{k} \mathbf{\Phi}_{k} \mathbf{x}_{k}\|_{2}^{2} + \mu \rho(\mathbf{x}) \}.$$
 (5)

The optimization can be achieved efficiently via iterative thresholding algorithm [5] and  $\mathbf{x}_0$  is the final output. The proposed method is verified on airborne SAR data, in which the ambiguity is numerically simulated. The result by the proposed method in Fig. 1a shows that the ambiguity (within the red rectangle) is suppressed quite well compared with the result by range doppler algorithm (RDA) in Fig. 1b.



Fig. 1: Simulation Results. (a)The proposed method. (b) RDA.

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