Autofocus for CS Based ISAR Imaging in the presence of Gapped Data

Elisa Giusti*, Sonia Tomei*, Alessio Bacci*, Marco Martorella* Fabrizio Berizzi*
*Department of Information Engineering, University of Pisa, Via G. Caruso 16, 56022, Pisa, Italy - Telephone: +39(0)502217673
Email: [elisa.giusti, marco.martorella, fabrizio.berizzi] @iet.unipi.it , [sonia.tomei, alessio.bacci]@for.unipi.it

***Abstract*—Compressive sensing (CS) has drawn the attention of the radar community since it can be used to improve radar imaging in ISAR applications. In particular, CS is a powerful tool to process gapped data, when the conventional Fourier based techniques fail. CS can be applied to the gapped data in the slow time domain after target motion compensation. As a consequence, autofocus is a crucial aspect to be considered prior to the application of CS. Two different autofocus algorithms will be introduced and then compared with each other.**

I. Introduction

Inverse Synthetic Aperture Radar (ISAR) is a well known technique that provides high-resolution radar images of moving targets. Since the ISAR image is composed of a number of dominant scatterers that is much smaller than the number of pixels in the image domain, an ISAR signal can be considered sparse in the image domain and it is then suitable for the application of Compressed Sensing (CS).

The advantages of the application of CS to ISAR processing have been investigated in the literature by considering different aspects. It has been demonstrated that the ISAR image reconstruction can be performed by processing data which have been acquired with a sampling rate lower than the Shannon-Nyquist requirement. Resolution enhancement can be obtained in both delay time and Doppler domain as shown in [1]. Finally, the CS can be used to process data that are not complete in the frequency domain [2], [3] and/or in the slow time domain [4]. In all the aforementioned cases, conventional Fourier reconstruction algorithms fail while ISAR reconstruction based on CS (CS-ISAR) provides good results.

In the current literature, it is assumed that the data used to form the image via CS-ISAR processing has already been motion compensated. Such an assumption is quite strong, as the motion compensation is typically an integral part of the ISAR processing. In this paper, the motion compensation problem is tackled in a particular case when CS is needed to obtain high quality ISAR images from gapped data in the slow time domain. In fact, such a data may not be complete over a CPI (Coherent Processing Interval) which is long enough to obtain the desired cross range resolution. In this case, the CS provides a useful tool able to fill the gaps in the slow time domain and get high resolution ISAR images. It is worth pointing out that the target motion estimation for motion compensation is fundamental in order to successfully apply the CS.

In the present paper, two autofocus algorithms based on the maximization of the image contrast will be proposed and then compared against each other. The problem formulation is in Section II, while the theoretical formulation will be provided for both approaches in Section III. Results based on simulations will be shown to assess the effectiveness of one against the other in Section IV.

II. Signal Model

In order to understand the importance of the autofocusing in the ISAR image formation process, the signal model will be given. The system geometry is defined in Figure 1, in which

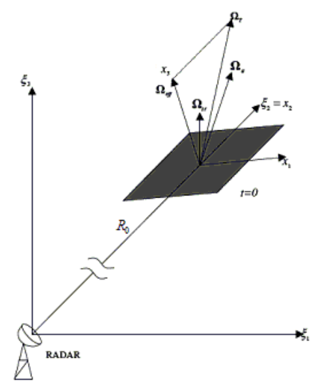

Figure 1. System geometry

$T_x(x_1, x_2, x_3)$ is the Cartesian reference system embedded on the target, $T_\xi(\xi_1, \xi_2, \xi_3)$ is the Cartesian reference system centered on the transmitter and R_0 is the radar-target distance. Ω_{eff} is the effective angular rotation vector which contributes to the ISAR image formation. Under the hypothesis of small variations of the aspect angle, the interactions among the scatterers can be neglected so that the baseband signal at the output of the matched filter, before motion compensation, is given by [5].

$$S\left(f,t\right) = W\left(f,t\right) \sum_{k=1}^{K} \sigma_k e^{-j\frac{4\pi f}{c} R_0(t)} \times e^{-j\frac{4\pi}{c} f \left(x_1^{(k)} cos(\Omega_{eff} t) + x_2^{(k)} sin(\Omega_{eff} t) \right)} \quad (1)$$

where σ_k is the complex amplitude of the k^{th} scatterer. $W(f,t)$ defines the region in the Fourier domain in which the signal is defined. By defining the spatial frequencies as

$$\begin{aligned} X_1(f,t) &= \frac{2f cos(\Omega_{eff}t)}{c} \approx \frac{2f}{c} \\ X_2(f,t) &= \frac{2f_0 sin(\Omega_{eff}t)}{c} \approx \frac{2f_0\Omega_{eff}t}{c} \end{aligned} \tag{2}$$

where the right-hand approximation in (2) holds if the variation of the aspect angle within the observation time, T_{obs}, is small, that is $\Omega_{eff}T_{obs} \ll 1$. If the last relationship holds true, then the signal is defined in a region given by a rectangular grid, so the two-dimensional Fast Fourier Transform (2D-FFT) can be used to form the ISAR image after motion compensation. At this point, the received signal can be rewritten as

$$\begin{aligned} S(f,t) = W(f,t) \sum_{k=1}^{K} \sigma_k e^{-j\frac{4\pi f}{c}R_0(t)} \\ \times e^{-j\frac{4\pi}{c}\left(f x_1^{(k)} + f_0 x_2^{(k)}\Omega_{eff}\right)} \end{aligned} \tag{3}$$

By defining the variables (τ, ν), representing the delay-time and the Doppler frequency respectively,

$$\begin{aligned} \tau &= \frac{2x_1}{c} \\ \nu &= \frac{2f_0\Omega_{eff}x_2}{c} \end{aligned} \tag{4}$$

the received signal at the output of the matched filter in (3) can be rewritten as

$$\begin{aligned} S(f,t) = CW(f,t) \sum_{k=1}^{K} \sigma_k e^{-j2\pi(f\tau_k + t\nu_k)} \\ \times e^{-j\frac{4\pi f}{c}R_0(t)} \end{aligned} \tag{5}$$

where $C = \frac{c^2}{4f_0\Omega_{eff}}$. In a real scenario,the signal domain, (f,t), and the image domain, (τ,ν), are both discrete domains so that the discrete variables can be defined as

$$\begin{aligned} f &= f_0 + m\Delta f & m &= 0,...,N_f - 1 \\ t &= nT_R & n &= 0,...,N_{st} - 1 \\ \nu &= d\Delta_\nu & d &= 0,...,D-1 \\ \tau &= d\Delta_\tau & q &= 0,...,Q-1 \end{aligned}$$

where Δf is the frequency step, T_R is the Pulse Repetition Interval (PRI), $\Delta_\nu = \frac{1}{DT_R}$ is the Doppler frequency resolution and $\Delta_\tau = \frac{1}{Q\Delta f}$ is the delay-time resolution. So, the discrete version of (5) is given by

$$\begin{aligned} S(m,n) = CW(m,n) \sum_{k=1}^{K} \sigma_k e^{-j2\pi\frac{mq_k}{Q}} e^{-j2\pi\frac{nd_k}{D}} \\ \times e^{-j\frac{4\pi m\Delta f}{c}R_0(n)} \end{aligned} \tag{6}$$

where

$$W(m,n) = (u(n) - u(n-N)) \cdot (u(m) - u(m-M)) \tag{7}$$

and $u(\bullet)$ is the unit step discrete function.
As it can be noticed, if the phase term depending on $R_0(t)$ in (6) can be neglected, i.e. the motion compensation is well performed, then the first line of (6) shows that the relation between the signal after motion compensation and the target reflectivity function is given by a 2D-FFT. So, supposing that the motion compensation is done, the first line of (6) can be rewritten in a matricial form as follows

$$\mathbf{S}_c = \mathbf{\Psi}_D \mathbf{I} \mathbf{\Psi}_R^T \tag{8}$$

where $\mathbf{\Psi_D}$ and $\mathbf{\Psi_R}$ are the Fourier matrices which define the ISAR image domain, $\mathbf{S}_c$ is the motion compensated signal and $\mathbf{I}$ is the ISAR image. It is worth pointing out that (8) clearly shows that the ISAR image is a representation of the motion compensated signal in a different domain, defined by the Fourier matrices $\mathbf{\Psi_D}$ and $\mathbf{\Psi_R}$. Secondly, (8) is true in case of complete data. For gapped data in the slow time domain, the basis matrix $\mathbf{\Psi_D}$ is not complete since all the elements on the rows corresponding to the gaps in the slow time are equal to zero. So, a different formulation can be written, in which the matrix $\mathbf{\Theta}_D$ accounts for the sensing operation on the matrix $\mathbf{\Psi}_D$

$$\mathbf{S}_c = \mathbf{\Theta}_D \mathbf{I} \mathbf{\Psi}_R^T \tag{9}$$

It is worth pointing out that in case of gapped data the effectiveness of conventional autofocusing techniques is reduced because of the lack of information in the data.

Referring to (8), the aim of the reconstruction problem is that of recovery the ISAR image $\mathbf{I}$ from the incomplete data $\mathbf{S}_c$. The reconstruction problem is performed via minimization of the $l_0 - norm$ as in (10), which is performed by the 2D-SL0 algorithm [6].

$$\min_{\mathbf{I}} \| \mathbf{I} \|_0 \quad \text{s.t.} \quad \mathbf{S}_c = \mathbf{\Theta}_D \mathbf{I} \mathbf{\Psi}_R^{\mathrm{T}} \tag{10}$$

III. PROPOSED AUTOFOCUSING ALGORITHMS

The algorithms proposed in this paper are variations of the ICBA (Image Contrast Based Algorithm)[5], which is an iterative algorithm that estimates $R_0(t)$ via the maximization of the Image Contrast (IC) (See Fig. 2).

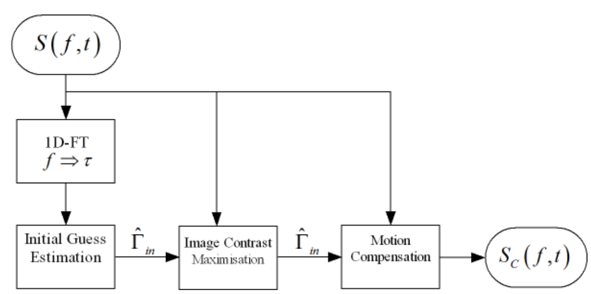

Figure 2. ICBA block diagram

IC is defined as

$$IC(\mathbf{\Gamma}) = \frac{\sqrt{E\left\{\left[|I(\mathbf{\Gamma})| - E\{|I(\mathbf{\Gamma})|\}\right]^2\right\}}}{E\{|I(\mathbf{\Gamma})|\}} \tag{11}$$

where $E\{\bullet\}$ is the spatial mean over the image coordinates, $I(\Gamma)$ is the image evaluated after motion compensation with fixed set of motion parameters $\mathbf{\Gamma}$ and $|I(\Gamma)|$ denotes the image intensity.

A polynomial is used to approximate $R_0(t)$ as in (12)

$$\hat{R}_0(t) = \sum_{p=0}^{P} \gamma_p t^p \tag{12}$$

where P is the polynomial order and it is generally no more than 3. The unknown polynomial coefficients γ_p are estimated by image contrast maximization,

$$\hat{\mathbf{\Gamma}} = \underset{\mathbf{\Gamma}}{argmax}\{IC(\mathbf{\Gamma})\} \tag{13}$$

where $\mathbf{\Gamma} = [\gamma_0, \gamma_1, ..., \gamma_P]$.

When the data is gapped, the conventional 2D-FFT cannot be used to form the image because the data is not complete and it will lead to distortions. In this case, a modified ICBA in which the image formation is performed via CS (CS-ICBA) can be applied, as in (10). In this case, CS reconstruction is performed at each iteration leading to very high computational time.

In order to overcome this issue, two different autofocusing algorithms are proposed in this paper, in which the CS reconstruction is avoided in the motion compensation process.

A. ICBA

This approach consists of applying the conventional ICBA based on 2D-FFT reconstruction to gapped data in the slow time domain. The distortions introduced by the 2D-FFT applied to gapped data can affect the convergence of the IC maximization problem. However, this approach is faster than the CS-ICBA and simulations have shown good results.

B. Multi-window ICBA

This approach is a modification of ICBA applied to multistatic ISAR [7]. In this case, the conventional 2D-FFT reconstruction is applied to each slow time window where the signal is present and the maximization is performed on the product of the contrast values

$$\hat{\mathbf{\Gamma}} = \underset{\mathbf{\Gamma}}{argmax}\{IC_{prod}(\mathbf{\Gamma})\} \tag{14}$$

where

$$IC_{prod}(\Gamma) = \prod_{n=1}^{N} IC_n(\Gamma) \tag{15}$$

and $n = 1, ..., N$ denotes the slow time window. It is worth noticing that on each slow time window, the data is complete and so the application of the conventional 2D-FFT image reconstruction does not lead to distortions.

IV. Simulation results

The ISAR data have been simulated using a target model of an aircraft consisting of 35 scatterers. The system parameters are shown in Table I. The data generated is motion compensated and the gapped data is generated by putting zero in the rows corresponding to the slow time gaps (Fig. 3(a)). The target motion is added after that in order to better evaluate the performance of the proposed algorithms. The target motion is modelled with a second order polynomial

$$R_0(t) = \gamma_0 + \gamma_1 t + \gamma_2 t^2 \tag{16}$$

It is important to note that the proposed algorithms estimate only γ_1 and γ_2 since γ_0 only leads to a constant error in the phase which does not cause distortion in the reconstructed image.

f_0	*Central Frequency*	$10GHz$
B	*Bandwidth*	$300MHz$
T_{obs}	*Observation Time*	$0.8s$
PRF	*Pulse Repetition Frequency*	$158.75Hz$

Table I
System parameters

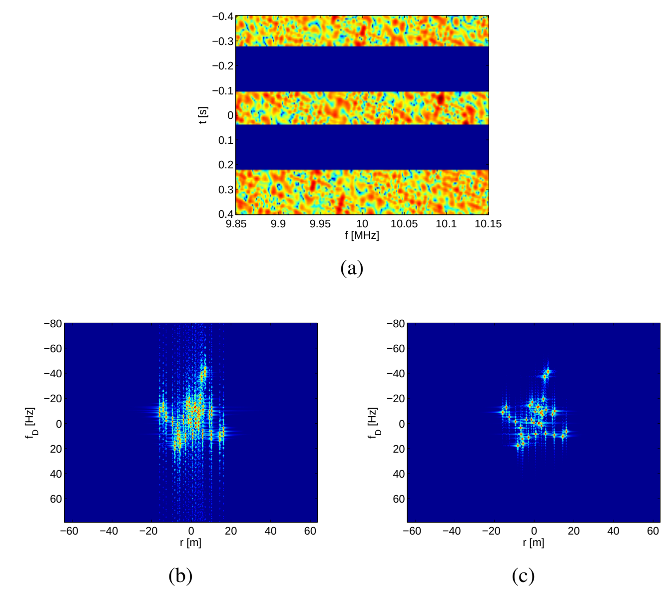

Figure 3. (a) Original gapped data in the slow time domain; Image reconstruction via 2D-FFT (b) and via CS (c)

The image reconstructed via conventional 2D-FFT in Fig. 3(b) is distorted and shows grating lobes due to the slow time gaps in the data. On the other hand, the quality of the image reconstructed via CS in Fig. 3(c) is really good. So the use of CS for image reconstruction seems to be a smart choice, but it is worth pointing out that the motion compensation is a crucial step that must be performed prior the application of CS in order to get good results. In fact, the application of the

CS to data which are not motion compensated leads to highly distorted images, such as in Fig.4.

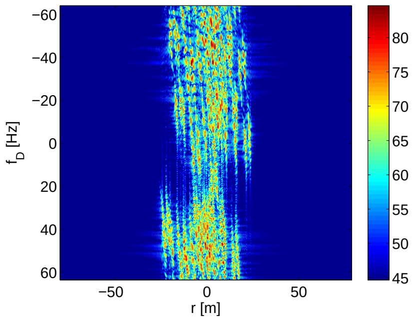

Figure 4. Image reconstruction via CS on non compensated gapped data

Fig.5 shows the results of the image reconstruction via CS after the application of the ICBA to the gapped data while Fig.6 shows the image reconstruction via CS after the application of the Multi-window ICBA to the gapped data. As it can be seen, both the approaches give good results even though it is important to underline that the first approach may not converge to the real maximum in the IC function, as it contains local maxima (Fig. 7(a) and Fig.7(b)). When it happens, the estimated coefficients of $R_0(t)$, that is $\hat{\gamma}_1$ and $\hat{\gamma}_2$, are not correct and the reconstructed image may be distorted.

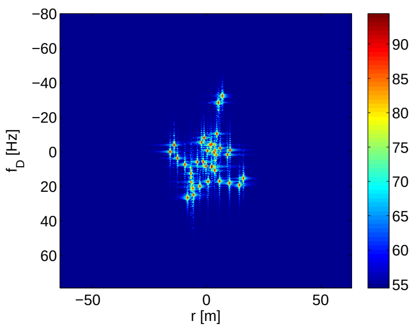

Figure 5. Image reconstruction via CS on gapped data with motion compensation via ICBA

	Real	*Estimated ICBA*	*Estimated via Multi-window ICBA*
γ_1	8	8.1298	8.1265
γ_2	1	1.0006	1.0161

Table II
REAL AND ESTIMATE TARGET VELOCITY (γ_1) AND ACCELERATION (γ_2)

V. CONCLUSIONS

In this paper the application of CS to ISAR gapped data in the slow time domain has been investigated. Specifically, two different algorithms have been proposed to motion compensate gapped data for CS based ISAR applications. Both methods

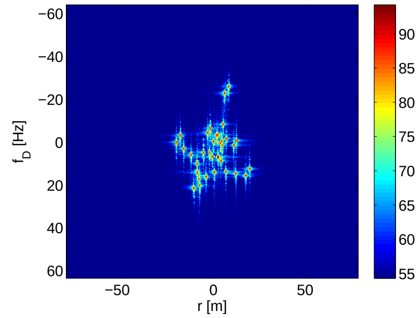

Figure 6. Image reconstruction via CS on gapped data with motion compensation via Multi-window ICBA

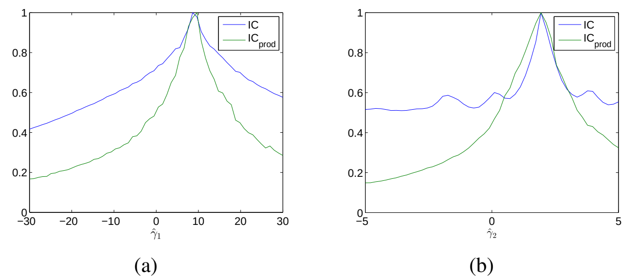

Figure 7. IC vs target velocity ($\hat{\gamma_1}$)(a) and acceleration ($\hat{\gamma_2}$)(b)

are based on variations of the ICBA technique. The effectiveness of the proposed methods has been demonstrated by using simulated data. The algorithms performances have been evaluated by means of error on the target motion parameters estimation.

REFERENCES

[1] L. Zhang, M. Xing, C.-W. Qiu, J. Li, J. Sheng, Y. Li, and Z. Bao, "Resolution enhancement for Inversed Synthetic Aperture Radar imaging under low snr via improved compressive sensing," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 48, no. 10, pp. 3824–3838, 2010.

[2] L. Zhang, Z.-J. Qiao, M. Xing, Y. Li, and Z. Bao, "High-resolution ISAR imaging with sparse stepped-frequency waveforms," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 49, no. 11, pp. 4630–4651, 2011.

[3] W. Qiu, E. Giusti, A. Bacci, M. Martorellla, F. Berizzi, H. Z. Zhao, and Q. Fu, "Compressive sensing for passive ISAR with DVB-T signal," in *Proc. of the International Radar Symposium 2013*, (Dresden, Germany), 2013.

[4] L. Zhang, Z. J. Qiao, M. D. Xing, J. L. Sheng, R. Guo, and Z. Bao, "High-resolution ISAR imaging by exploiting sparse apertures," *Antennas and Propagation, IEEE Transactions on*, vol. 60, no. 2, pp. 997–1008, 2012.

[5] M. Martorella, F. Berizzi, and B. Haywood, "Contrast maximisation based technique for 2-D ISAR autofocusing," *Radar, Sonar and Navigation, IEE Proceedings -*, vol. 152, no. 4, pp. 253–262, 2005.

[6] A. Ghaffari, M. Babaie-Zadeh, and C. Jutten, "Sparse decomposition of two dimensional signals," in *Acoustics, Speech and Signal Processing, 2009. ICASSP 2009. IEEE International Conference on*, pp. 3157–3160, 2009.

[7] S. Brisken, M. Martorella, T. Mathy, C. Wasserzier, and E. Giusti, "Multistatic isar autofocussing using image contrast optimization," in *Radar Systems (Radar 2012), IET International Conference on*, pp. 1–4, 2012.