Sparse Delay-Doppler Image Reconstruction under Off-Grid Problem

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Pulse-Doppler radar has been successfully applied to surveillance and tracking of both moving and stationary targets. For efficient processing of radar returns, delay-Doppler plane is discretized and FFT techniques are employed to compute matched filter output on this discrete grid. However, for targets whose delay-Doppler values do not coincide with the computation grid, the detection performance degrades considerably. Especially for detecting strong and closely spaced targets this causes miss detections and false alarms. Although compressive sensing based techniques provide sparse and high resolution results at sub-Nyquist sampling rates, straightforward application of these techniques is significantly more sensitive to the off-grid problem. Here a novel and OMP based sparse reconstruction technique with parameter perturbation, named as PPOMP, is proposed for robust delay-Doppler radar processing even under the off-grid case. In the proposed technique, the selected dictionary parameters are perturbed towards directions to decrease the orthogonal residual norm.

In compressive sensing (CS) formulation, a sampled version of the measurement is adapted to a linear matrix-vector relationship in delay-Doppler domain with parameter discritization:

$$\boldsymbol{y}_s = \boldsymbol{\Phi}\,\boldsymbol{\Psi}\,\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

In 1, each column of $\boldsymbol{\Psi}$ corresponds to a specific (τ, ν) pair and $\boldsymbol{\Phi}$ is the sensing matrix. In the following, dictionary and sensing matrix will be considered together as the basis $\boldsymbol{A}$. Standard CS problem is to find a sparse $\boldsymbol{x}$, given $\boldsymbol{A}$ and $\boldsymbol{y}_s$.

In general, a target with parameters (τ_T, ν_T) may not be located at the grid node but is positioned within the grid area with an unknown perturbation from the grid node. Our goal is to perturb the grid parameters and hence the column vectors in $\boldsymbol{A}$, so that a better fit to the measurements can be accomplished. This goal can be formulated as the following optimization problem:

$$\min_{\alpha_i, \delta\tau_i, \delta\nu_i} \left\| \boldsymbol{b} - \sum_{i=1}^{k} \alpha_i\, \boldsymbol{a}(\tau_i + \delta\tau_i, \nu_i + \delta\nu_i) \right\|_2 \quad s.t. \quad |\delta\tau_i| < \Delta_\tau/2, \quad |\delta\nu_i| < \Delta_\nu/2. \tag{2}$$

Solution of the problem in (2) provides the perturbation parameters $(\delta\tau_i, \delta\nu_i)$ and the representation coefficients α_i for the k^{th} iteration of the OMP. Since the required optimization is non-convex, here we propose to use a gradient descent optimization of the cost function. Therefore starting from the grid nodes, the columns of $\boldsymbol{A}$ will be gradually perturbed until a convergence criteria is met. To simplify the iterations further, α_i's and $(\delta\tau_i, \delta\nu_i)$'s will be sequentially updated in the following way. First initialize $\tau_{i,1} = \tau_i$, and $\nu_{i,1} = \nu_i$, $i = 1, \ldots, k$, to grid centers and obtain an initial representation coefficient vector $\boldsymbol{\alpha}_1$ with the least-squares solution on the parameter points. Starting from $l = 1$ until convergence, perform updates $\tau_{i,l+1} = \tau_{i,l} + \delta\tau_{i,l}$ and $\nu_{i,l+1} = \nu_{i,l} + \delta\nu_{i,l}$ where

$$\{\delta\tau_{i,l}, \delta\nu_{i,l}\} = \arg\min_{\delta\tau_i, \delta\nu_i} \left\| \boldsymbol{b} - \sum_{i=1}^{k} \alpha_{i,l}\, \boldsymbol{a}(\tau_{i,l} + \delta\tau_i, \nu_{i,l} + \delta\nu_i) \right\|_2$$

However obtaining solution to the above constrained nonlinear optimization problem is not practical for radar applications. Gradient descend based perturbation of the parameters can be utilized for the solution. Linearization of the above cost function around $(\tau_{i,l}, \nu_{i,l})$ significantly reduces the complexity of the optimization. Calculation of the gradient direction requires a matrix vector multiplication which can be performed significantly faster than solving nonlinear least squares problem.

Figure 1 shows the convergence of the gradient based iterations to the actual parameter points. Note that the separation of these two targets is closer than a grid size corresponding to the classical Rayleigh resolution limit both in delay and Doppler axis. While a matched filter won't be able to resolve these two targets, the proposed PPOMP technique could identify their actual positions accurately.

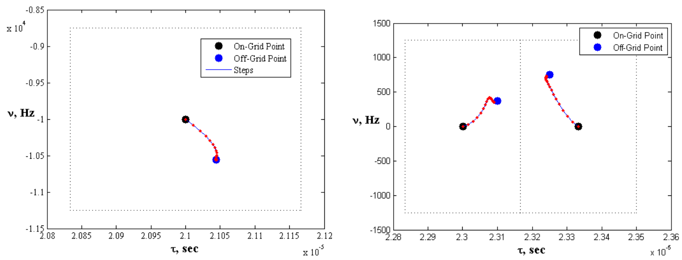

Figure 1: Gradient based steps taken within the PPOMP.

In order to measure the performance of the proposed technique, we treat the given radar scene as a 2-D Gaussian Mixture Model with delay-Doppler parameters being the means of the Gaussians and the normalized reflection coefficients being the mixture weights. By this way, it is possible to compute the Kullback-Leibler Divergence(KLD) between two given radar scene, namely the actual and the reconstructed one.

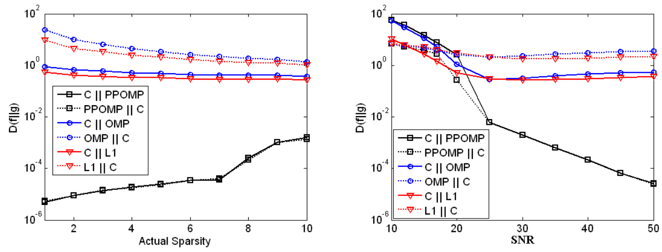

Figure 2: Kullback-Liebler Divergences between the correct and reconstructed target scenes.

Compared to the standard OMP technique, proposed method provides significantly lower errors for a wide range of sparsity levels and SNRs. Furthermore, due to the lower complexity of its implementation, PPOMP technique is more feasible in radar applications than the convex optimization based techniques.