On the Robustness of Bayesian Compressive Sensing for Directions-of-Arrival Estimation

Paolo Rocca, Giacomo Oliveri, and Andrea Massa
ELEDIA Research Center
DISI, University of Trento
Via Sommarive 5, 38123, Povo, Trento
andrea.massa@ing.unitn.it

***Abstract*—Compressive Sensing (CS) has demonstrated to be particularly adapt for dealing with the directions-of-arrival (DoAs) estimation of electromagnetic signals impinging on an array of sensors. Unlike deterministic CS methods, the Bayesian CS (BCS) allows to overcome some theoretical limitations of the CS, such to enable a reliable and versatile DoAs estimation tool able to work with different array, noise and signals configurations.**

Keywords-Bayesian Compressive Sensing; directions-of-arrival estimation; planar antenna array

I. INTRODUCTION

Compressive Sensing methodologies have been widely used with prominent results in several electromagnetic applications. Among them, it is worth mentioning the microwave imaging [1-5], antenna arrays synthesis and design [6-8], array diagnosis [9, 10], and directions-of-arrival estimation [11-14].

CS strategies can be used for signal recovery when the relationship between the data and the unknowns is linear and the signal is sparse or can be made sparse according to a representation with suitable basis functions. In the specific case of the DoAs estimation, the problem is intrinsically sparse since only a few angular directions, amongst the many that can be considered through a fine discretization of the observation angular domain, are characterized by an incoming signal. On the other hand, the problem is clearly non-linear because the unknown DoAs are embedded within the exponentials of the so-called steering vectors [15]. To avoid this problem, CS methods are aimed at recovering the sparse signal vector whose non-zero entries are then associated to the corresponding steering vectors such to determine the unknown DoAs.

Thanks to such a reformulation of the original problem, several approaches based on deterministic CS have been proposed [11, 12] and effectively compared with state-of-the art approaches. More recently, a Bayesian version of the CS, namely the Bayesian CS, has been introduced [16] and suitably customized to deal with DoAs estimation problems [13, 14]. Unlike deterministic CS-based approaches that are aimed at minimizing the l_1-norm of the signal vector while forcing the data fitting though a l_2-norm constraint, in the BCS the problem is formulated as the retrieval of the maximally sparse solution that maximizes the a-posteriori probability of fitting the acquired data samples [16]. Although both CS and BCS can recover the DoAs within a single snapshot (i.e., with the voltages measured at the output of the receiving array elements at a single time instant), the precision and reliability of the results are strongly related to the condition of the scenario at hand (e.g., power of the signal with respect to the environmental noise, number of signals with respect to number of sensors).

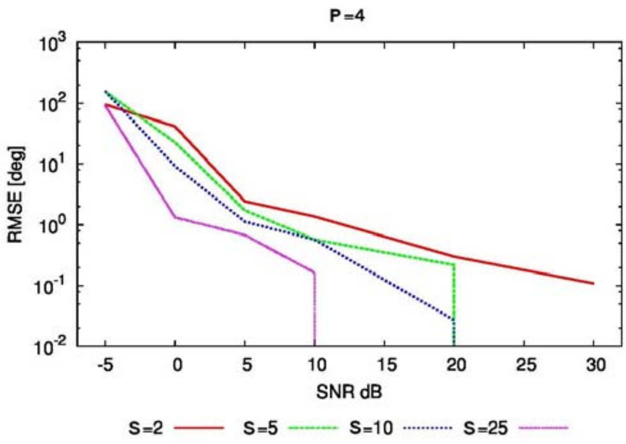
To cope with these problems, a different Bayesian CS approach has been proposed, namely the multi-task BCS (MT-BCS) [17], which is able to deal with sparse problems whose solutions are correlated. The potentialities of such a method have been investigated in the framework of the DoAs estimation problems in [13, 14] where the retrieval of the signal directions have been carried out exploiting the information acquired at multiple consecutive snapshots. Thanks to use of more data, the MT-BCS has shown being more robust than the single-snapshot BCS approach and able to address complex problems characterized by challenging electromagnetic scenarios with low signal-to-noise ratio (SNR) or large number of signals arriving on the antenna. The price to pay is in this case the velocity of the DoAs retrieval. In occasion of the conference, the last advances at the ELEDIA Research Center on the MT-BCS for the robust DoAs estimation will be presented and discussed.

II. MATHEMATICAL FORMULATION

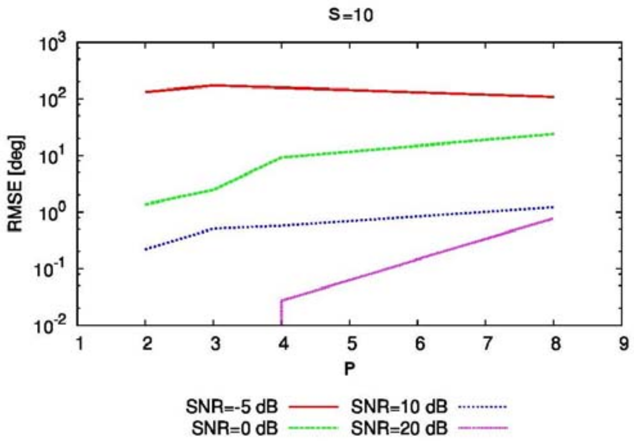
Let us consider an antenna array made of M sensors. A set of P narrow-band plane waves are impinging on the array from directions ϑ_p , $p = 1,...,P$. From a mathematical viewpoint, the relation between the measurable data (i.e., the open circuit voltages at the sensor output, $\underline{d} = \{d_m, m = 1,..., M\}$) and the unknown DoAs at the *s*-th snapshot of acquisition is [13]

$$\underline{d}^{(s)} = \underline{\underline{\Lambda}}(\vartheta)\,\underline{f}^{(s)} + \underline{\eta}^{(s)} \qquad (1)$$

where $\underline{\underline{\Lambda}}(\vartheta)$ and $\underline{f}^{(s)}$ are the matrix of the steering vectors and the signal vector [15], respectively. Moreover, $\underline{\eta}^{(s)}$ is the vector of the noise, supposed in this case additive and having Gaussian distribution with zero mean.



(*a*)



(*b*)

Figure 1. Behavior of the RMSE (*a*) versus the SNR with *P*=4 signals and *S*={2, 5, 10, 25} snapshots and (*b*) versus *P* with *S*=10 snapshots and SNR={-5, 0, 10, 20}*dB*.

The DoAs estimation is carried out by means of the MT-BCS by discretizing the angular domain with a fine grid having a number of samples much larger than P such that $\underline{f}^{(s)}$ turns out being sparse. The solution of the MT-BCS approach is given by the following expression [13]

$$\hat{\underline{f}} = \frac{1}{S}\sum_{s=1}^{S}\left\{\left[\underline{\underline{\Delta}}^T(\vartheta)\underline{\underline{\Delta}}(\vartheta) + diag(\underline{h})\right]^{-1} + \underline{\underline{\Delta}}^T(\vartheta)\underline{d}^{(s)}\right\} \quad (2)$$

where $\underline{h}$ is the so-called hyper-parameter vector used to statistically-correlate (i.e., $\underline{h}^{(s)} = \underline{h}, \forall s$) the probability distribution of the solutions amongst the different snapshots [17]. Since many coefficients of $\hat{\underline{f}}$ are not exactly zero because of the presence of the noise, the estimated DoAs $\hat{\vartheta}_p$, $p = 1,...,P$ are those of the steering vectors whose corresponding entries of the estimated signal vector $\hat{\underline{f}}$ have values not close to zero.

III. Numerical Results

As representative results, Figure 1 shows the results of the MT-BCS when using a $M = 5\times 5 = 25$ array with half-wavelength-spaced isotropic sensors. Supposing *P*=4, the root-mean square error (RMSE) versus the SNR is reported in Fig. 1(*a*) when varying the number of available snapshots *P*. In Fig. 1(*b*), the analysis is performed versus *P* for different SNR values while fixing *S*=10.

References

[1] G. Oliveri, *et al.*, “A bayesian compressive sampling-based inversion for imaging sparse scatterers,” *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 10, pp. 3993-4006, Oct. 2011.

[2] L. Poli, *et al.*, “Microwave imaging within the first-order Born approximation by means of the contrast-field Bayesian compressive sensing,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 6, pp. 2865-2879, Jun. 2012.

[3] L. Poli, *et al.*, “Imaging sparse metallic cylinders through a Local Shape Function Bayesian Compressive Sensing approach,” *J. Opt. Soc. America A*, vol. 30, no. 6, pp. 1261-1272, 2013.

[4] L. Poli, *et al.*, “Bayesian compressive sensing approaches for the reconstruction of two-dimensional sparse scatterers under TE illumination,” *IEEE Trans. Geosci. Remote Sens.*, vol. 51, no. 5, pp. 2920-2936, May. 2013.

[5] L. Poli, *et al.*, "MT–BCS-based microwave imaging approach through minimum-Norm current expansion," *IEEE Trans. Antennas Propag.*, vol. 61, no. 9, pp. 4722-4732, Sep. 2013.

[6] G. Oliveri and A. Massa, “Bayesian compressive sampling for pattern synthesis with maximally sparse non-uniform linear arrays,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 2, pp. 467-481, Feb. 2011.

[7] G. Oliveri, *et al.*, “Complex-weight sparse linear array synthesis by Bayesian Compressive Sampling,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 5, pp. 2309-2326, May 2012.

[8] F. Viani, *et al.*, “Compressive Sensing pattern matching techniques for synthesizing planar sparse arrays,” *IEEE Trans. Antennas Propag.*, vol. 61, no. 9, pp. 4577-4587, Sep. 2013.

[9] M. D. Migliore, “A compressed sensing approach for array diagnosis from a small set of near-field measurements,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 2127–2133, Jun. 2011.

[10] G. Oliveri, *et al.*, “Reliable diagnosis of large linear arrays - A Bayesian compressive sensing approach,” *IEEE Trans. Antennas Propag.*, vol. 60, no. 10, pp. 4627-4636, Oct. 2012.

[11] D. Malioutov, *et al.*, “A sparse signal reconstruction perspective for source localization with sensor arrays,” *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010-3022, Aug. 2005.

[12] A. C. Gubruz, *et al.*, “Bearing estimation via spatial sparsity using compressive sensing,” *IEEE Trans. Aerospace Electron. Syst.*, vol. 48, no. 2, pp. 1358-1369, Apr. 2012.

[13] M. Carlin, *et al.*, “Directions-of-arrival estimation through Bayesian compressive sensing strategies,” *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3828-3838, Jul. 2013.

[14] M. Carlin, *et al.*, “Bayesian Compressive Sensing as applied to directions-of-arrival estimation in planar arrays,” *J. Electrical Computer Eng.*, vol. 2013, pp. 1-12, 2013.

[15] Z. Chen, *et al.*, *Introduction to Direction-Of-Arrival Estimation*. Norwood, MA: Artech House, 2010.

[16] S. Ji, *et al.*, “Bayesian compressive sensing,” *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2346-2356, Jun. 2008.

[17] S. Ji, *et al.*, “Multi-task compressive sampling,” *IEEE Trans. Signal Process.*, vol. 57, no. 1, pp. 92–106, Jan. 2009.