Bayesian compressive sensing based blind DOA estimation for multiple antennas*

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Abstract: *This paper discusses the direction of arrival (DOA) estimation problem for multiple antennas. A spatial domain compression scheme is proposed to compress the redundant signal of the received antennas array, where a Bernoulli distribution random weight matrix instead of a Gaussian matrix is acted as the measurement matrix. An angle sparse model is introduced to our scheme. The formulated recovery problem is then solved based on the convex programming using sparse Bayesian learning (SBL), which do not need the number of the sources. The proposed scheme is very applicable where the antenna receiver array is very large. With less data, the proposed scheme can provide super DOA estimation performance. Simulation results verify the usefulness of our scheme.*

Keywords: *Blind DOA estimation; Bayesian compressive sensing; sparse bayesian learning.*

1. Introduction

Direction of arrival (DOA) estimation is a fundamental task in radar detection that has been recently investigated in [1, 2]. Related theory points out that the numbers of the antenna array is proportional to the degrees of detection, the additional degrees of freedom are well-qualified for overcoming fading effect, enhancing spatial resolution, strengthening parameter identifiability and also improving target detection performance which has been applied in phased array radar and multi-in multi-out (MIMO) radar. However, the additional antenna channel also means that more signal sampling, transition, storage and processing unit are required , which would be a huge challenge to traditional radar system.

In radar detection, targets can be considered highly sparse in the background scene, which is consistent to the compressed sensing (CS) theory. CS theory pointed out that if the signal is compressible or sparse in a transform domain, then it can be recovered exactly with high probability from fewer measurements via l1-norm optimization[3]. The CS radar system can achieve superior spatial resolution as compared to traditional radar system thus caused much attention[4–9]. The CS based DOA estimation methods have been proposed in both time domain[6, 7] and spatial domain[8, 9].

In this paper, we present a general scene for array receiving antenna in which reducing system complexity considerably while keeping good performance of estimating target parameters. The array signal is compressed with a Bernoulli distribution random weight matrix, which is circuit feasibility to the radar system. An angle sparse model is introduced to the compressed signal, with the angle information of the targets are reconstructed through sparse Bayesian learning (SBL) algorithm, of which the global minimum is at the sparsest solution to the recovery problem[10, 11]. Simulation result shows that the proposed approach can accomplish super-resolution by using far fewer samples than existing methods.

The rest of the paper is organized as follows. In section 2, we present the signal model for the proposed scheme. In section 3, we present the DOA estimation method with SBL. Simulation results are given in section 4. Finally, we provide concluding remarks in section 5.

2. Signal model

2.1. System architecture

Consider a general antennas array model, as shown in Figure 1. Suppose that a uniform linear antenna array consisting of N elements, the distance of the adjacent elements is no more than half of the received wavelength. It is also assumed that there are K non-coherent targets appear in the far-field of receive arrays and the k th target is at azimuth angle θ_k . The receiving signal matrix $\mathbf{X}(t) = [x_1(t); x_2(t); \ldots; x_N(t)]$ takes the form

$$\mathbf{X}(t) = \mathbf{A}(\theta)\mathbf{S}^*(t) + \mathbf{V}^*(t) \tag{1}$$

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where $x_i(t)$ is the ith antenna receive signal; $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)]$ is the direction matrix; $\mathbf{a}(\theta_k) = [1, e^{j\omega}, \ldots, e^{j(N-1)\omega}]^T$ is the receive steering vector for θ_k , where $\omega = -2\pi d sin\theta_k/\lambda$, and λ is the wave length; $\mathbf{S}^*(t) = [s_1^*(t); s_2^*(t); \ldots; s_K^*(t)]$ is the source matrix, $s_k^*(t)$ is the transmitting signal of the source k ; $\mathbf{V}^*(t)$ is the received additive white Gaussian noise.

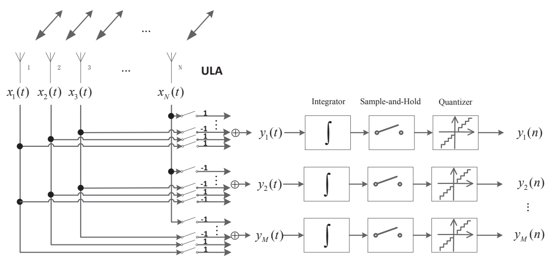


Figure 1. Receive signal model

The received signal of the antenna array is redundant, the high-dimensional signal is really challenges our system. The CS theory inspires that a $(-1, 1)$ distributed $M \times N$ Bernoulli random weight matrix $\mathbf{\Phi}$ been applied to the received signal, and the measured signal $\mathbf{Y}(t) = [y_1(t); y_2(t); \ldots; y_M(t)]$ is

$$\mathbf{Y}(t) = \mathbf{\Phi}\mathbf{X}(t) = \mathbf{\Phi}\mathbf{A}(\theta)\mathbf{S}^*(t) + \mathbf{V}(t) \tag{2}$$

with $V(t)$ denotes the measured noise matrix.

2.2. Sparse represention

Let Y be the discrete-time waveform of $Y(t)$, by discretizing the angle space as $\mathbf{B}(\theta) = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \ldots, \mathbf{b}(\theta_P)]$ and $P \gg K$, (2) can be written as

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{B}(\theta)\mathbf{S} + \mathbf{N} = \mathbf{\Theta}\mathbf{S} + \mathbf{V} \tag{3}$$

where $\mathbf{\Theta} = \mathbf{\Phi}\mathbf{B}(\theta)$ is the sense matrix, $\mathbf{S} = [\mathbf{s}_1(n); \mathbf{s}_2(n); \ldots, \mathbf{s}_P(n)]$ where

$$\mathbf{s}_p(n) = \begin{cases} \mathbf{s}_k^*(n) & \text{if } b(\theta_p) = a(\theta_k) \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{4}$$

Suppose that the received noise $\mathbf{N}$ is incoherent with $\mathbf{\Theta}\mathbf{S}$. As the steering vector $\mathbf{b}(\theta_p)$ can be regard as discrete single-frequency sinusoidal signal with frequency $\omega = -2\pi d sin\theta_p/\lambda$, the base matrix $\mathbf{B}(\theta)$ can be considered as a Fourier matrix, the incoherent measurement matrix $\mathbf{\Phi}$ ensures the sense matrix $\mathbf{\Theta}$ satisfy the restricted isometry property (RIP)[3].

2.3. Blind DOA estimation method

The the covariance matrix $\mathbf{R}$ can be expressed as

$$\mathbf{R} = \mathbf{\Theta}\mathbf{S}\mathbf{S}^H\mathbf{\Theta}^H + \mathbf{V}\mathbf{V}^H = \mathbf{\Theta}\mathbf{\Lambda}\mathbf{\Theta}^H + \sigma\mathbf{I} \tag{5}$$

which can be estimated with L snapshots by $\overset{\wedge}{\mathbf{R}}_y = \frac{1}{L}\sum_{l=1}^{L}\mathbf{Y}(l)\mathbf{Y}^H(l)$ .Define the vectorization operation $\boldsymbol{vec}$ as $\boldsymbol{vec}([\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_N}]) = [\mathbf{r_1}; \mathbf{r_2}; \ldots; \mathbf{r_N}]$,which rearranges the matrix by column vector, where $\mathbf{r_i}$ is a column vector. Rearrange the covariance matrix

$$\boldsymbol{vec}(\mathbf{R}) = \boldsymbol{vec}(\mathbf{\Theta}\mathbf{\Lambda}\mathbf{\Theta}^H) + \boldsymbol{vec}(\sigma\mathbf{I}) \tag{6}$$

with the property $\boldsymbol{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\boldsymbol{vec}(\mathbf{B})$ where $\otimes$ represents the Kronecker product, rewrite (6) as

$$\boldsymbol{vec}(\mathbf{R}) = (\mathbf{\Theta}^* \otimes \mathbf{\Theta})\boldsymbol{vec}(\mathbf{\Lambda}) + \boldsymbol{vec}(\sigma\mathbf{I}) \tag{7}$$

where $\boldsymbol{vec}(\mathbf{\Lambda})$ is a sparse vector with K nonzero elements corresponding to the Kronecker product of steering vector in $(\mathbf{\Theta}^* \otimes \mathbf{\Theta})$. It is indicate that there is a $P^2 \times P$ selection matrix $\mathbf{Q}$ and a $P \times 1$ sparse vector $\mathbf{w}$ fulfills $\boldsymbol{vec}(\mathbf{\Lambda}) = \mathbf{Qr}$, where the elements in $\mathbf{w}$ represents the elements on the diagonal of $\mathbf{\Lambda}$.

With the property $(\mathbf{A} \otimes \mathbf{B})\mathbf{U} = \mathbf{A} \circ \mathbf{B}$, where $\mathbf{U}$ is a selection matrix and $\circ$ stands for the Khatri-Rao product of the matrix, which equals to the Kronecker product by column. (7) can be rewrite as

$$\boldsymbol{vec}(\mathbf{R}) = (\mathbf{\Theta}^* \circ \mathbf{\Theta})\mathbf{w} + \boldsymbol{vec}(\sigma\mathbf{I}) = \mathbf{\Psi}\mathbf{w} + \mathbf{n} \tag{8}$$

3 The SBL based recovery method

3.1. The sparse recovery problem

Let $\mathbf{y} = \boldsymbol{vec}(\mathbf{R}) = \mathbf{\Psi}\mathbf{w} + \mathbf{n}$, our object is recovery $\mathbf{w}$ from $\mathbf{y}$. The nonzero location in $\mathbf{w}$ stands for the DOA of the source, the DOA estimation problem from (8) can be formulated as the convex problem

$$\overset{\wedge}{\mathbf{w}} = arg\ min \| \mathbf{w} \|_0 , \ \ s.t. \ \mathbf{y} = \mathbf{\Psi}\mathbf{w} + \mathbf{n} \tag{9}$$

the operation $\|\mathbf{w}\|_0$ denote the l_0-norm of the sparse vector $\mathbf{w}$. In general, this problem is NP-hard. It has been proved that the l_0-norm is equal to the l_1-norm under certain conditions. Usually a relaxed optimization problem via constraint weighted l_1-norm replacing (9). The optimization function can be rewritten as

$$\overset{\wedge}{\mathbf{w}} = arg\ min \| \mathbf{y} - \mathbf{\Psi}\mathbf{w} \|_2^2 + \| \mathbf{w} \|_1 \tag{10}$$

,where $\| \mathbf{w} \|_1$ denote the l_1-norm of the vector $\mathbf{w}$ and η is a tradeoff parameter balancing estimation quality.

3.2. SBL recovery

The sparse Bayesian framework assumes an independent zero-mean Gaussian noise with variance σ^2 , the probability density function (PDF) of the vector $\mathbf{y}$ is

$$p(\mathbf{y}|\mathbf{w},\sigma^2) = (2\pi\sigma^2)^{-M/2} exp\left\{ \frac{\|\mathbf{y}-\mathbf{\Psi}\mathbf{w}\|_2^2}{2\sigma^2} \right\} \quad (11)$$

The prior over the parameters supposed subject to a Gaussian prior

$$p(\mathbf{w}|\boldsymbol{\alpha}) = (2\pi)^{-P/2} \prod_{p=1}^{P} \alpha_p^{1/2} exp\left\{ -\frac{\alpha_p w_p^2}{2} \right\} \quad (12)$$

where the key to model the sparsity is the use of the P independent hyperparameters $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_P]^T$, which moderate the strength of the prior. The posterior parameter distribution is also Gaussian after given $\boldsymbol{\alpha}$, and $p(\mathbf{w}|\mathbf{y},\boldsymbol{\alpha}) = N(\mathbf{w}|\boldsymbol{\mu},\mathbf{\Sigma})$ with the mean given by

$$\boldsymbol{\mu} = \sigma^{-2}\mathbf{\Sigma}\mathbf{\Psi}^H\mathbf{y} \quad (13)$$

and the covariance

$$\mathbf{\Sigma} = (\mathbf{\Gamma} + \sigma^{-2}\mathbf{\Psi}\mathbf{\Psi}^H)^{-1} \quad (14)$$

where $\mathbf{\Gamma} = diag(\boldsymbol{\alpha})$.

If we initially set $\boldsymbol{\alpha}$ to a given non-zero vector, with the iteration of the algorithm, which along with (13) and (14), many parameters α will be driven to infinity while the α corresponding to the DOA of the targets retain relatively small.

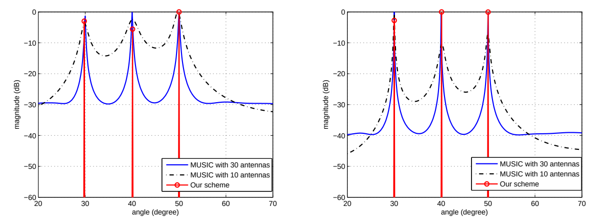
3.3. Scheme analysis

If all the signal $\mathbf{X}(t)$ have been sampled, there requires at least N analog-to-digital converters (ADC) and $N \times M$ data storage unit, while our scheme only requires M ADC and $M \times L$ data storage unit. As all the data $\mathbf{X}_{N\times L}$ can be restored from $\mathbf{Y}_{M\times L}$, the effective aperture of the antenna in our scheme is N instead of M , which means that our scheme keeps the effective aperture of the antenna as well as reduce the number of the sampled data. And our spatial domain compressed scheme retains space for improving the estimation performance with traditional method, such as signal accumulation in time domain, changes compression ratio adaptively to different application scenarios, etc.

The cost function can be globally minimized using a variety of optimization algorithms, such as OMP, FOCUSS and SBL, among which Bayesian algorithms generally achieve the best recovery performance. SBL is one important family of Bayesian algorithms, it can be considered as an l_1-weighted algorithm. The maximum a posteriori probability criterion helps SBL to pursuit the optimal regularization parameter, thus adaptively reach the best performance, especially faced with strong correlation of the atoms.

4 Simulation results

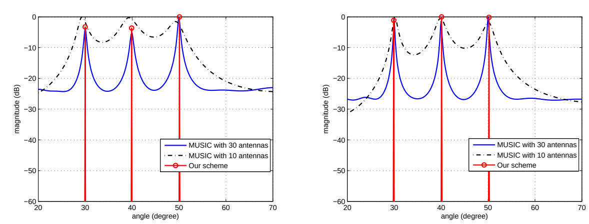
Simulation conditions: uniform linear array with total antenna $N = 30$, compressing rate $r = \frac{M}{N} = \frac{10}{30} = 0.33$, K sources located at angles $30°, 40°, 50°$ separately, the snapshot is 200, and the signal power are all 1.



(a) $SNR = 10dB$ (b) $SNR = 20dB$

Figure 2. Simulation1 result

Simulation 1: The estimation results of our scheme with $SNR = 10dB$ and $20dB$ separately. Figure 2 shows the angle estimation results of our scheme compared with 10 and 30 antennas MUSIC algorithms. It can be seen that our scheme performs as well as the uncompressed MUSIC scheme.

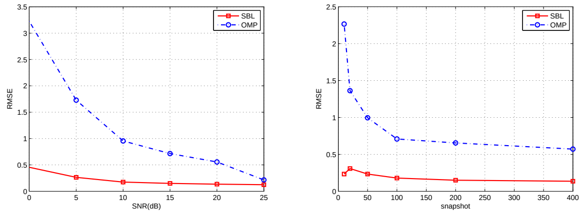


(a) $L = 10$ (b) $L = 40$

Figure 3. Simulation2 result

Simulation 2: The estimation results of our scheme with snapshot $L = 10$ and $L = 40$ separately, $SNR = 15dB$. Figure 3 shows the angle estimation results of our scheme compared with 10 and 30 antennas MUSIC. From which proves that our scheme perform well in the condition of smaller snapshot.

Simulation 3: The performance of our scheme with different SNR and snapshot. Define root mean squared error (RMSE) as $\frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{T}\sum_{i=1}^{T}(\hat{\theta}_{i,k} - \theta_k)^2}$, where T is the number that count successful recovery K targets in 1000 Monte Carlo simulations. $\hat{\theta}_{i,k}$ is the estimate DOA of $\theta_{i,k}$. Figure 4 shows the RMSE of the SBL and OMP algorithm with different SNR and snapshot. From which we can see that the selected SBL algorithm perform much good than the OMP algorithm.



(a) RMSE with differernt SNR

(b) RMSE with differernt L

Figure 4. Simulation3 result

5 Conclusion

In this paper, we proposed a Bayesian compressive sensing based blind DOA estimation scheme which can reduce the complexity of traditional radar system while keep good estimation performance. The proposed spatial domain compression spatial scheme utilizes the sparsity of the targets in the background scene to obtain the initial estimation of DOA, and keeps the aperture of initial antenna array. Simulation results proved that the Bayesian based sparse recovery method would achieve super performance in radar parameter estimation.

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