Estimation of Moving Target Parameters using Compressive Sensing Methods

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***Abstract*—The parameter estimation of a distributed target with several moving elements is an important question in radar based security applications. In this paper, a solution of this problem is proposed by using an incoherent radar sensor network as well as signal processing methods known from the area of Compressive Sensing. Thereby, three different algorithms are evaluated: Orthogonal Matching Pursuit (OMP), SIMPLEX and nonlinear Convex Optimization. It can be shown that a relative small number of radar channels is sufficient to determine positions and velocity vectors of many point scatterers included in the analyzed scene.**

I. Introduction

Radar based sensors operating at millimeter wave frequencies provide powerfull tools in the area of security applications. Thereby, the precise knowledge of the target's motion parameters is a crucial requirement for signal processing using SAR techniques. During scene reconstruction, a certain phase history is assumed for each point target, based on the sensor position over time. For a moving target, this assumption is not valid anymore, so an unambiguous solution is not possible. In order to resolve this ambiguity, various autofocus algorithms have been developed in the past.

In case of a distributed target with several different velocity components a motion estimation with conventional methods is not possible due to the large number of unknown parameters, so more advanced algorithms must be applied. The fast developing field of Compessive Sensing (CS) provides a possible solution to this problem. Using CS methods the ambiguity of scene reconstruction can be resolved with an "overwhelming" probability by selecting a solution vector which is optimal with respect to certain additional requirements. For instance, one can look for a scene containing the lowest possible number of point scatterers (ℓ_0-norm minimization).

The present paper presents a theoretical approach for parameter estimation of a distributed moving scene by means of a multichannel radar network and a signal processing based on Compressive Sensing methods.

II. Signal Model

A. Generic Geometry

In the presented work a 2-dimensional generic measurement geometry corresponding to Fig. 1 is assumed. The analyzed scene contains N_p independent point targets with a velocity vector assigned to each. The scene is surrounded

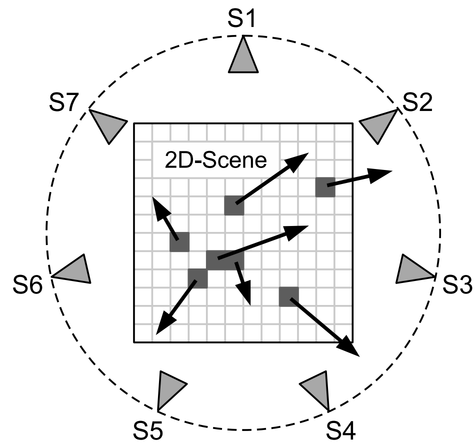


Fig. 1. Generic measurement geometry consisting of a 2-dimensional scene surrounded by 7 radar sensors (S1 to S7); the velocity vector of each point scatterer is indicated by an arrow

by N_s radar sensors with broad antenna beams and thus no directivity. In the simplest case, each sensor consists of a transmitter and a receiver in monostatic geometry, but a multistatic or even a MIMO configuration are also imaginable. Further, no interaction between the single sensors and no multiple reflections between point targets are assumed. Because of the large distance between two sensors there is also no coherent phase relation between the channels.

B. Measurement Data

For simulation of the multichannel radar system a simple stepped-frequency approach was chosen. During the measurement time each sensor transmits a series of N_t pulses with N_f discrete frequency steps in each. Summarized, the measurement data consists of

$$M = N_s N_f N_t \tag{1}$$

complex samples. Assuming a constant velocity of the n-th point target its position $\vec{p}_n$ can be expressed as:

$$\vec{p}_n(t) = \vec{p}_{0,n} + \vec{v}_n t \tag{2}$$

with t being the time, $\vec{p}_{0,n}$ the position for $t = 0$ and $\vec{v}_n$ the velocity vector of the n-th scatterer. Using the position $\vec{p}_m$ of the m-th sensor the range history $R_{m,n}$ can be calculated for each combination of (m, n):

$$R_{m,n}(t) = |\vec{p}_{0,n} - \vec{p}_m + \vec{v}_n t| \approx R_{0,m,n} + v_{r,m,n}\, t \tag{3}$$

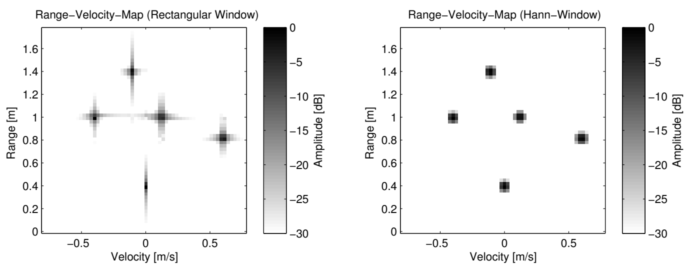


Fig. 2. Example of a Range-Velocity-Map with 5 point scatterers calculated using (a) Rectangular and (b) Hann window functions

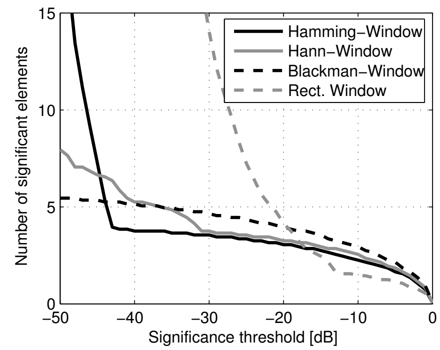


Fig. 3. Average number of significant elements in the Point Spread Function of a harmonic signal weighted with different window functions

with $R_{0,m,n}$ representing the distance between m-th sensor and n-th scatterer for $t = 0$ and $v_{r,m,n}$ being the corresponding radial velocity.

For a given time t and a given frequency f the measurement data from a single scatterer can be written as:

$$D_{m,n}(f,t) \approx A_n \cdot \mathrm{e}^{-j2\pi f T_{0,m,n}} \cdot \mathrm{e}^{-j2\pi f_{d,m,n}\, t} \tag{4}$$

with

- A_n: complex amplitude of the n-th scatterer,
- $T_{0,m,n} = 2R_{0,m,n}/c_0$: propagation time for $t = 0$,
- $f_{d,m,n} \approx 2f_c v_{r,m,n}/c_0$: Doppler frequency,
- f_c: center frequency of the transmitted signal.

Using equation 4 the received signal of the m-th sensor can be calculated:

$$D_m(f,t) = \sum_{n=1}^{N_p} D_{m,n}(f,t). \tag{5}$$

It should be mentioned that some assumptions like a small relative bandwidth and a short measurement time have been made during this derivation. Nevertheless, the proposed method is not restricted to these assumptions as long as the measurement data can be compressed to Range-Doppler-Maps corresponding to section III.

III. Range-Velocity-Maps

As shown in the previous section, the measurement data of each radar sensor consists of $N_t \times N_f$ complex samples. According to equation 4 the variables t and f are located in two independent exponential terms, so a Fast Fourier Transform (FFT) can be calculated along the t and f-axes. After this operation all point scatterers of the analyzed scene are separated depending on their sensor distances $R_{0,m,n}$ and their radial velocities $v_{r,m,n}$. This way, the measurement data is consequently transformed into N_s single Range-Velocity-Maps corresponding to the number of radar channels. Fig. 2a shows an example of a RV-Map with 5 point targets.

The maximum separation of point targets is the main intention for the calculation of RV-Maps previous to scene reconstruction. A further important reason is data reduction as a first step to a sparse signal representation. By applying a significance threshold to a RV-Map all elements below the threshold can be neglected, which allows an economical memory use. To keep the number of non-zero elements as low as possible, the measurement data can be weighted using different window functions before Fourier transform. Fig. 3 shows the average number of non-zero elements depending on the significance threshold and the used window function. It is obvious that the Hamming window is the best choice if all elements below -40 dB can be neglected. For a lower threshold also the Hann window might be preferred (see Fig. 2b).

As mentioned above, the phases of a RV-Map don't contain useful information, as the distances between sensors are very large relative to the signal wavelength and the real-world targets usually cannot be represented by ideal point scatterers. The processing steps of a RV-Map are summarized below:

- weighting with 2-dimensional Hamming window,
- 2-dimensional Fast Fourier Transform,
- amplitude calculation by dismissing the signal phase.

IV. CS-Solution

Compressive Sensing is a rapidly developing research area, which tries to overcome the limitations given by the Shannon-Nyquist sampling theorem. The main idea behind CS is that a signal can successfully be reconstructed from an insufficient set of measurement data if its information content is only low enough. As the measure of information is strongly related to the number of vector elements, this statement is equivalent to the requirement for a sparse signal representation. A general CS problem can be written as:

$$D = \Phi S \tag{6}$$

with S being the unknown signal (in our case corresponding to scene reflectivity), Φ representing the $M \times N$ sensing matrix with $M < N$ and D containing the measurement data. The linear system given by equation 6 is underdetermined, so an unique unambiguous solution doesn't exist. The CS based approach looks for a particular solution with a minimized ℓ_0 respective ℓ_1-norm:

$$S_{L0} = \min_S \{\|S\|_0 \;:\; D = \Phi S\} \tag{7}$$

$$S_{L1} = \min_S \{\|S\|_1 \;:\; D = \Phi S\}. \tag{8}$$

TABLE I. LIST OF SIMULATION PARAMETERS

Parameter	Value	Description
f_c	94 GHz	Signal center frequency
B	5 GHz	Bandwidth
dT	1 ms	Time between two pulses
N_f	48	Number of frequency steps
N_t	30	Number of pulses
N_s	3, 5, 7, 9, 12	Number of sensors
N_p	1 – 30	Number of point targets
N_r	900	Spatial discretization (30×30)
N_v	577	Velocity discretization

In case of a noisy measurement the constraints can be modified to allow a certain reconstruction error ε^2:

$$S_{L1} = \min_{S} \left\{ \|S\|_1 \ : \ \|D - \Phi S\|_2^2 \leq \varepsilon^2 \right\}. \tag{9}$$

Some early work on sparse signal reconstruction is described in [1]–[3], further results are presented in [4]–[8].

A. Orthogonal Matching Pursuit

The Matching Pursuit Algorithm was firstly introduced by Zhang in 1993 [9] and later extended by Pati *et al.* to Orthogonal Matching Pursuit [10]. This "greedy" approach tries to reconstruct the signal piece by piece until no significant improvement can be achieved. This algorithm can be easily implemented and provides excellent behaviour in terms of stability and convergence. Unfortunately, no guarantees for the reconstruction quality can be given. In the present case, OMP was slightly modified to deal with non-negative numbers.

B. SIMPLEX Algorithm

The SIMPLEX algorithm was introduced by Dantzig [11] in 1947 and efficiently solves a linear optimization problem of the type:

$$x_{opt} = \max_{x} \left\{ c^T x \ : \ \Phi x \leq b, \ x \geq 0 \right\}. \tag{10}$$

This approach maximizes the objective function $c^T x$ under the constaints $\Phi x \leq b$ and $x \geq 0$. By taking the absolute value of both sides of equation 6 one yields:

$$|D| = D_a \leq \Phi_a S_a = |\Phi| \cdot |S|. \tag{11}$$

Using this expression the optimization problem 8 can be rewritten corresponding to the SIMPLEX formalism:

$$S_{a,opt} = \min_{S_a} \left\{ \|S_a\|_1 \ : \ \Phi_a S_a \geq D_a, \ S_a \geq 0 \right\}. \tag{12}$$

The described algorithm was implemented using the GPL software package lp_solve, which can be accessed from many programming enviroments [12].

C. Nonlinear Convex Optimization

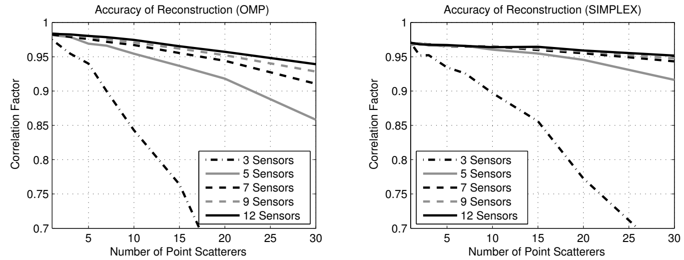
The first two CS solutions presented in this paper deal with the optimization problem given by equation 8. In case of noisy measurement data, additional constraints must be added to equation 9 to match the requirement for non-negative numbers (see section III):

$$S_{a,opt} = \min_{S_a} \left\{ \|S_a\|_1 \ : \ \|D_a - \Phi_a S_a\|_2^2 \leq \varepsilon^2, \ S_a \geq 0 \right\}. \tag{13}$$

To solve this nonlinear optimization problem, the Log-Barrier method can be applied [13], [14]. The main idea behind this concept is to approximate the constraints in equation 13 by differentiable logarithmic functions. In doing so, the solution can be found by minimizing the objective function

$$\begin{aligned} u(S_a) &= \sum_{i=1}^{N} S_{a,i} - \frac{1}{\tau} \Bigg(\log \left(\varepsilon^2 - \|\Phi_a S_a - D_a\|_2^2 \right) \\ &+ \sum_{i=1}^{N} \log(S_{a,i}) \Bigg) \end{aligned} \tag{14}$$

with $S_{a,i}$ being the i-th element of the vector S_a and the constant τ representing a weighting value. The expression 14 can be minimized in an iterative way using a simple gradient algorithm.



(a) (b)

Fig. 4. Average accuracy of CS-reconstruction as correlation factor between original and calculated scene for (a) OMP and (b) SIMPLEX algorithms

V. SIMULATION

A. Simulation Parameters

To validate the system design described in section II as well as the CS-based algorithms proposed in section IV a series of simulations was performed. Table I contains a list of the corresponding simulation parameters.

The 2-dimensional measurement geometry defined by Fig. 1 leads to a 4-D solution space with the dimensions x, y, v_x and v_y. Although the solution vector contains only few non-zero elements, the total number of base functions can be calculated from $N_r \times N_v$ and corresponds to 519300 columns of Φ. Further, the number of data samples is defined by equation 1 and counts 17280 in the case of 12 sensors. This way, the sensing matrix Φ consists of nearly $9 \cdot 10^9$ elements, which only can be handled using sparse data structures.

B. OMP Results

A series of simulations based on randomly generated moving scenes was performed for each combination of parameters shown by Table I. The average reconstruction quality was calculated as the correlation factor between original and reconstructed scenes (see Fig. 4a). It is obvious that the algorithm shows a better performance with an increasing number of sensors and a decreasing scene complexity.

Additionally, a specific pattern consisting of 10 point scatterers was analyzed. Fig. 5 shows the original scene as well as the OMP reconstruction results for 3, 7 and 12 sensors. In this presentation, each point target is represented by an arrow indicating its velocity vector. The amplitude of a scatterer is encoded by color.

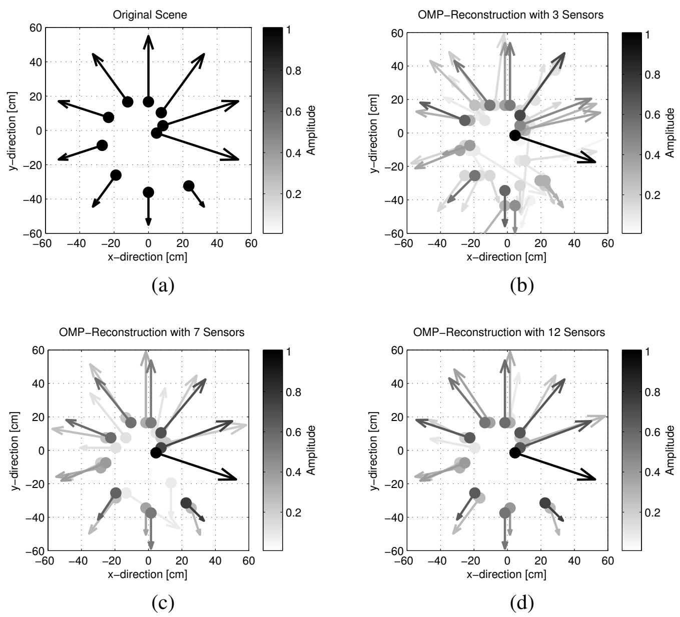


Fig. 5. Reconstruction of a generic moving scene using the OMP algorithm: (a) original scene, (b)–(d) reconstructed with 3, 7 and 12 sensors

C. SIMPLEX Results

The average quality of SIMPLEX reconstruction based on 100 randomly generated moving scenes is presented by Fig. 4b. The algorithm shows a significant better performance compared with OMP, especially if the analyzed scene contains many point scatterers.

Moreover, the number of iteration steps as well as the computing time strongly depend on the number of sensors and scatterers. Empirical data show the following relationship:

- Iteration steps $\sim N_p^2 N_s$,
- Computing time $\sim N_p^3 N_s^2$.

D. Convex Optimization Results

The accuracy of scene reconstruction from noiseless data using nonlinear Convex Optimization is very close to those of the SIMPLEX method (results not shown here). Anyway, by adding noise to the measurement data the algorithms shows an unexpected behaviour: the solution vector is not sparse anymore and gets a "noise floor" around the true target positions (see Fig. 6). A possible explanation is that the algorithm is explicitly allowed to make reconstruction errors, which are not restricted to the amplitude of true targets.

VI. Conclusion

The results of this paper show that the reconstruction of distributed moving targets using Compressive Sensing methods is possible, even if the single radar sensors are not coherently linked to each other. While the OMP algorithm is fast and easy to implement, the SIMPLEX approach provides a better reconstruction quality. In case of noisy data, nonlinear Convex Optimization is the method of choice.

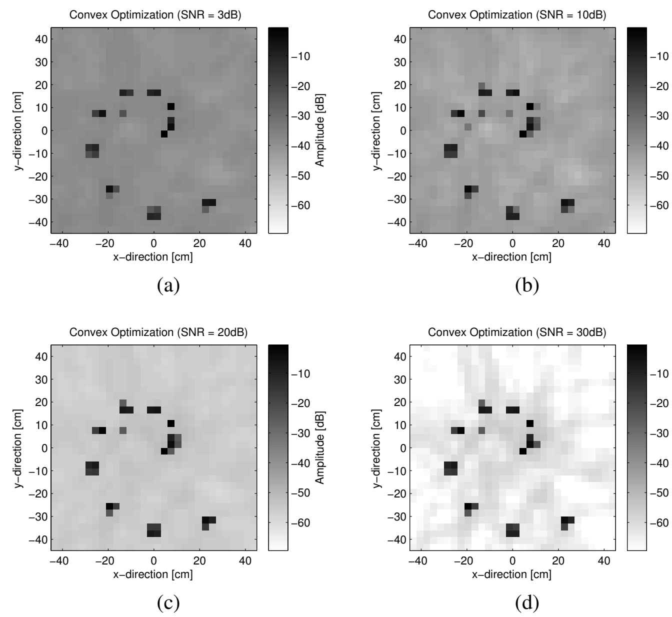


Fig. 6. Amplitude of scene reconstruction based on noisy measurement data using nonlinear Convex Optimization: (a) SNR = 3 dB, (b) SNR = 10 dB, (c) SNR = 20 dB, (d) SNR = 30 dB

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