Estimation of Moving Target Parameters using Compressive Sensing Methods

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Outline

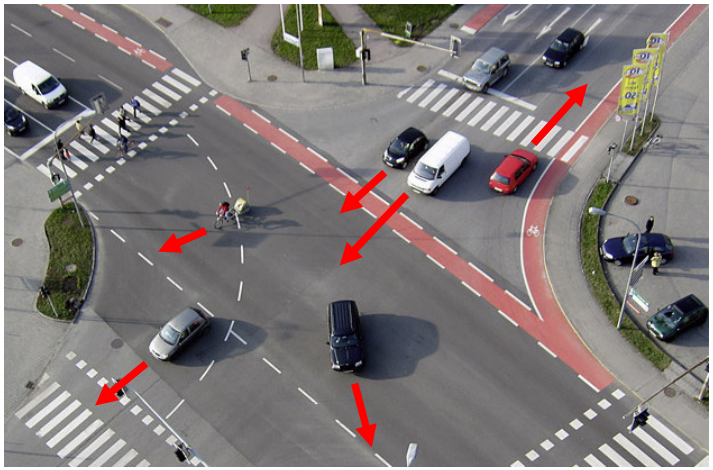
- Introduction and motivation
- Simulated geometry and signal model
- Formulation of the reconstruction problem
- Solution with Orthogonal Matching Pursuit
- Solution with SIMPLEX algorithm
- Solution with nonlinear Convex Optimization
- Summary

Introduction

- Many potential targets of radar based sensors contain a large number of independently moving elements with parameters (x_l, y_l, v_{xl}, v_{yl}, …)
- Conventional image reconstruction methods fail, as no unambiguous solution exists
- <u>Observation:</u> the most scenes can be described as sparse (the number of strong scatterers is relatively low)
- → Collected radar data contain much less information than is suggested by its bandwidth
- <u>Possible solution:</u> Compressive Sensing

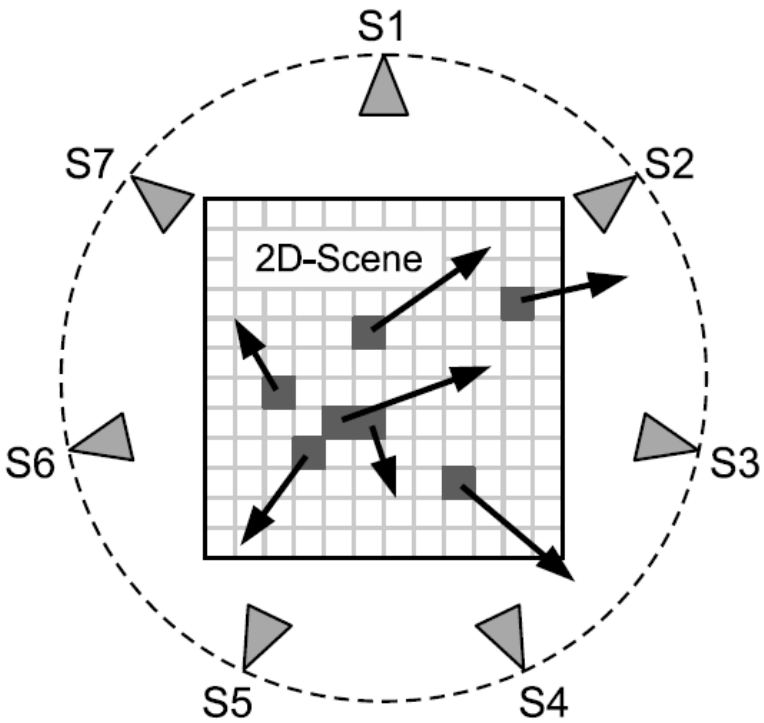
3D-Example

2D-Example



Simulated Geometry

- Generic 2-dimensional scene with N_s radar sensors located around the target
- Scene contains N_p moving point scatterers
- Each sensor transmits a series of N_t pulses with N_f discrete frequency steps
- → Set of measurement data contains $M = N_s \cdot N_f \cdot N_t$ complex samples
- <u>Assumptions:</u>
  - no coherency between sensors
  - velocities are nearly constant during the measurement time



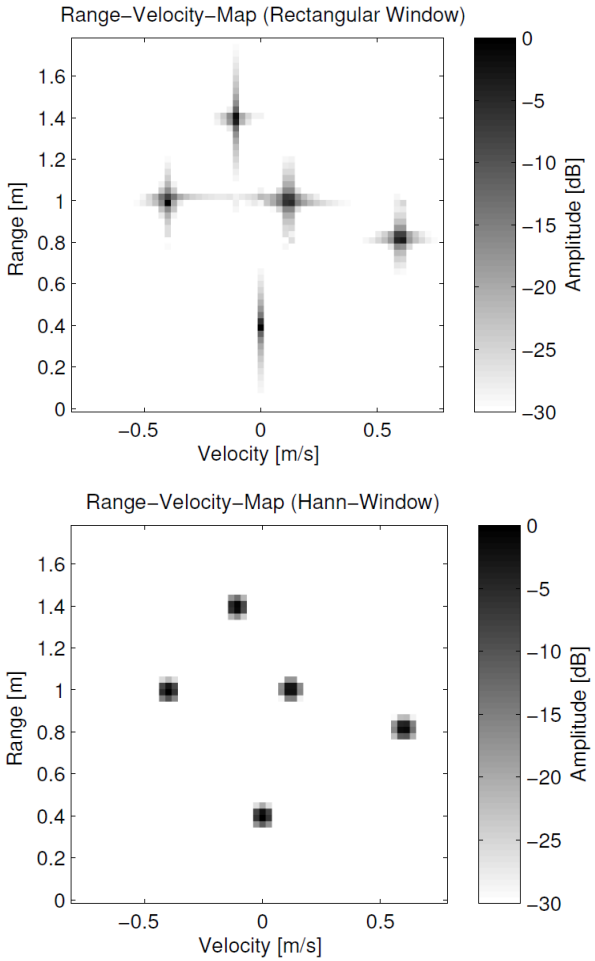
Generic Measurement Geometry

Signal Model

- Position of n-th scatterer: $\vec{p}_n(t) = \vec{p}_{0,n} + \vec{v}_n t$
- Distance between n-th scatterer and m-th sensor: $R_{m,n}(t) = |\vec{p}_{0,n} - \vec{p}_m + \vec{v}_n t|$
- Linear approximation: $$R_{m,n}(t) \approx |\vec{p}_{r,m,n}| + \frac{\langle \vec{p}_{r,m,n}, \vec{v}_n \rangle}{|\vec{p}_{r,m,n}|} t = R_{0,m,n} + v_{r,m,n}\, t$$
- Measured signal: $$D_{m,n}(f,t) \approx A_n \exp\left(-j2\pi f T_{0,m,n}\right) \exp\left(-j2\pi f_{d,m,n}\, t\right)$$
- with propagation time: $T_{0,m,n} = 2R_{0,m,n}/c_0$
- and Doppler frequency: $f_{d,m,n} \approx 2 f_m v_{r,m,n}/c_0$
- Signal of the m-th sensor: $$D_m(f,t) = \sum_{n=1}^{N_p} D_{m,n}(f,t)$$
- $\rightarrow$ Scatterers can be separated depending on their distances and radial velocities relative to the sensor!

Range-Velocity-Maps

- Calculation using 2-dimensional FFT
- RV-maps describe a moving scene from the perspective of a particular sensor
- Scatterers are separated depending on their ranges and radial velocities
- Data contains no directional information
- Side-lobes can be reduced by applying window functions (e.g. Hann window) to measurement data
- Number of non-zero elements is significantly reduced → „Sparsity“
- RV-maps are the basis for further (CS-based) image reconstruction steps



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Problem Formulation

- Measurement data of all sensors can be written as a single Vector D:

$$D = \begin{pmatrix} D_1 \\ D_2 \\ \vdots \\ D_{N_s} \end{pmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_{N_s} \end{bmatrix} \cdot \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{pmatrix} = \Phi \cdot S$$

- 2D Fourier transform as well as filtering with Window functions can be expressed as multiplication with matrix Ψ:

$$\Psi D = (\Psi \Phi) S$$

- Inequality of Cauchy-Schwarz:

$$|\Psi D| \leq |\Psi \Phi| \cdot |S| \quad \longrightarrow \quad D_a \leq \Phi_a S_a$$

- Linear optimization problem:

$$S_{L1} = \min_{S_a} \{ \|S_a\|_1 \ : \ \Phi_a S_a \geq D_a, \ S_a \geq 0 \}$$

- Introducing of reconstruction error E:

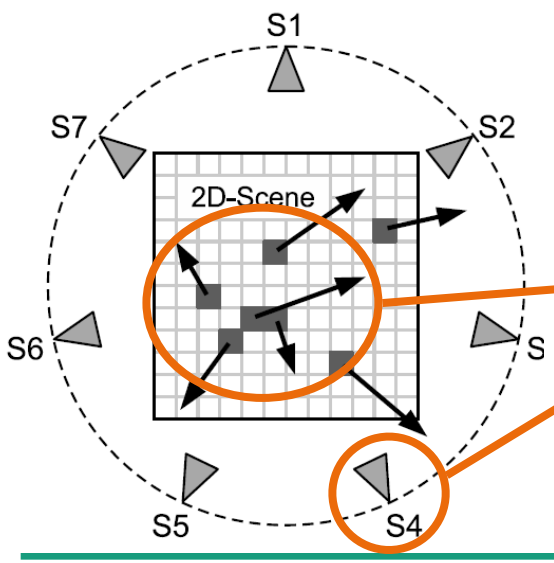
$$D_a = \Phi_a S_a + E$$

- Nonlinear optimization problem:

$$S_{L1} = \min_{S_a} \left\{ \|S_a\|_1 \ : \ \|E\|_2^2 \leq \varepsilon^2, \ S_a \geq 0 \right\}$$

Simulation Parameters

- Simulation of 100 randomly generated scenes with 1 – 30 moving point scatterers and CS reconstruction based on data of 3 – 12 sensors
- Statistical analysis of simulation results
- Additional: Simulation and reconstruction of a test pattern
- Evaluation of 3 algorithms: OMP, SIMPLEX, nonlinear convex optimization



Parameter	Value	Description
f_c	94 GHz	Signal center frequency
B	5 GHz	Bandwidth
dT	1 ms	Time between two pulses
N_f	48	Number of frequency steps
N_t	30	Number of pulses
N_s	3, 5, 7, 9, 12	Number of sensors
N_p	1 – 30	Number of point targets
N_r	900	Spatial discretization (30×30)
N_v	577	Velocity discretization

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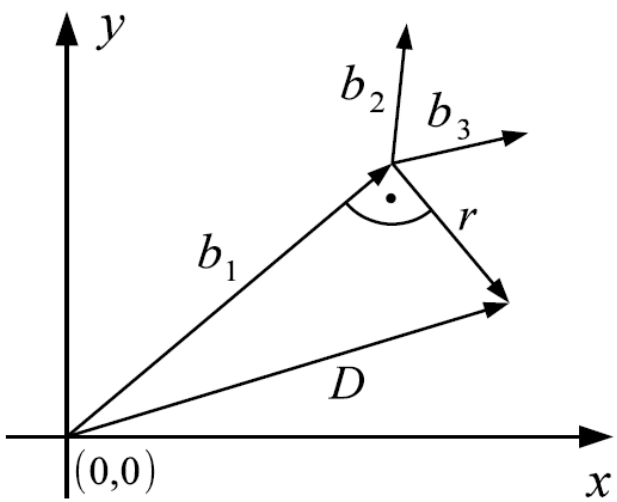
Orthogonal Matching Pursuit

- Iterative approach, belongs to the "greedy" algorithms
- Selection of one base function (column of sensing matrix) during each iteration step and termination if no improvement is possible anymore
- Suboptimal method, but easy implementation
- Adoption to deal with positive-valued measurement data and positive-valued sensing matrix:
  - original: $\max_i |\langle r, b_i \rangle| \quad \text{mit} \quad i \in [1, N]$
  - changed: $\max_i \langle r, b_i \rangle \quad \text{mit} \quad i \in [1, N]$



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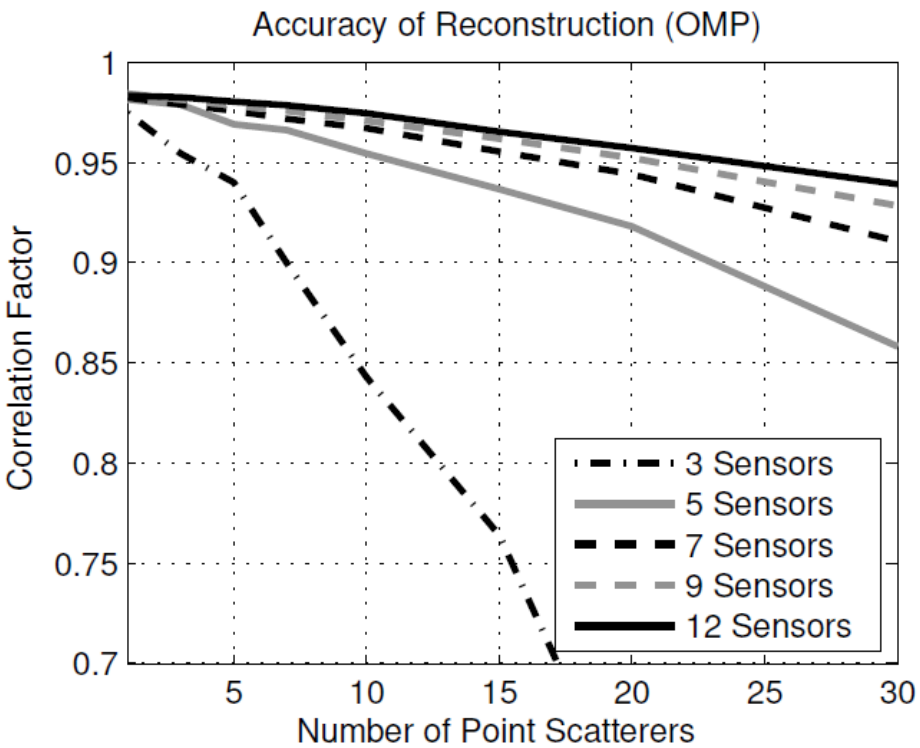
OMP: Random Distributions

- Reconstructed scene is correlated with the original distribution of moving scatterers
- Reconstruction quality is increasing with number of sensors (N_s) and decreasing with scene complexity (number of point targets N_p)
- Reliable reconstruction seems to be possible with 5 or more sensors
- Statistical evaluation shows strong variation of reconstruction quality
- Nearly no difference between results obtained with 9 or 12 sensors



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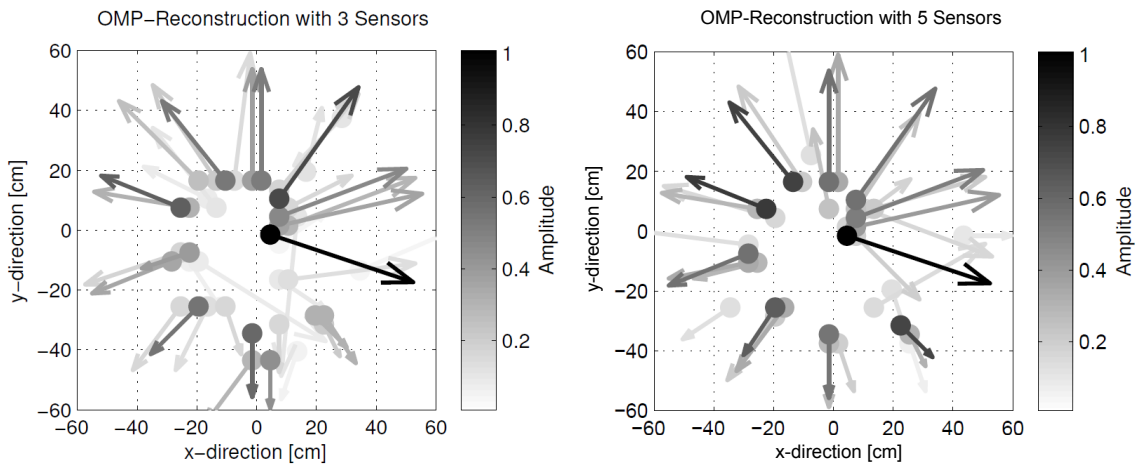
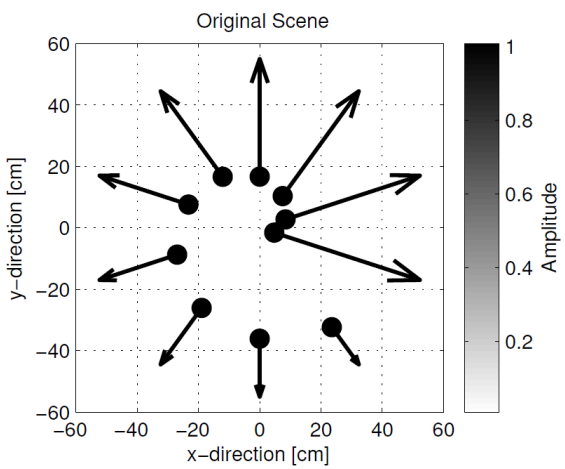
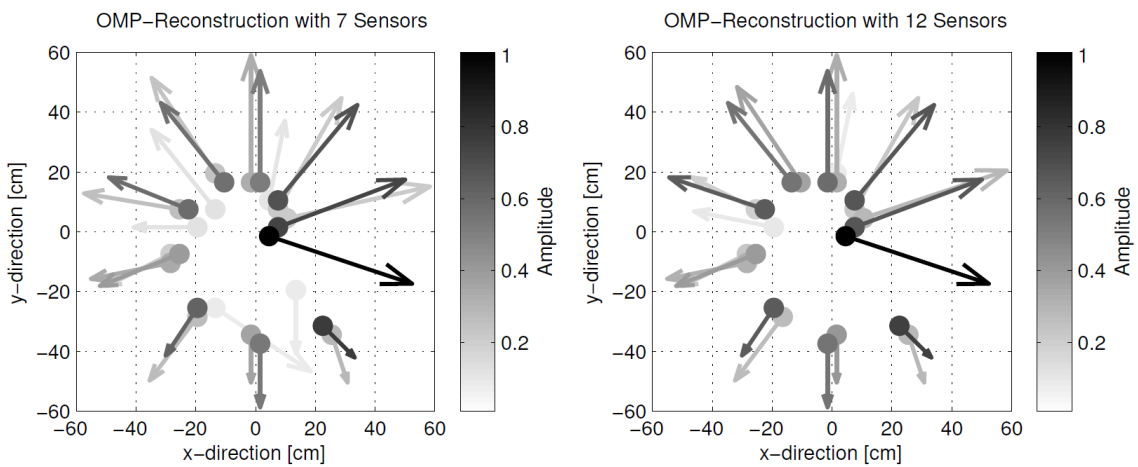
OMP: Test Pattern

- Generic test scene with 10 moving targets
- Arrow → Velocity
- Color → Amplitude
- Quality is increasing with number of sensors (N_s)







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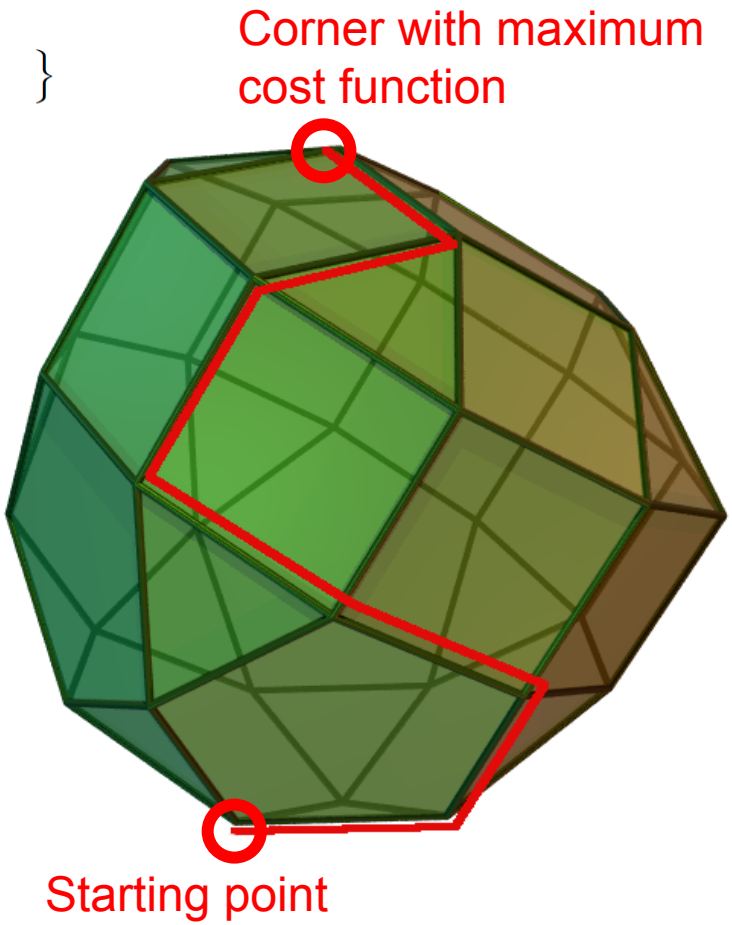
SIMPLEX Algorithm

$$S_{opt} = \max_{S}\{ \underbrace{C \cdot S}_{\text{Cost Function}} : \underbrace{\Phi \cdot S \leq D}_{1^{st}\text{ constraint}} , \underbrace{S \geq 0}_{2^{nd}\text{ constraint}} \}$$

- Scene consists of *N* pixels
  → N-dimensional solution space
- Each inequality of the 1st constraint divides solution space into two half-spaces
- Together the inequalities define a polytope in the solution space
- <u>Goal:</u> find a corner of the polytope with the maximum cost function
- the SIMPLEX algorithm moves along the edges until an improvement of the solution is not possible



Source: Wikipedia

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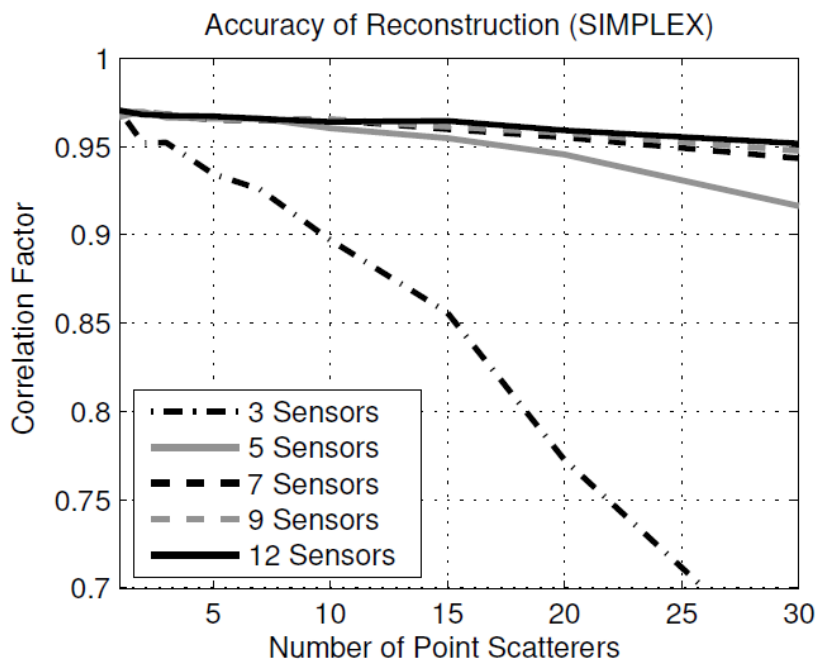
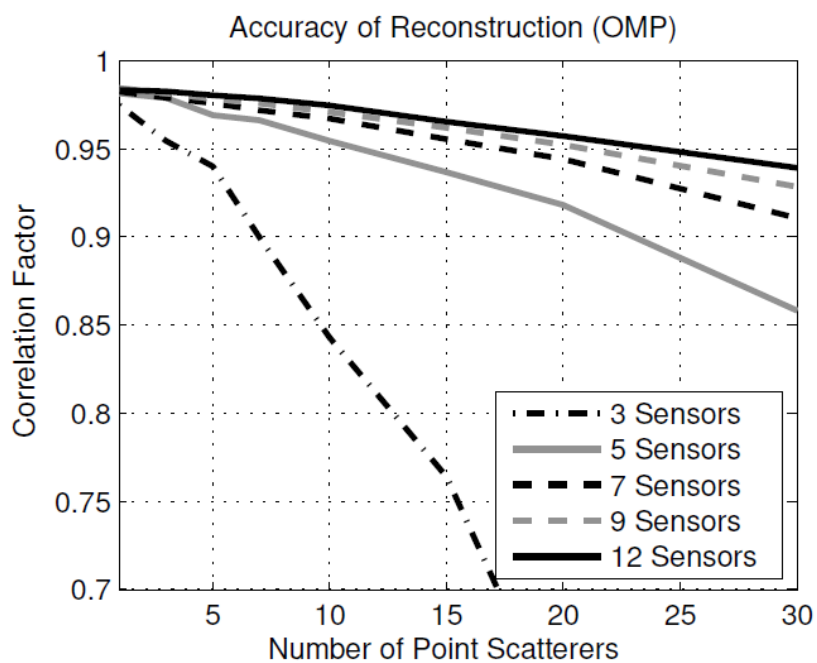
SIMPLEX: Random Distributions

- Similar behavior compared to Matching Pursuit
- Reconstruction quality is significantly better for complex scenes





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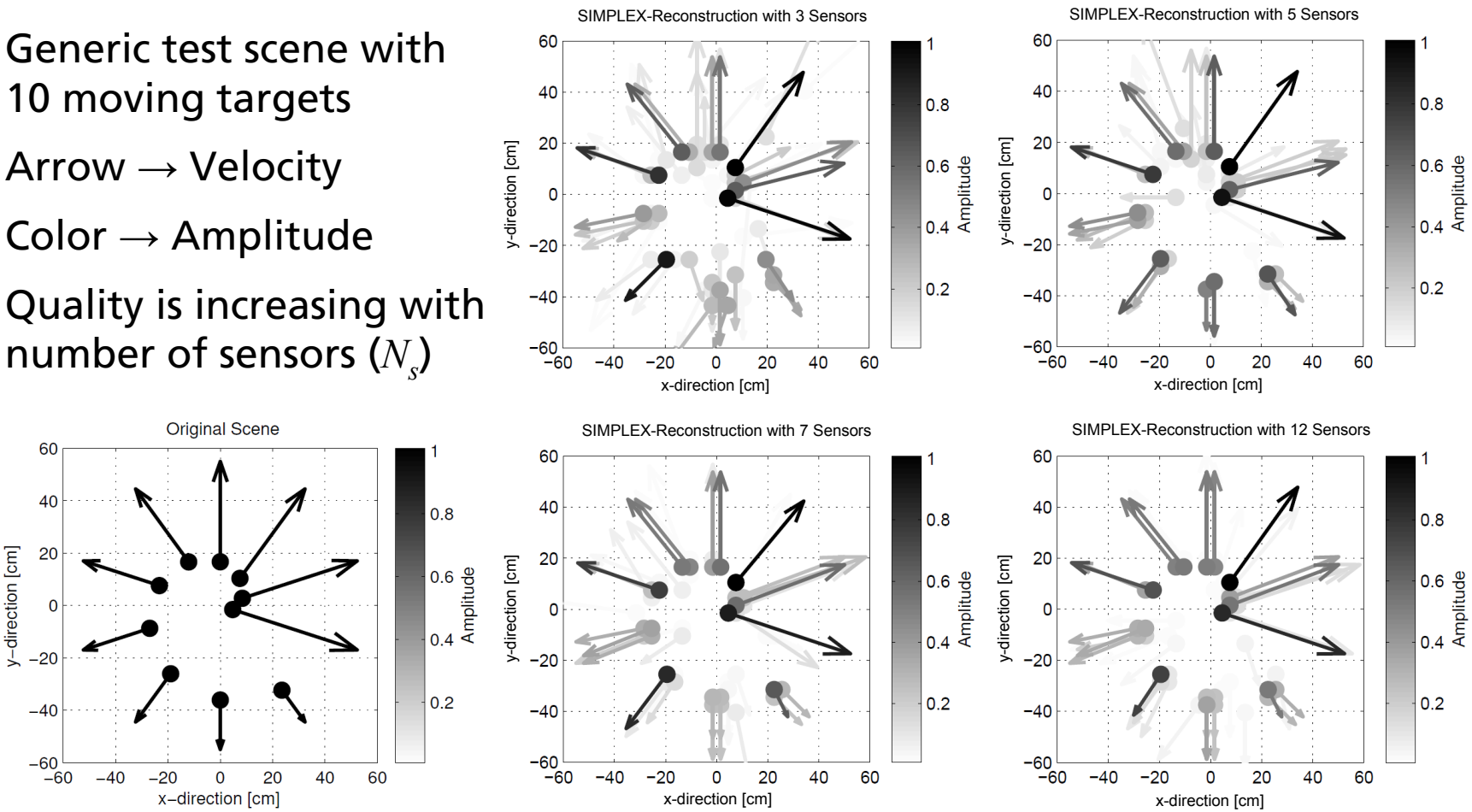
SIMPLEX: Test Pattern

- Generic test scene with 10 moving targets
- Arrow → Velocity
- Color → Amplitude
- Quality is increasing with number of sensors (N_s)



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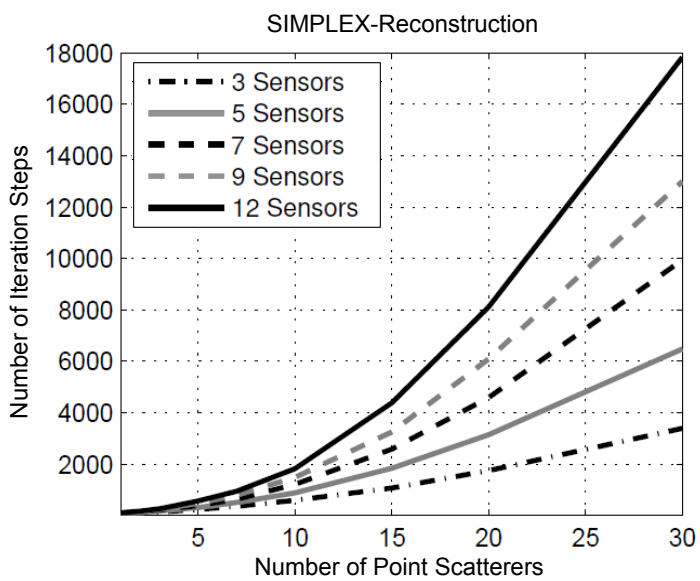
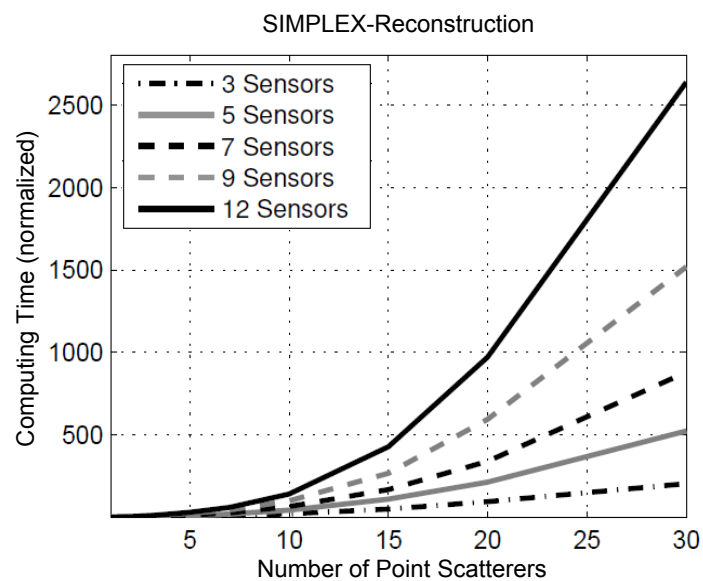
SIMPLEX: Complexity

- The complexity and computing time of the SIMPLEX algorithm show exponential dependency on the number of constraints M (in worst-case), simulation shows following behavior:
  - Number of iteration steps $\sim N_p^2 N_s$
  - Average computing time $\sim N_p^3 N_s^2$





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Nonlinear Convex Optimization

- Simulation of noisy measurement data
- Starting point: $D_a \leq \Phi_a S_a$
- In case of noisy measurement data, a reconstruction error ε can be allowed: $D_a = \Phi_a S_a + E$
- → nonlinear optimization problem:

$$S_{L1} = \min_{S_a} \left\{ \|S_a\|_1 \ : \ \|\Phi_a S_a - D_a\|_2^2 \leq \varepsilon^2, \ S_a \geq 0 \right\}$$

- Reconstruction using linear methods (e.g. SIMPLEX) not possible
- Express the optimization problem as differentiable steady function $u(S_a)$
- Optimize this function using e.g. the Gradient algorithm
- <u>Possible approach:</u> Log-Barrier algorithm
- Initial solution can be found e.g. using Matching pursuit

Log-Barrier Algorithm

- each optimization problem of the typ $x_{opt} = \min_x \{u_0(x) \ : \ u_i(x) \leq 0, \ i \in [1, M]\}$ can also be expressed as

$$x_{opt} = \min_x \left\{ u_0(x) + \sum_{i=1}^{M} L(u_i(x)) \right\} \qquad \text{with} \qquad L(u) = \begin{cases} 0 & u \leq 0 \\ \infty & u > 0 \end{cases}$$

- Logarithmic approximation of L: $\widehat{L}(u) = -\frac{1}{\tau} \log(-u)$

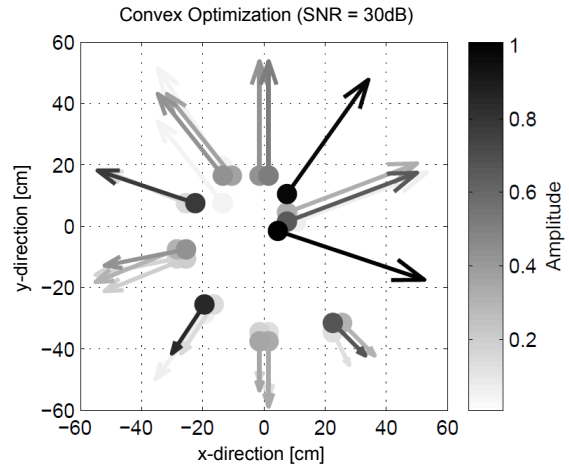
- Steady target function: $u(x) = u_0(x) + \sum_{i=1}^{M} \widehat{L}(u_i(x)) = u_0(x) - \frac{1}{\tau} \sum_{i=1}^{M} \log(-u_i(x))$

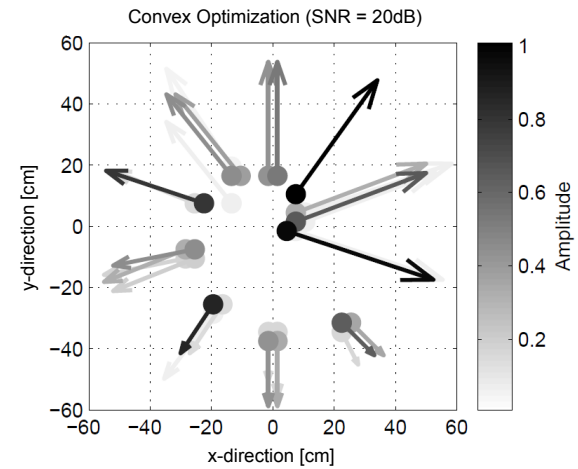
- Principle applied to present optimization problem:

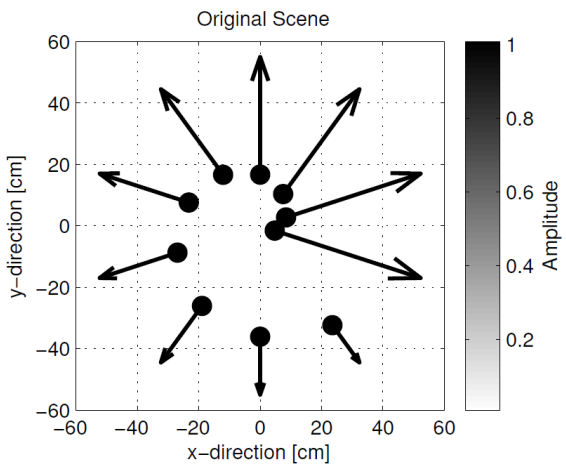
$$u(S_a) = \sum_{i=1}^{N} S_{a,i} - \frac{1}{\tau} \left(\log\left(\varepsilon^2 - \|\Phi_a S_a - D_a\|_2^2\right) + \sum_{i=1}^{N} \log(S_{a,i}) \right)$$

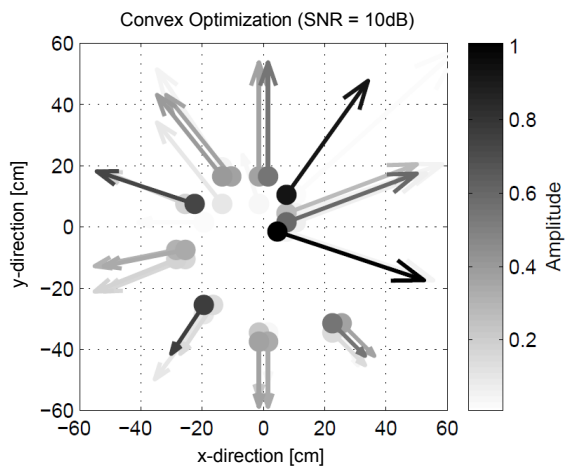
Nonlinear Convex Optimization

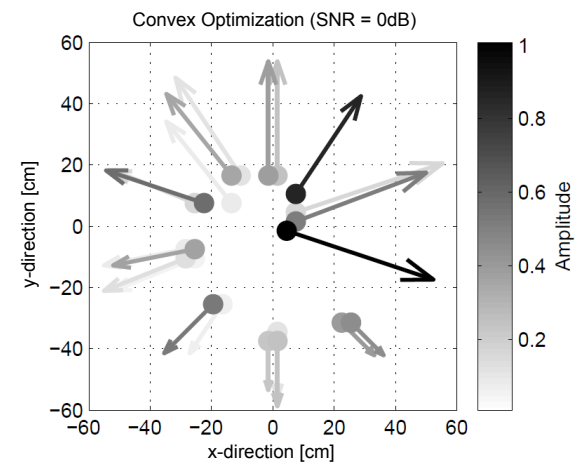
- Simulation with SNR of 0, 10, 20 and 30dB
- Reconstruction of noisy data is still possible!
- Some small details may be lost











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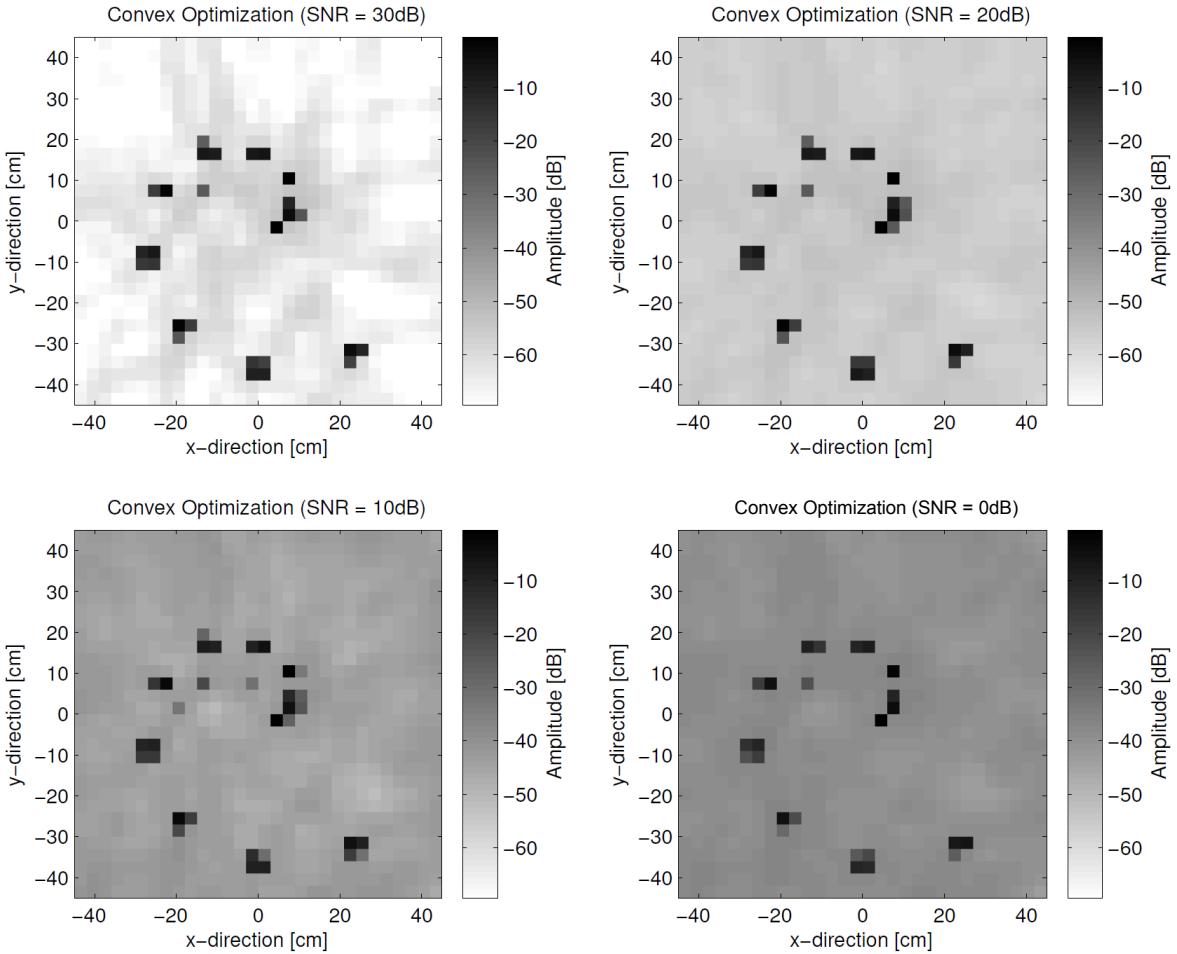
Nonlinear Convex Optimization

- Dynamics: 37dB – 63dB
- → very effective noise suppression
- With decreasing SNR a "noise floor" appears around the true targets
- → image dynamics is decreasing
- <u>possible explanation:</u> the algorithm allows reconstruction errors, which are not restricted to "true" targets



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Summary

- motion parameters of complex moving scenes can be estimated using a network of incoherently operating radar sensors as well as CS-based signal processing techniques
- Assumption: the scene contains only a limited number of significant elements
- A relatively small number of radar sensors is required (5 – 10)
- Coherency between sensors is not necessary
- Depending on the scenario, different reconstruction algorithms can be applied
- For high SNR linear optimization (SIMPLEX) or greedy methods may be used
- In case of low SNR nonlinear optimization methods show better performance
- Reconstruction is computationally heavy in terms of required memory and computing time, so real-time application is not possible (yet)

Thank You!