University of L'Aquila (Italy)

Electrical Engineering Faculty

CoSeRa Bonn 2013

Tomographic SAR Inversion by The Generic Log-Barrier Algorithm the Second Order Cone Programming Approach.

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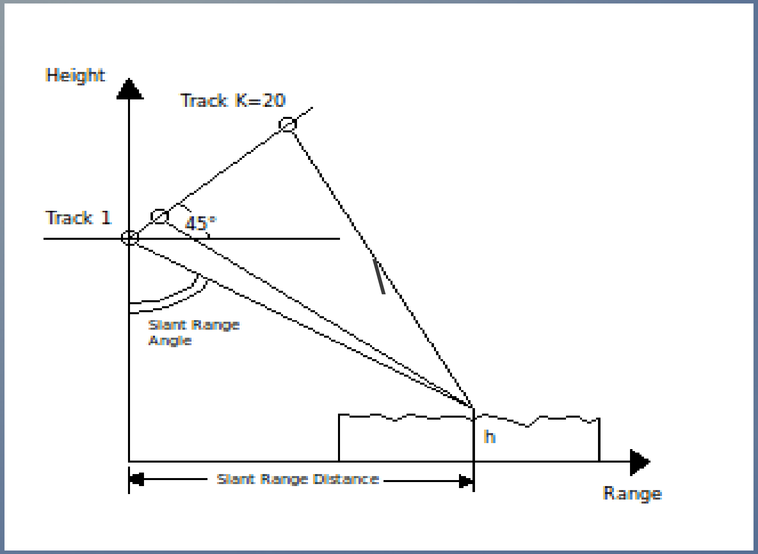
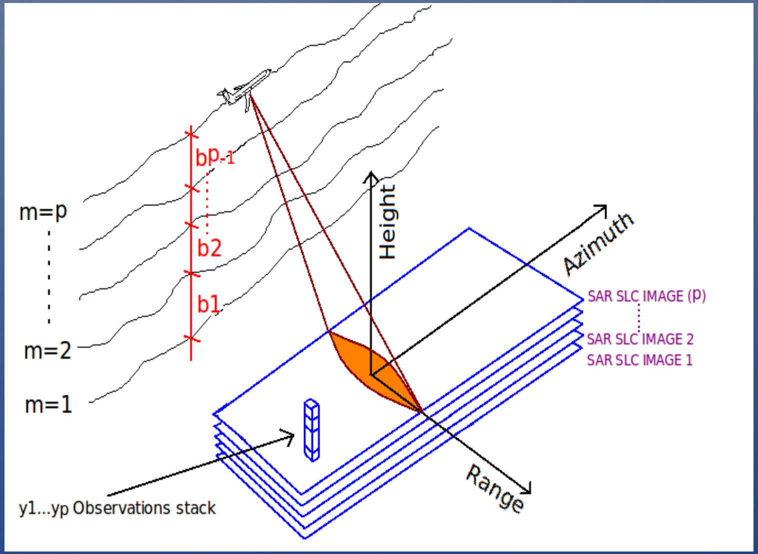
LIST OF ARGUMENTS

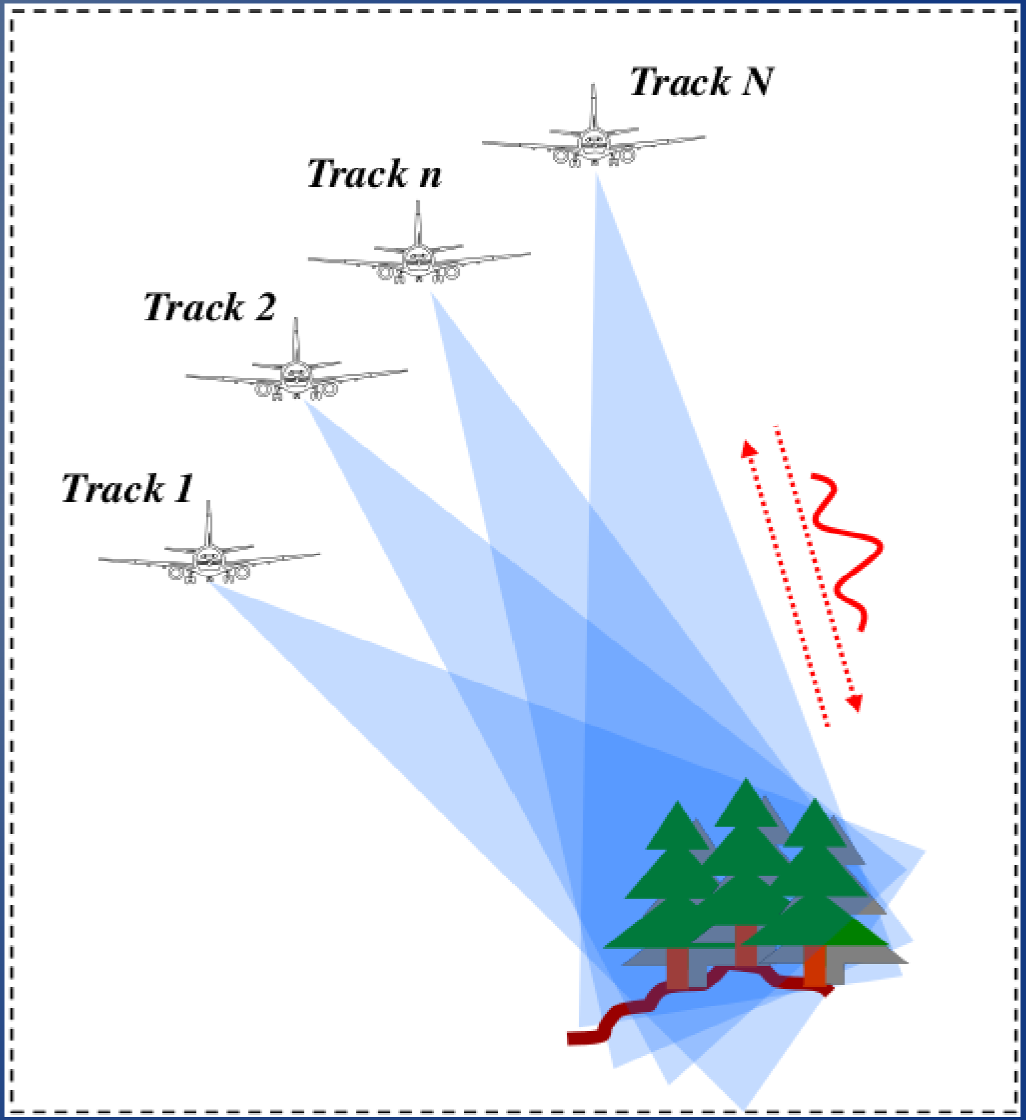
- Tomographic SAR analysis by spectral estimators;
- Under-sampled Tomographic SAR data-sets;
- Validation of Compressed Sensing (CS) signal processing for mixed environment by the Second Order Cone Programming (SOCP).

Multi-Baseline Geometry Acquisition

- SAR Tomography;







- Statistical De-correlation Effects in Repeat-Pass Tom-SAR acquisition Campaign;

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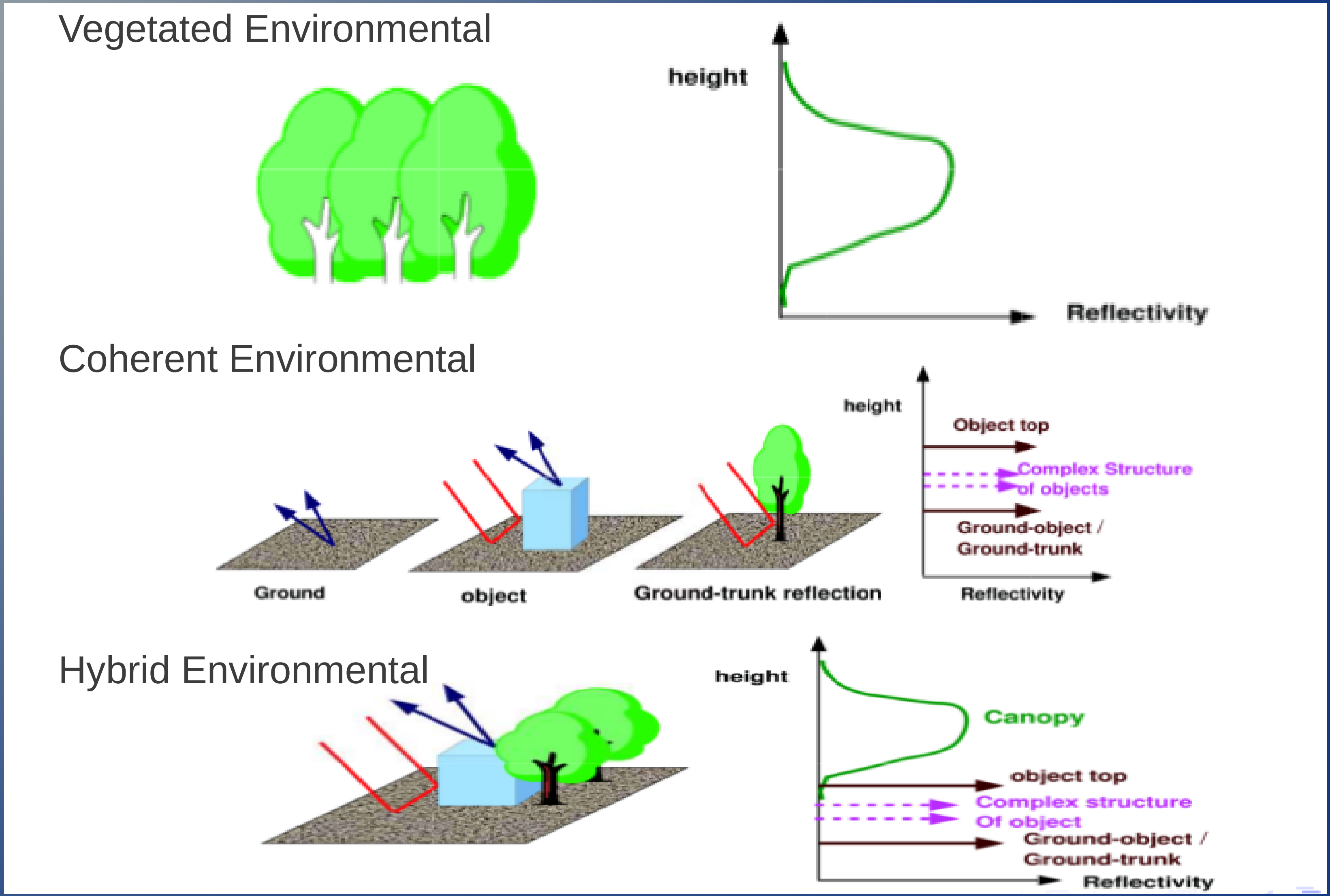
MB-PolInSAR Environments



[1] Yue Huang, Laurent Ferro-Famil, Fabrizio Lombardini: Improved Tomographic SAR Focusing using Automatic Baseline Error Compensation, ESA PolInSAR 2012.

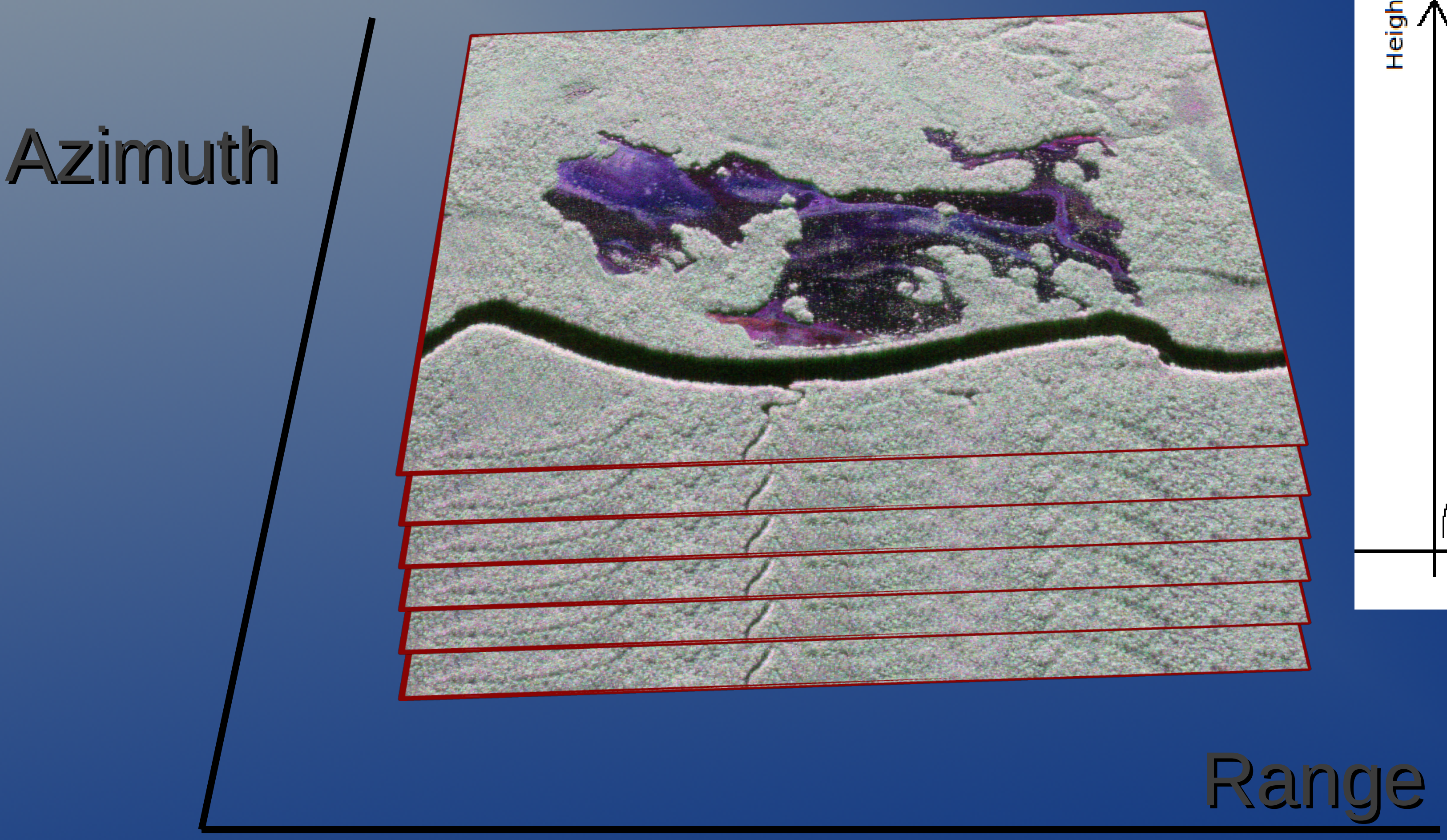
Tomographic Problem Solution

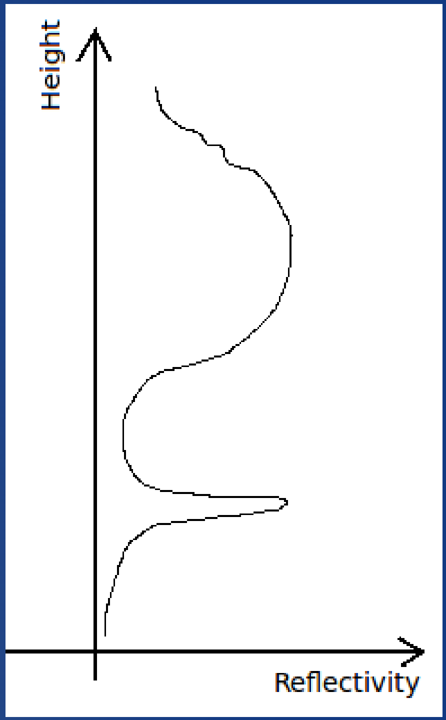
- y=A*x
- y= Known Observation Parameter;
- A=Geometric Built, Steering Matrix;
- x=Unknown reflectivity function, (vertical reflectivity profile);

$$y=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_P \end{bmatrix} \text{ with } A=\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{1N} \\ a_{12} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{P1} & a_{P2} & \cdots & a_{PN} \end{bmatrix}$$

$$a_{i,k} = e^{-j\frac{4\pi}{\lambda} r_{i,k}}$$

$$y = Ax \qquad y \in \mathbb{C}^{P} \quad A \in \mathbb{C}^{P\times N} \quad x \in \mathbb{C}^{N}$$





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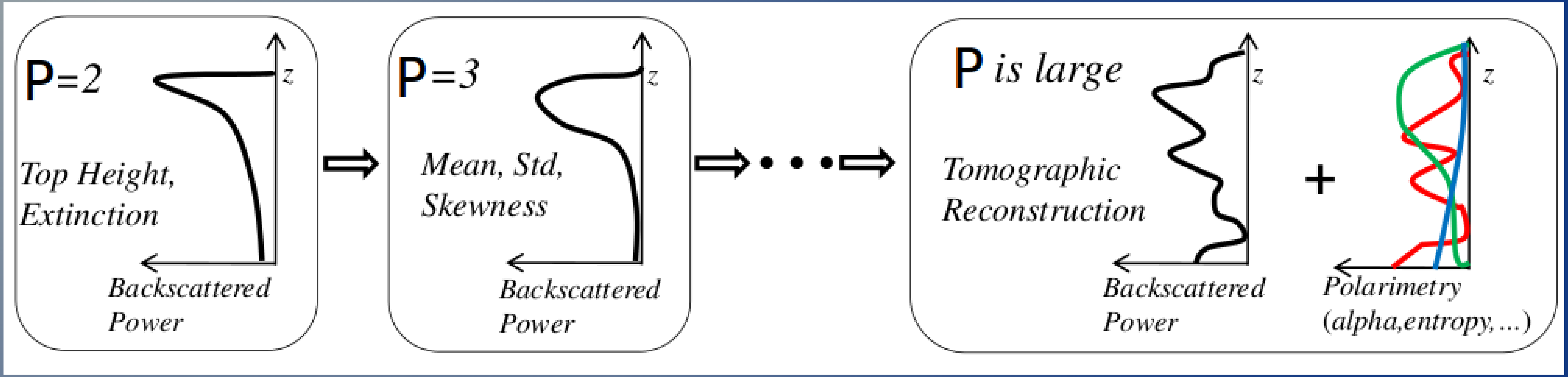
Tomographic Problem Solution



- A is not directly invertible, P<N
- Direct Method: $\boxed{A^{-1} = A^{\dagger}}$
- Indirect Method, (Covariance Matrix Estimation);
- This leads to poor results for many practical applications

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Covariance Matrix Estimation

- Tomographic Processing Procedure: Covariance Matrix Estimation;

$$\boldsymbol{R} = \frac{1}{L}\sum_{j=1}^{L} \mathrm{b}(j)\, \mathrm{b}^{\dagger}(j)$$

L=Number of independent Looks

Covariance Matrix Estimation

$\Leftrightarrow$

$$\mathbf{R}_k = \begin{bmatrix} \gamma_k(1,1) & \gamma_k(1,2) & \cdots & \gamma_k(1,\mathrm{P}) \\ \gamma_k(2,1) & \gamma_k(2,2) & & \gamma_k(2,\mathrm{P}) \\ \vdots & & \ddots & \vdots \\ \gamma_k(\mathrm{P},1) & \gamma_k(\mathrm{P},2) & \cdots & \gamma_k(\mathrm{P},\mathrm{P}) \end{bmatrix}$$

k=1.....Range Length

P=Observation Stack length

Beamforming: $\hat{\mathrm{x}} = argmax\left\{a^{\dagger}\,\boldsymbol{R}\,a\right\};$

Capon: $\hat{\mathrm{x}} = argmax\left\{\dfrac{1}{a^{\dagger}\boldsymbol{R}^{-1}a}\right\};$

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Compressed Sensing

An Optimization Problem seeks to find the best solution so that is true:

LP $\rightarrow$ $$\min \| x \|_1 \text{ subject to } Ax = y$$

SOCP $\rightarrow$ $$\min \| x \|_1 \text{ subject to } \| Ax - y \|_2 \leqslant \epsilon$$

CVX – Convex Optimization

Inequality Constrained Problem

$$\textit{minimize}\, f_0(x)$$

$$\text{subject to}\, f_i(x) \leqslant 0$$

$$\text{and}\ \ y = Ax$$

$x = (x_1 \ldots x_N)$ optimization variable

$f_0 : \mathbb{C}^N \rightarrow \mathbb{C}$ objective function

$f_i : \mathbb{C}^N \rightarrow \mathbb{C}\,;\ i = 1 \ldots m$ constraint functions

CVX – Convex Optimization

$f_0, \cdots, f_m : \in \mathbb{C}^n \rightarrow \mathbb{C}$ are convex and twice continuously differentiable

$A \in \mathbb{C}^{PxN}$ with **Rank** A=P<N continuously differentiable

$\hat{x}, f(\hat{x}) = \hat{p}$ is the optimal solition

$$\Longleftrightarrow$$

$\exists D$ s.t. $y = Ax$ and $f_i(x) \leqslant 0;\ i = 1 \cdots m.s.t.$

CVX – Convex Optimization

Duality:

Karush-Kuhn-Tucker (KKT) Condition:

any optimization problem with differentiable objective and constraint functions for which strong duality obtains, any pair of primal and dual optimal points must satisfy the following

$$\exists \lambda^* \in \mathbb{C}^m, v \in \mathbb{C}^P \; s.t.$$

$$Ax^* = y, \;\; f_i(x^*) \leqslant 0 \;;\; i = 1, \cdots, m$$

$$\lambda^* \succeq 0$$

$$\nabla f_0(x^*) + \sum_{i=1}^{m} \lambda_i \nabla f_i(x^*) + A^T v^* = 0$$

$$\lambda_i^* f_i(x^*) = 0 ; \;\; i = 1 \cdots m$$

CVX – Convex Optimization

Interior Point Methods
Applying Newton Methods

Inequality Constrained Problem

Equality Constrained Problem

Use of Log-Barrier Method

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CVX – Convex Optimization

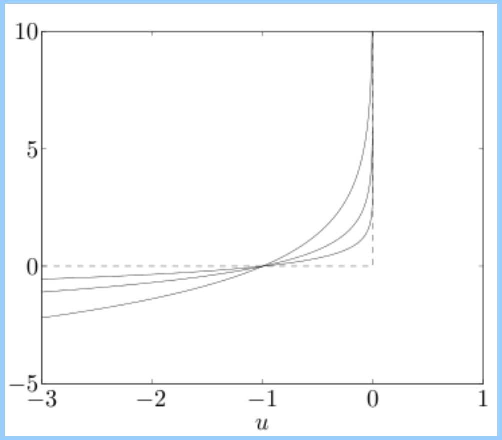
- The Log-Barrier Algorithm for SOCP

The best Equality Constrained Problem is:

minimize $f_0(x)+\sum_{i=1}^{m} I_{-}(f_i(x))$ subject to $y=Ax$

where $I_{-}:\mathbb{R}\to\mathbb{R}$ has the following indicator function for the non-positive reals

$$I_{-}=\begin{cases} 0\,; u\leqslant 0 \\ \infty\,; u>0 \end{cases}$$



No Inequality Constraint
But is not differentiable

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CVX – Convex Optimization

- The Log-Barrier Algorithm for SOCP

$\boldsymbol{I}_-$ is approximated into: $\hat{\boldsymbol{I}}_-$

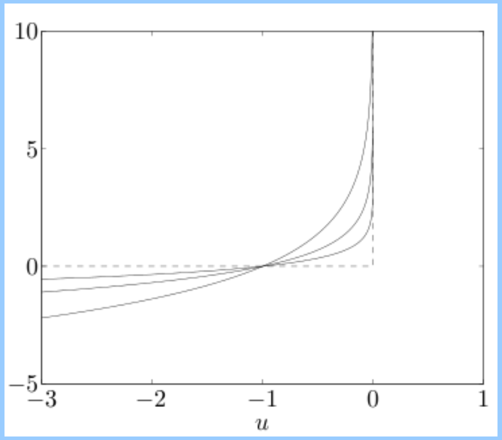
$$\hat{I}_- = -(1/t)\log(-u)$$

$$\text{minimize } f_0(x) + \sum_{i=1}^{m} \left[-(1/t)\log(-f_i(x)) \right]$$

$$\text{subject to } y = Ax$$

for the non-positive reals

$$\phi = \left[-(1/t)\log(-f_i(x)) \right]$$



The Log-Barrier is twice differentiable

CVX – Convex Optimization

- The Log-Barrier function is twice differentiable

$$g_x = \nabla \phi(x) = \sum_{i=1}^{m} \frac{1}{-f_i(x)} \nabla f_i(x)$$

$$H_x = \nabla^2 \phi = \frac{1}{\tau} \sum_{i=1}^{m} \frac{1}{f_i(x)^2} \nabla f_i(x)^T + \sum_{i=1}^{m} \frac{1}{-f_i(x)^2} \nabla^2 f_i(x)$$

- Approximation of the Log-Barrier Problem

$$f_0(x+\Delta x) \approx x + \langle g_x | \Delta_x \rangle + \frac{1}{2} \langle H_x \Delta_x | \Delta_x \rangle = q(x+\Delta x)$$

Solution: find Δ_x that minimizes $q(x+\Delta_x)$

subject to Ax=y $\textit{is the solution of the following linear equiation set:}$

$$\tau \begin{pmatrix} H_x & A_0^T \\ A_0 & 0 \end{pmatrix} \begin{pmatrix} \Delta_x \\ v \end{pmatrix} = -\tau g_x$$

CVX – Convex Optimization

- We have used a “matrix free” solver such as Conjugate Gradients (C.G.)
- The following two algorithms results has been opposed:
- Min-l1 with bounded residual correlation (Dantzig Selector) LP;

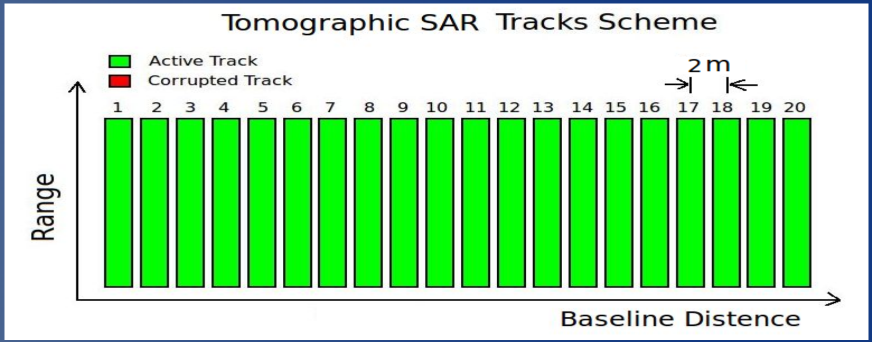
$$\boxed{min\|x\|_1 \text{ subject to } \|A^*(Ax-y)\|_\infty \leqslant \gamma}$$

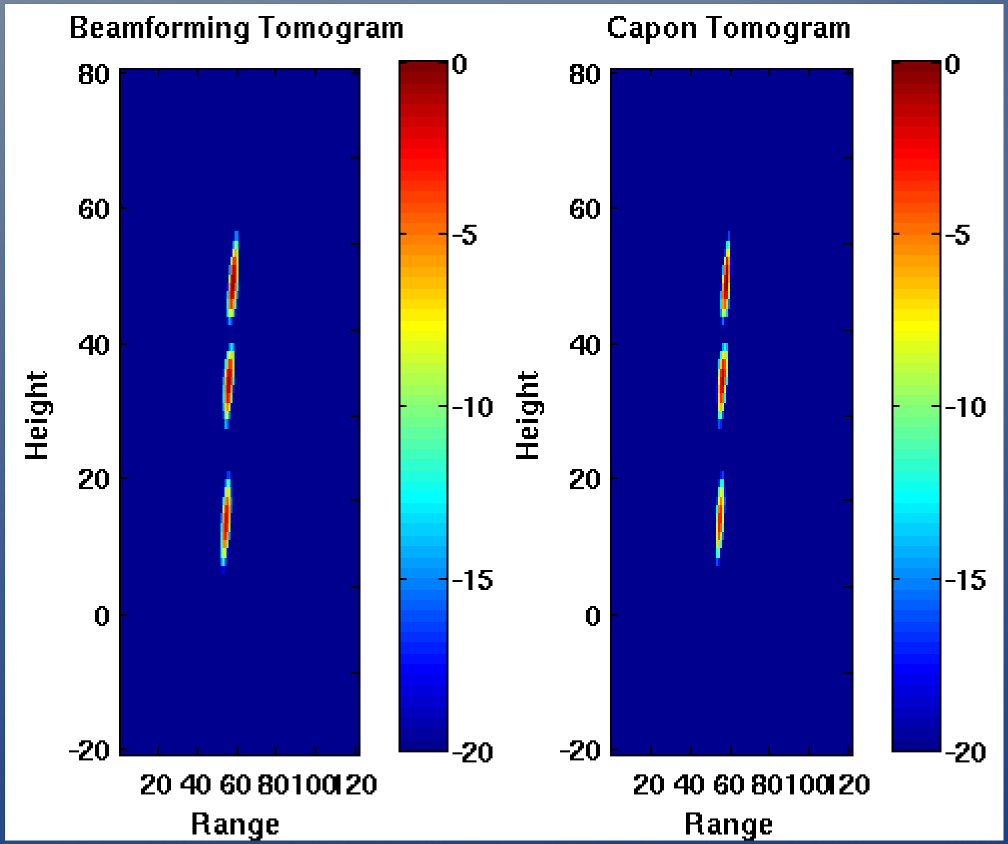
- Min-l1 with quadratic constraints SOCP

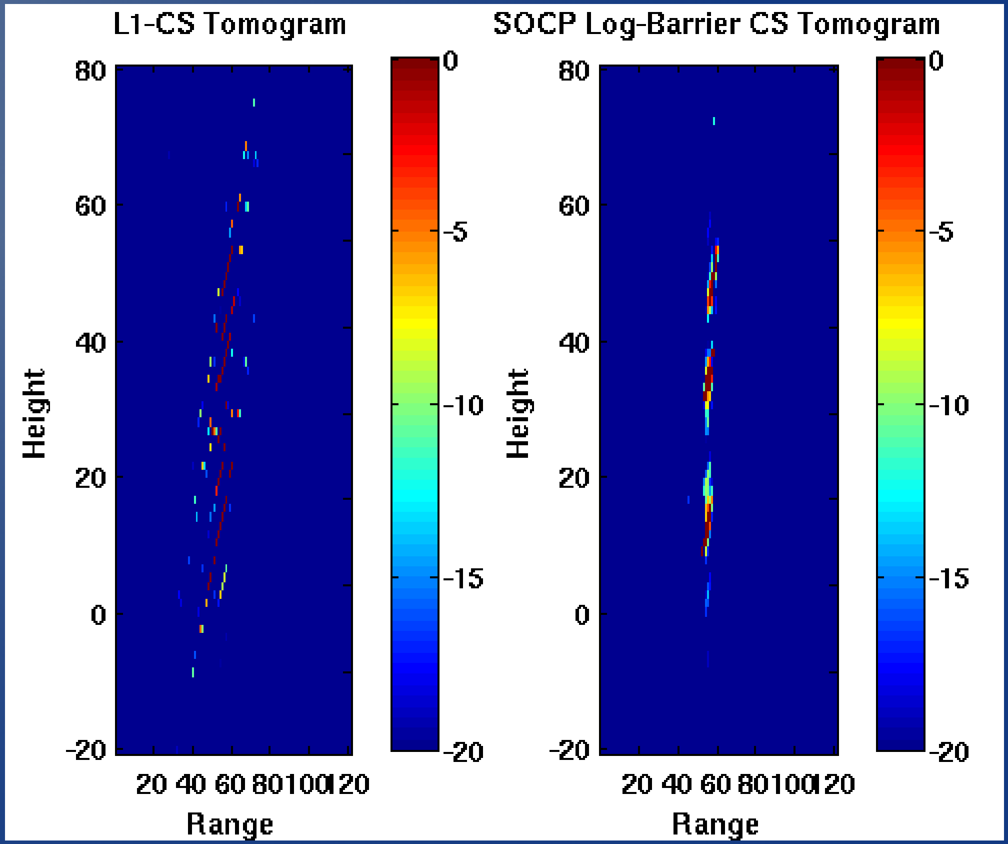
$$\boxed{min\|x\|_1 \text{ subject to } \|(Ax-y)\|_2 \leqslant \epsilon}$$

CVX – Convex Optimization

- Validation of the Log-Barrier algorithm to L1
- Point Scatterers







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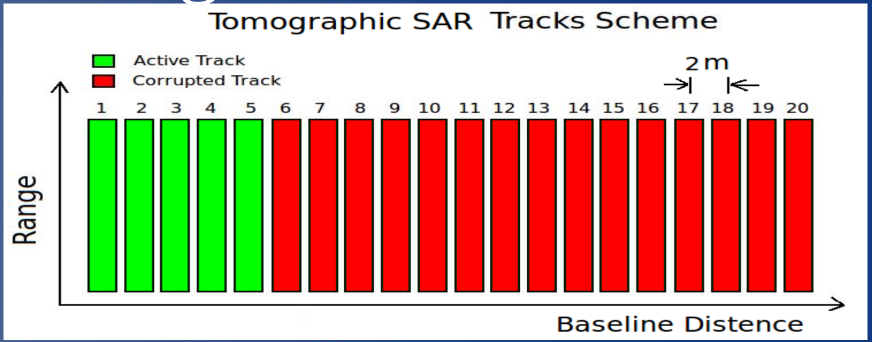
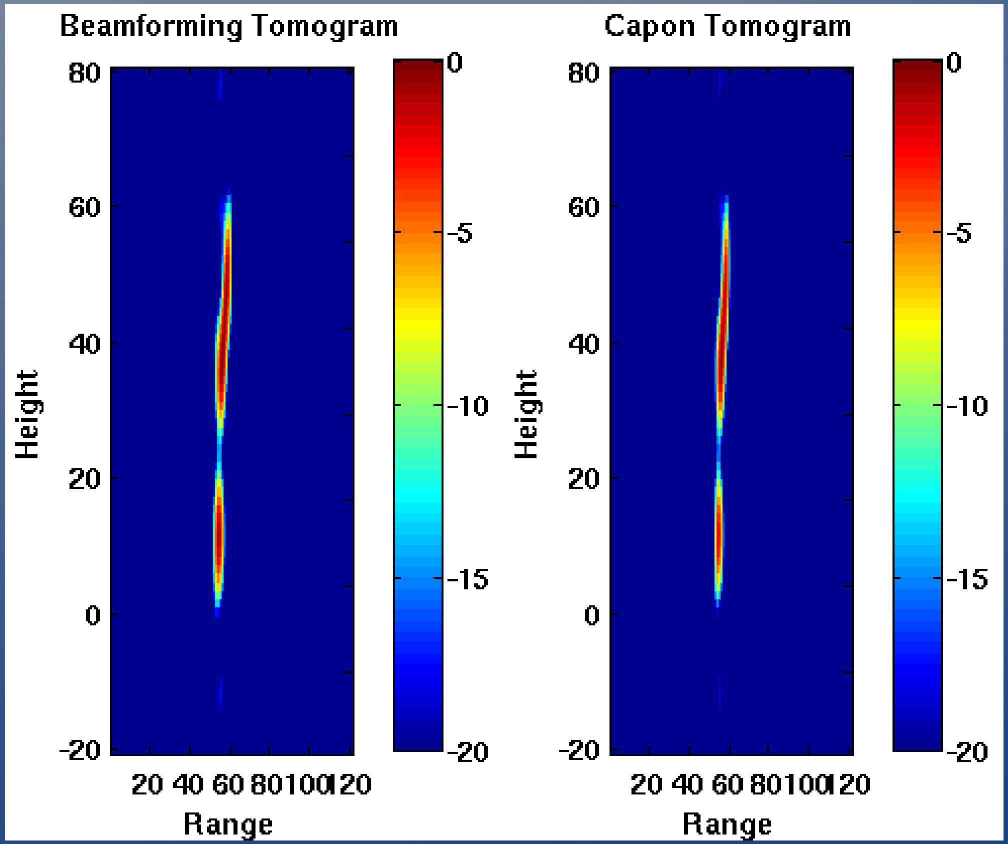
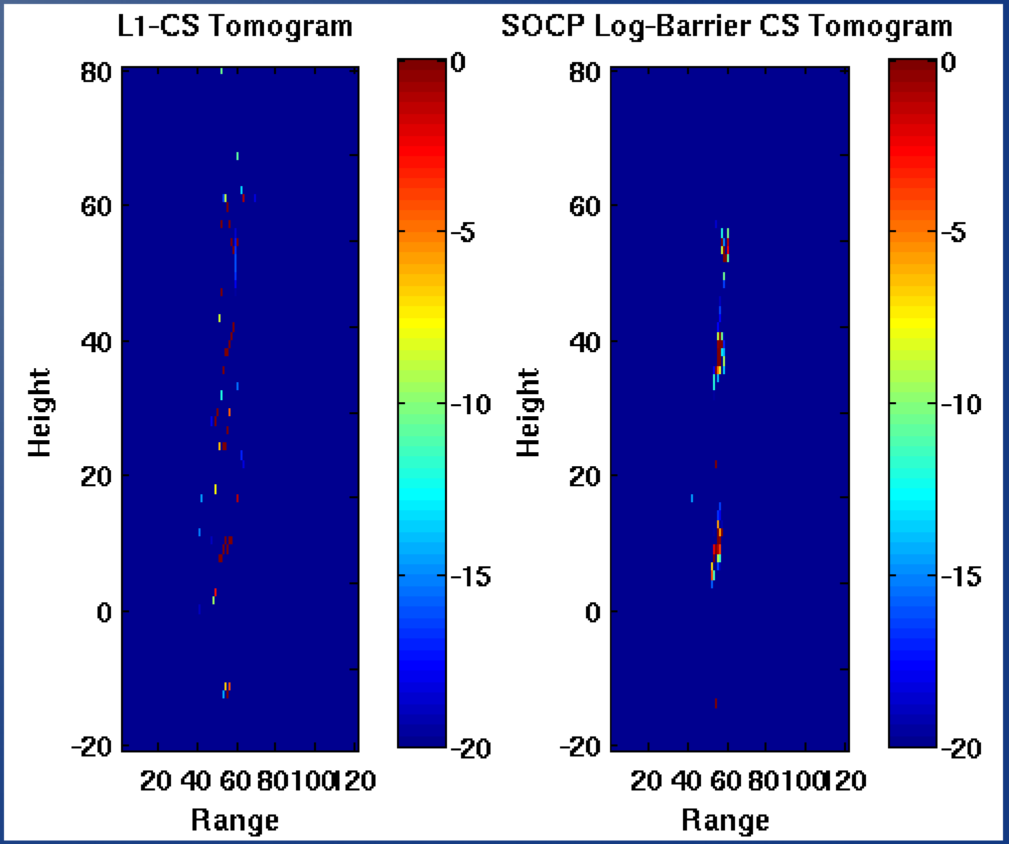
CVX – Convex Optimization

- Validation of the Log-Barrier algorithm
- Point Scatterers







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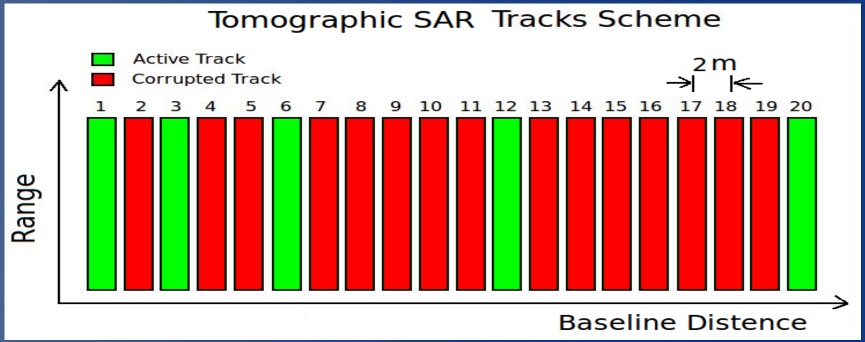

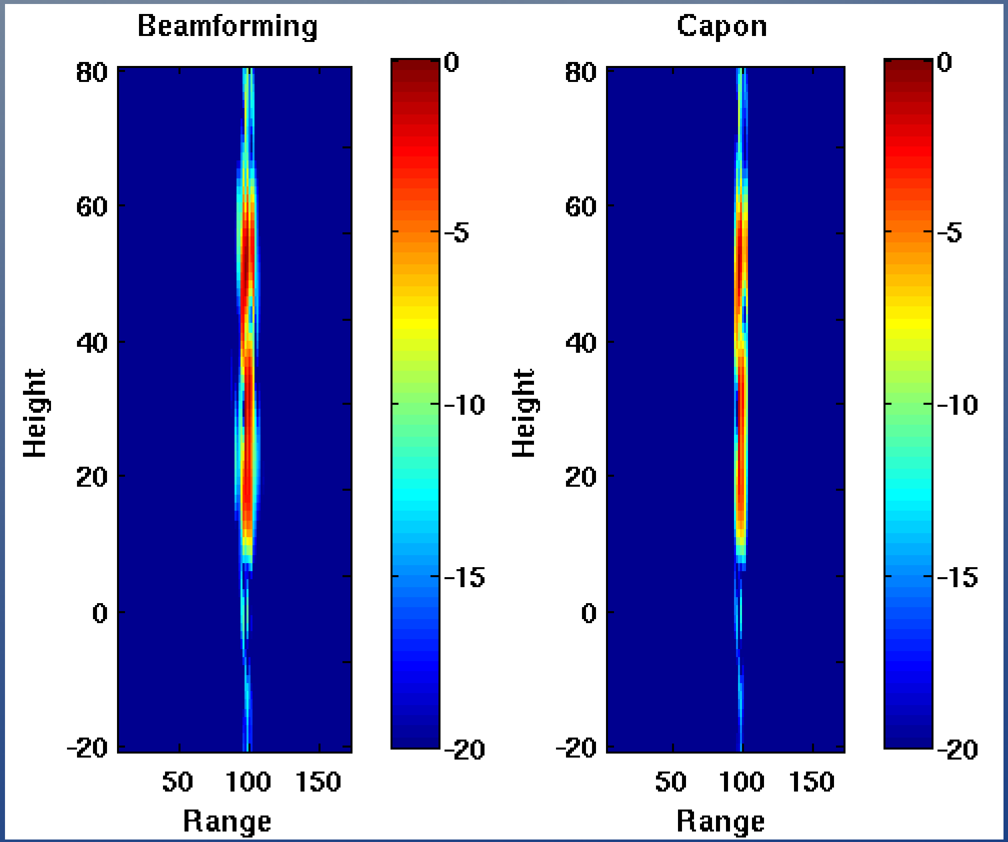

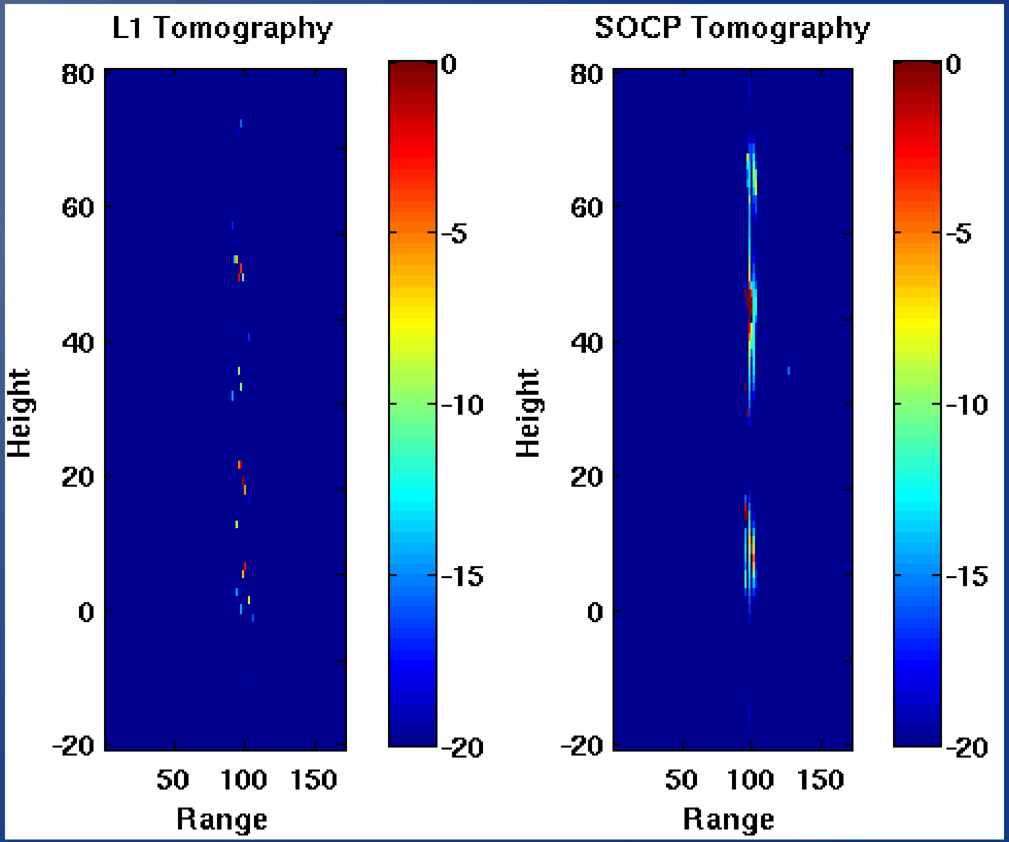

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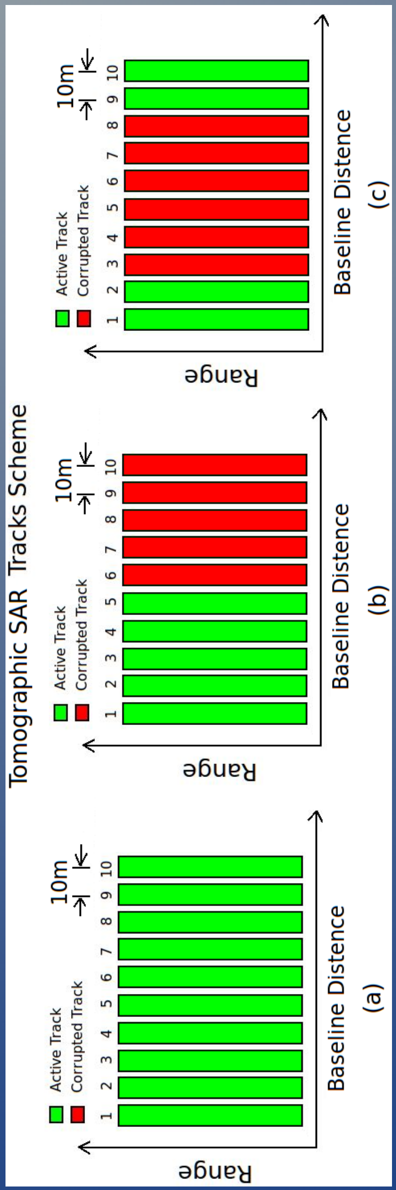
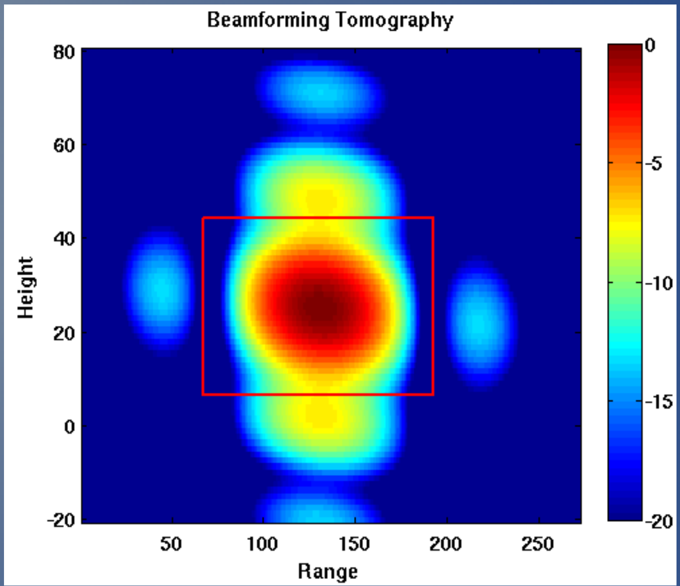
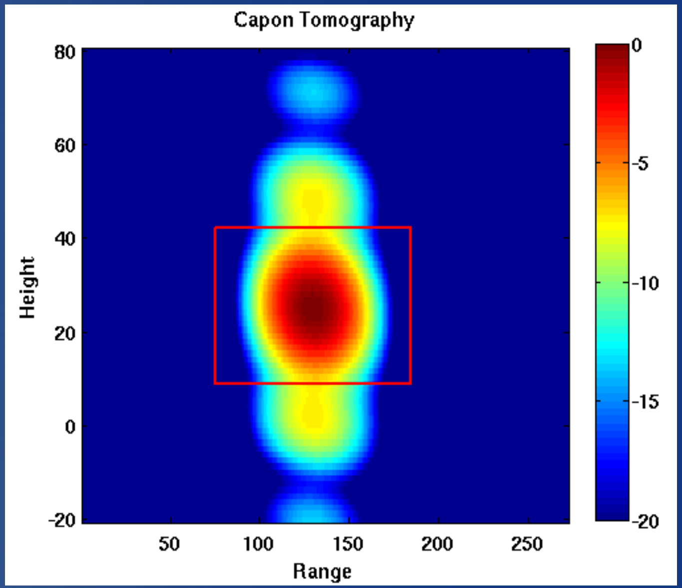
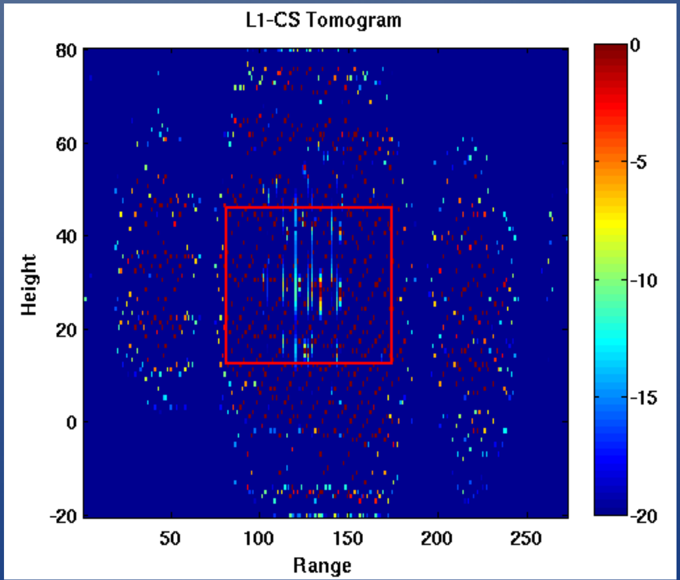
CVX – Convex Optimization

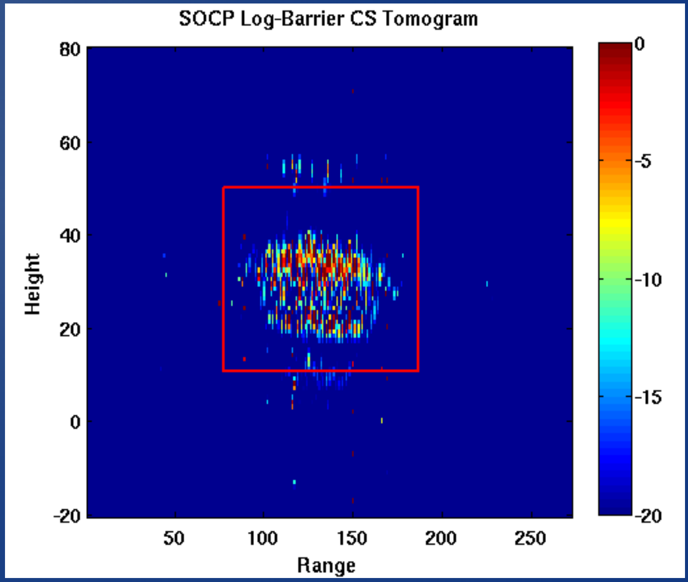
- Distributed Scatterer - Case (a)











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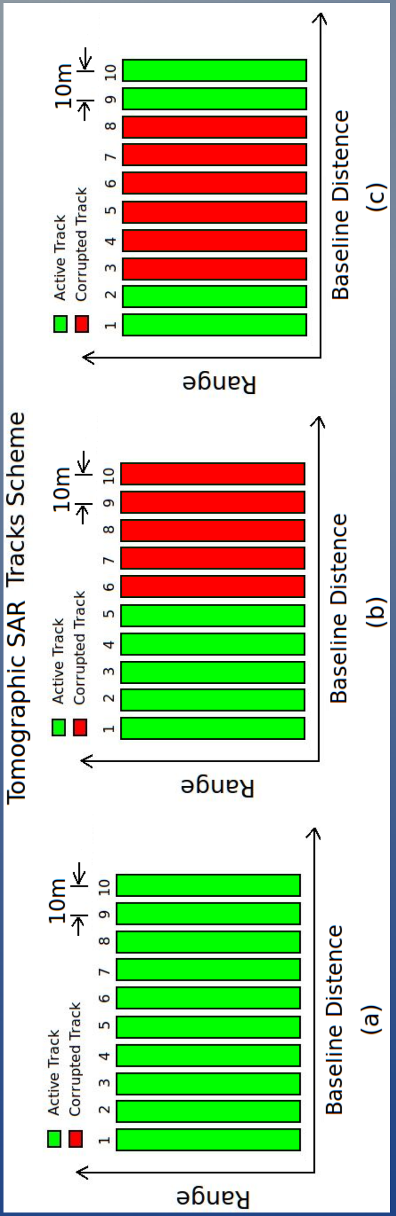
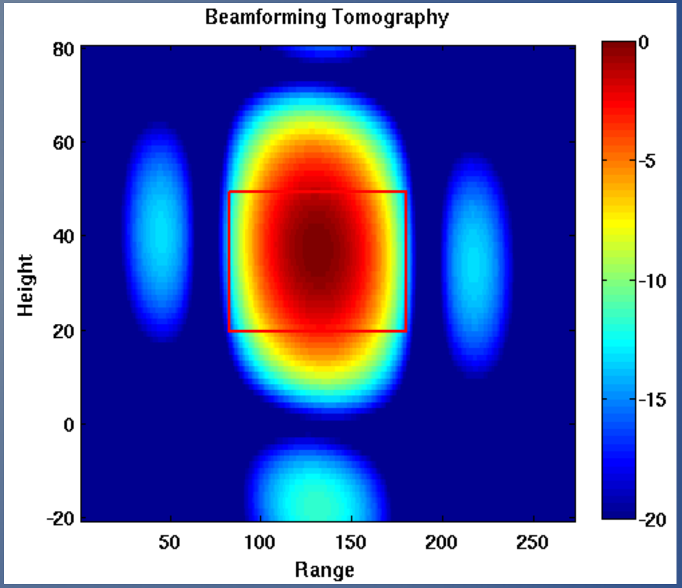
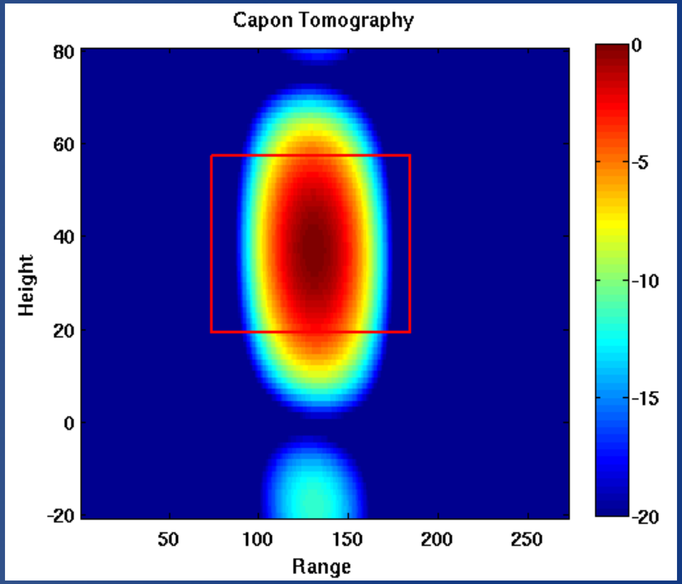
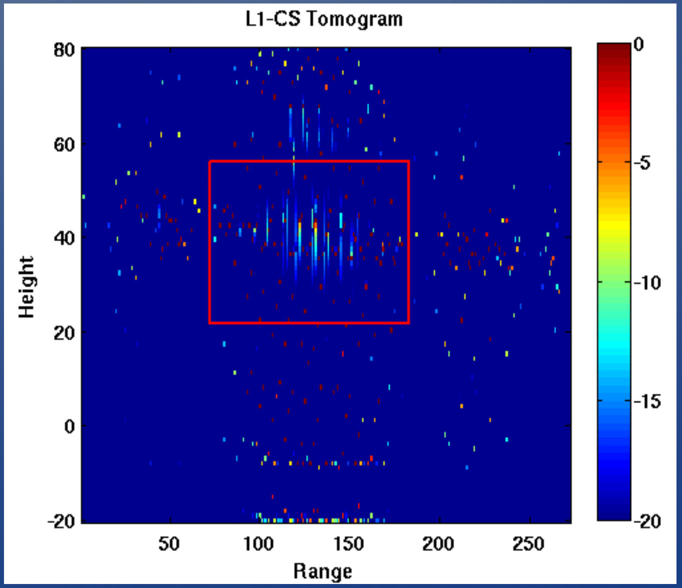
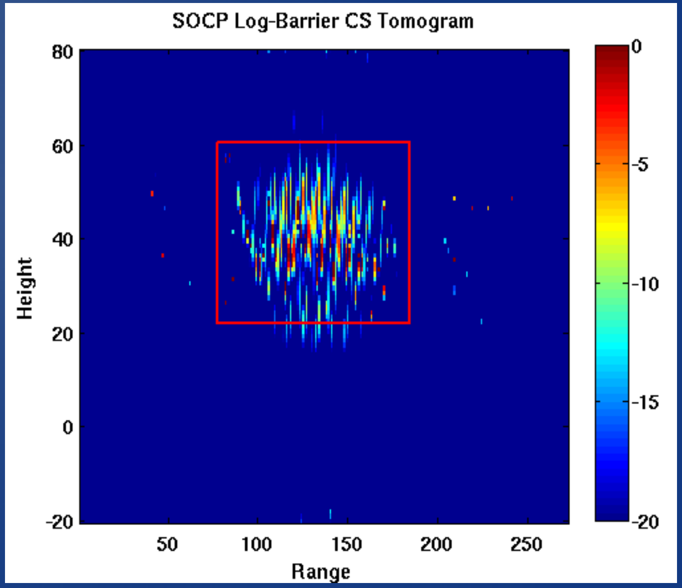
- Distributed Scatterer - Case (b)











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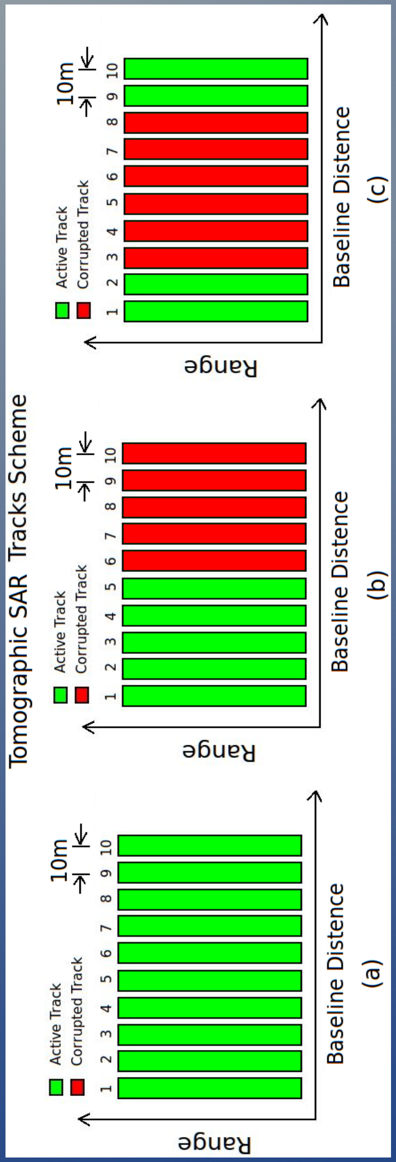
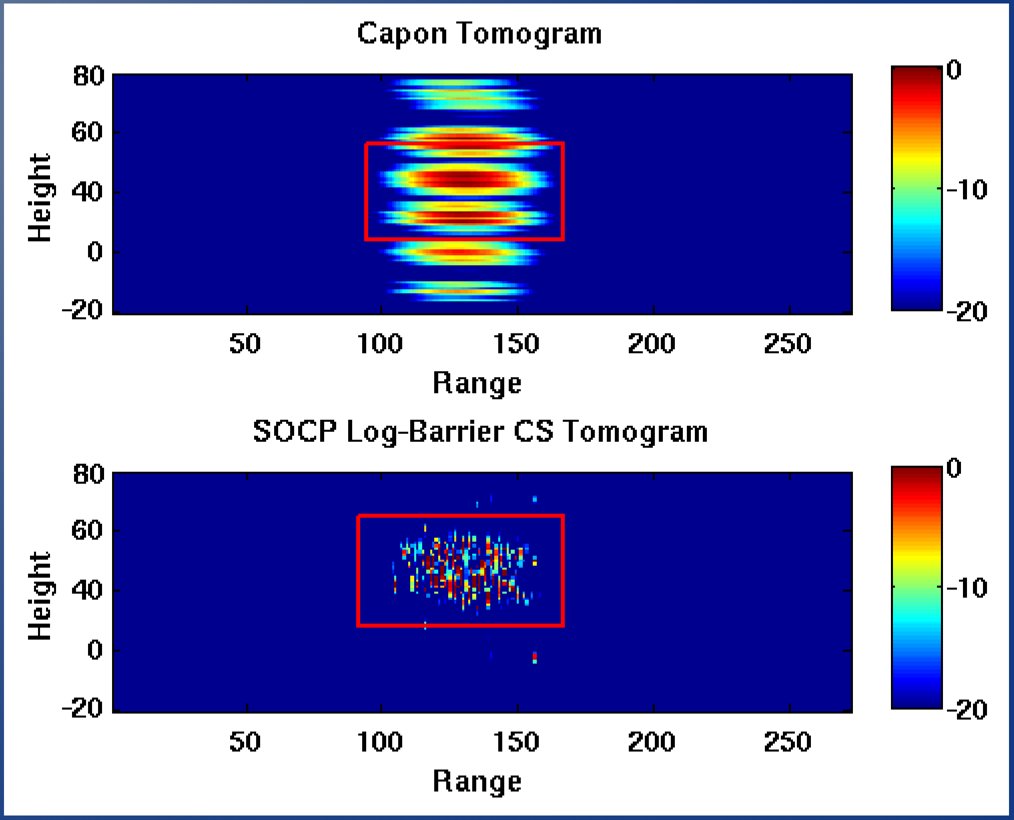


- Distributed Scatterer Case (c)



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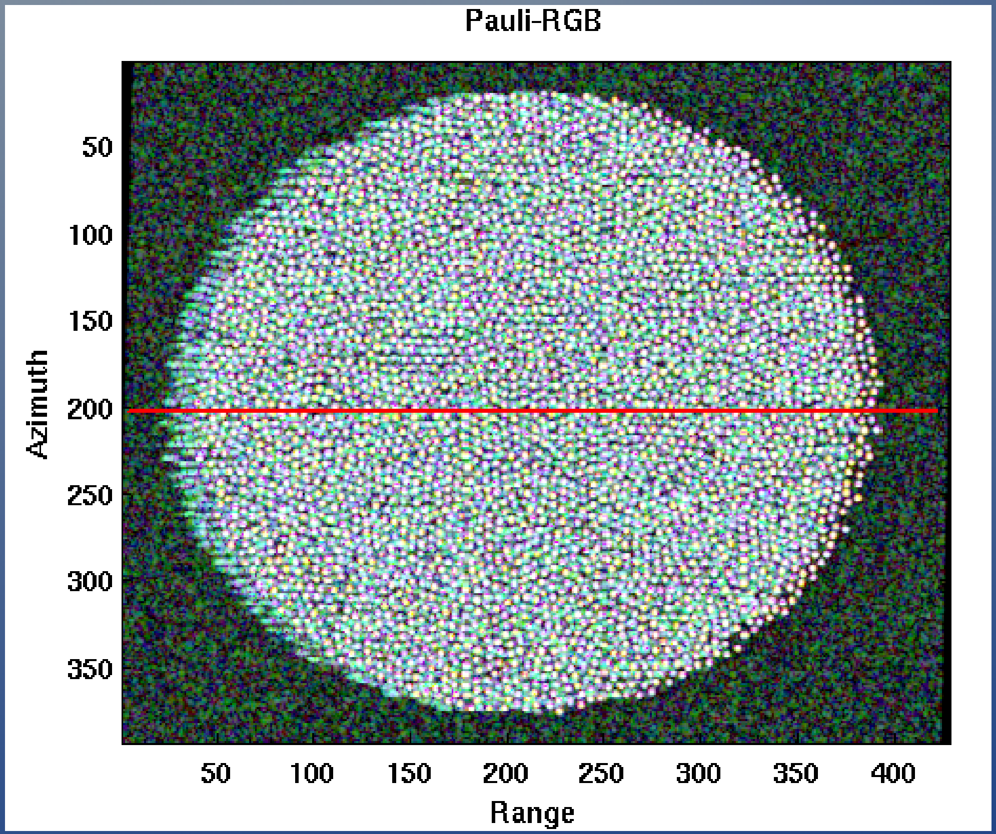
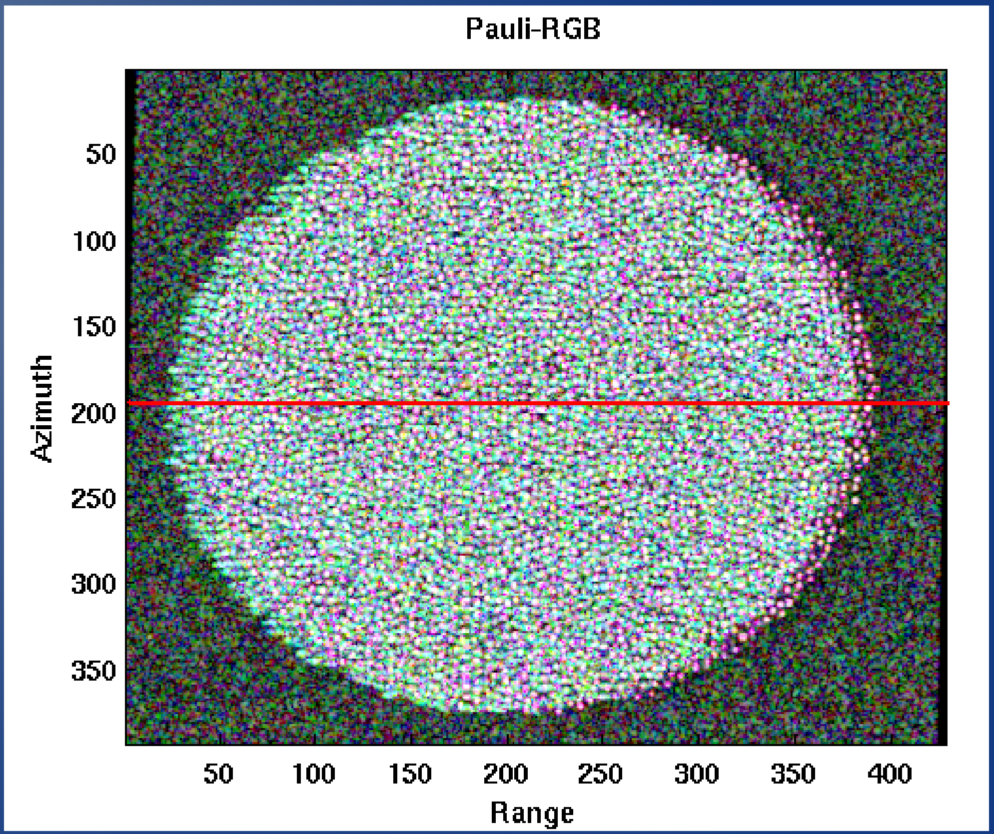
CVX – Convex Optimization

- Realistic PolSARPro Vegetation Environment

P-band



L-band



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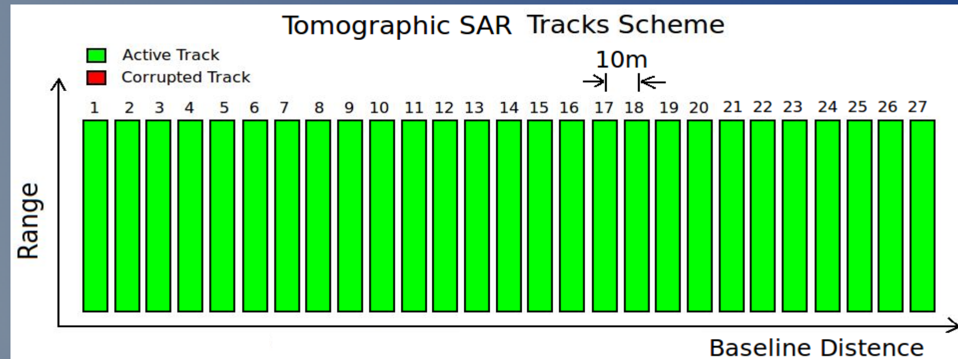

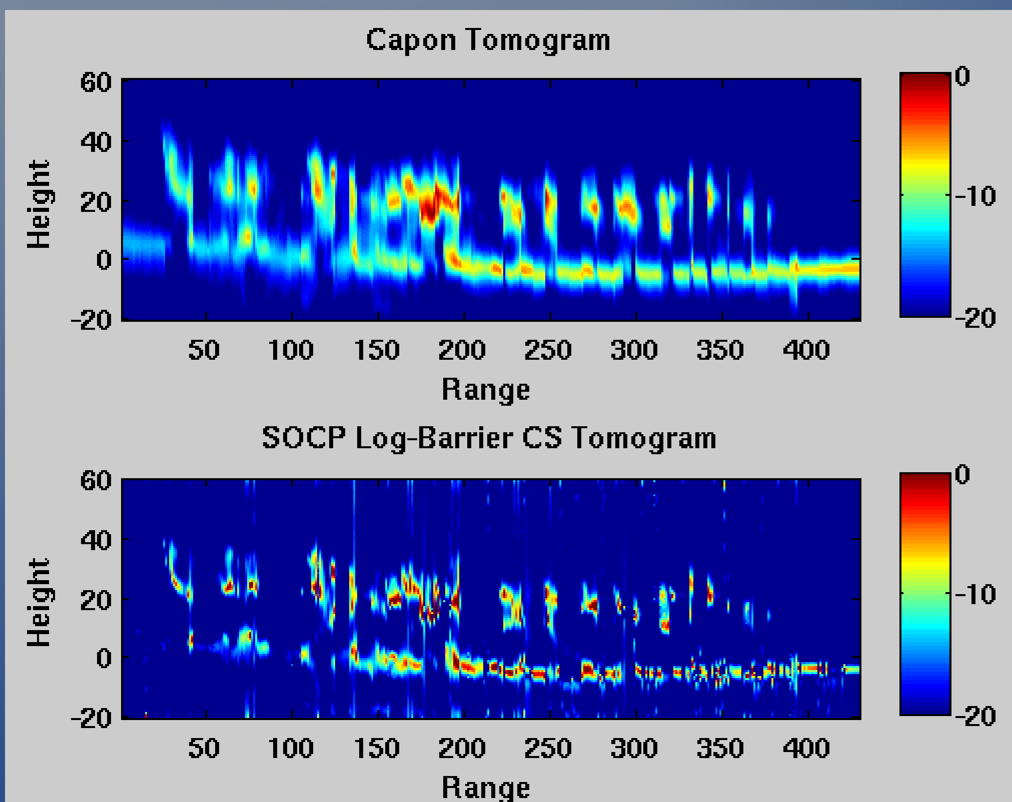

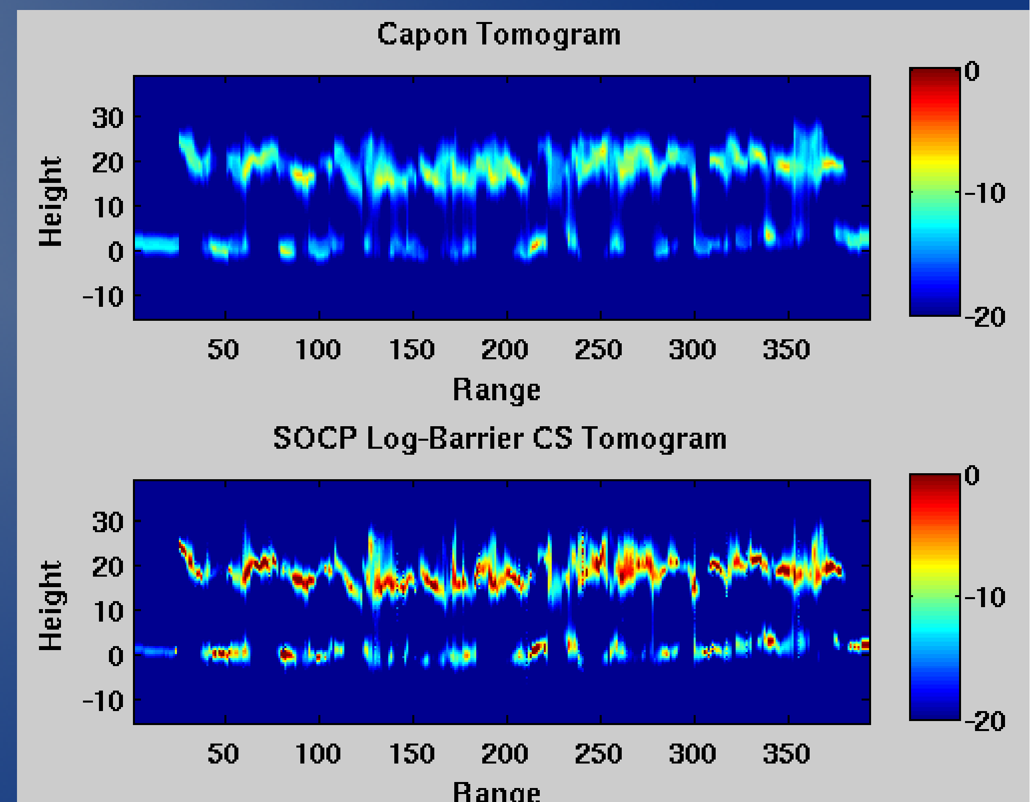

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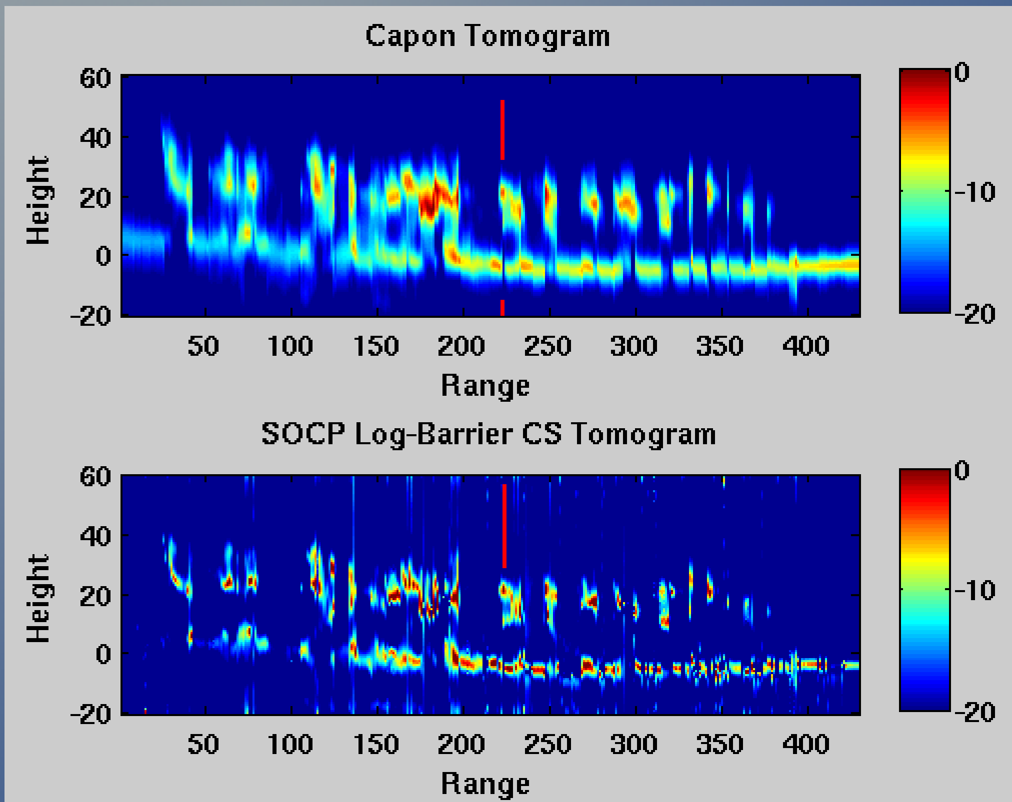
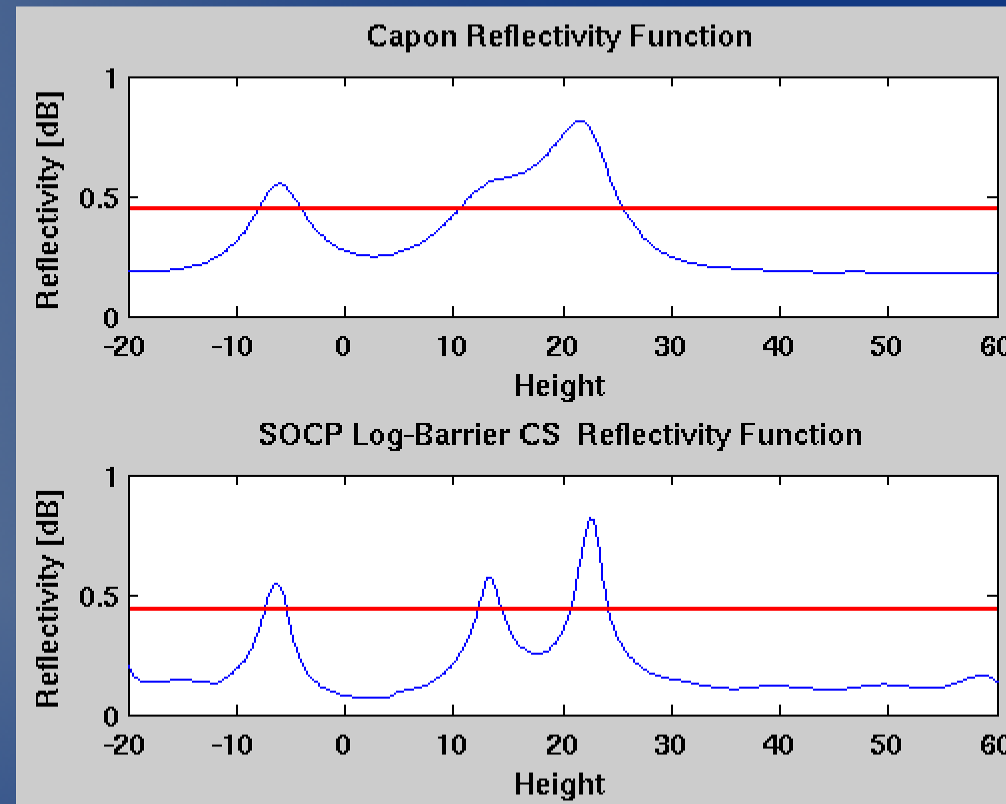




Capon Filter: Two layer definition

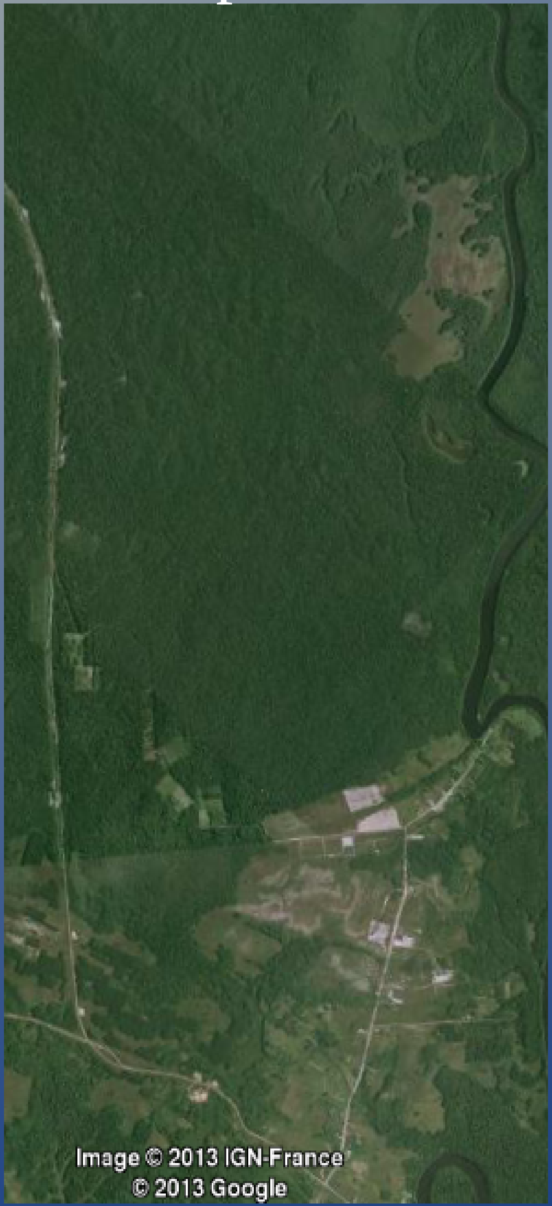
SOCP: Three layer definition

Airborne Real Data-Set

- Mission: TROPISAR-2009 (SETHI ONERA ESA)
- Geometry:
- 6 Tracks;
- Full POL;
- P and L-Band;
- Vertical Baseline 50 ft;
- Height 4000m.

Real data P-band

optical



radar

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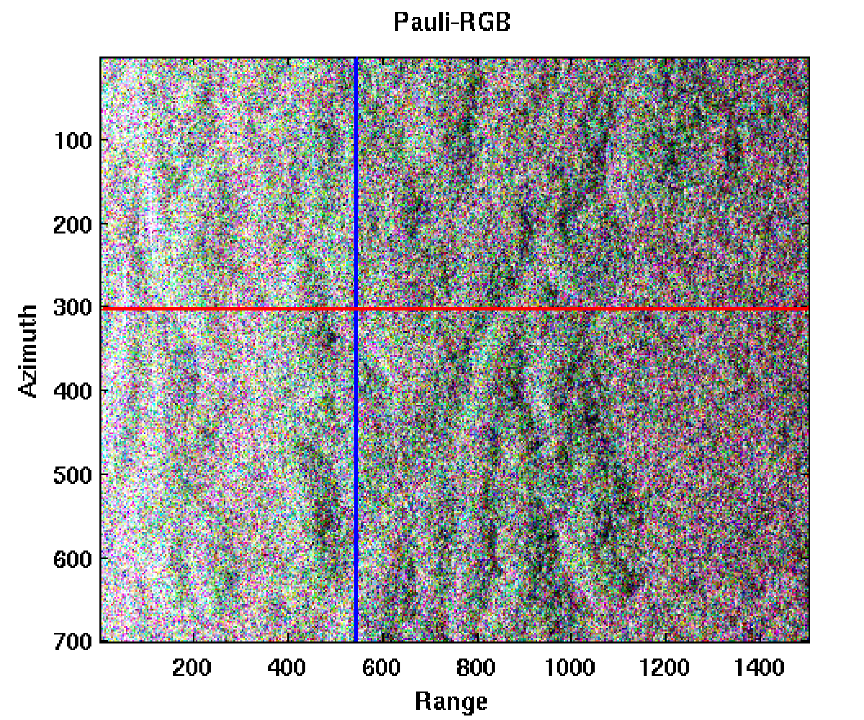
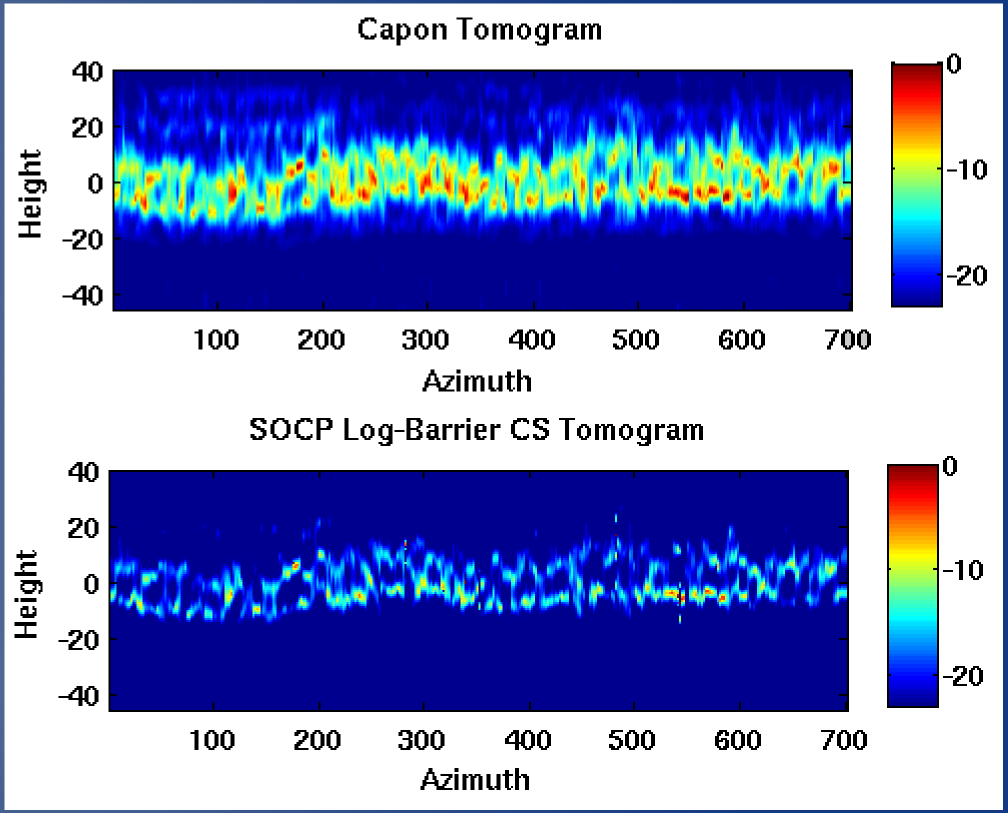
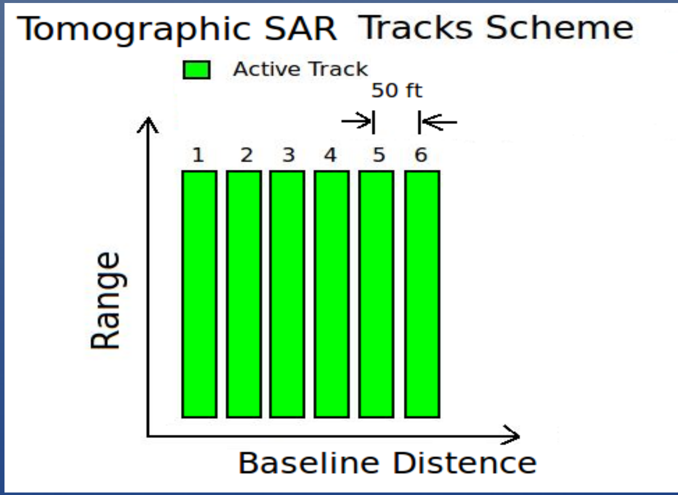
CVX – Convex Optimization

- Blue line case Tomogram







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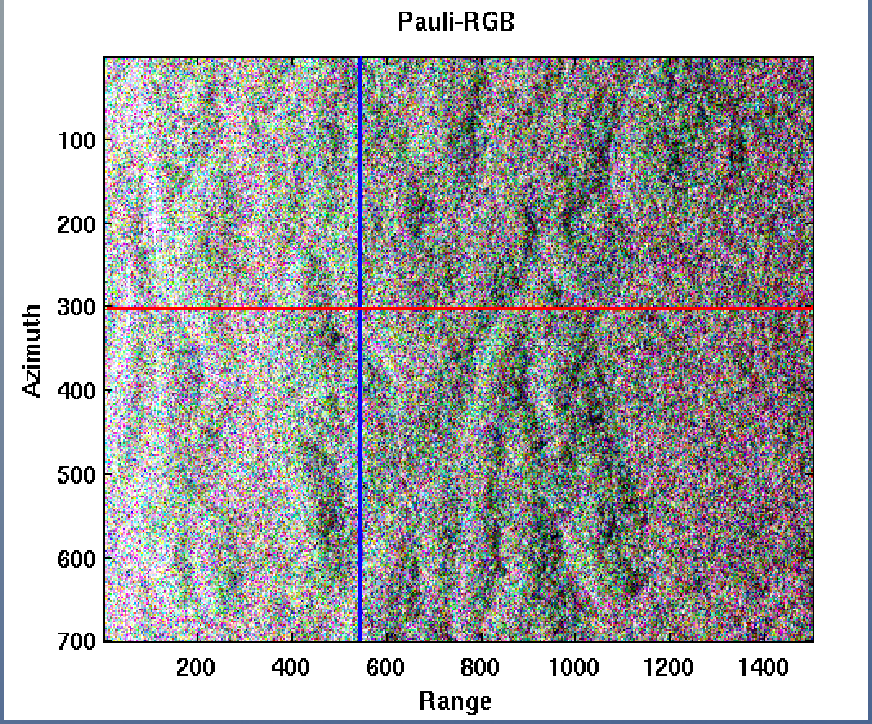
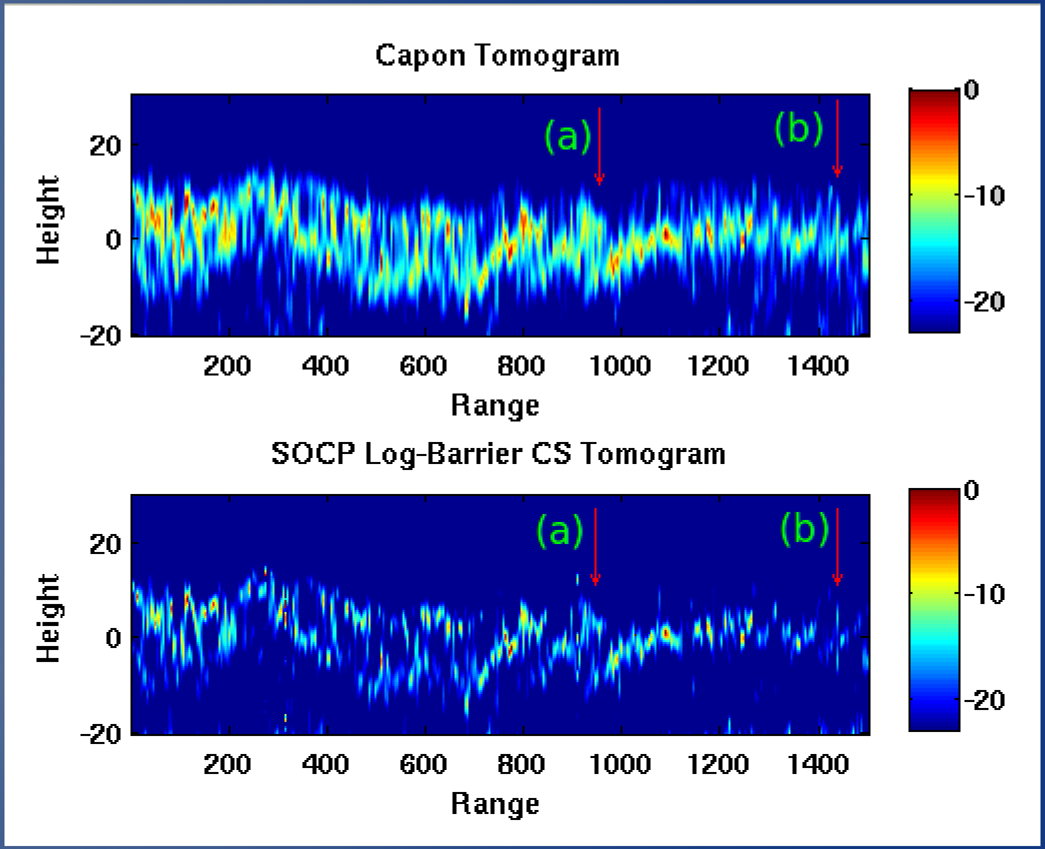
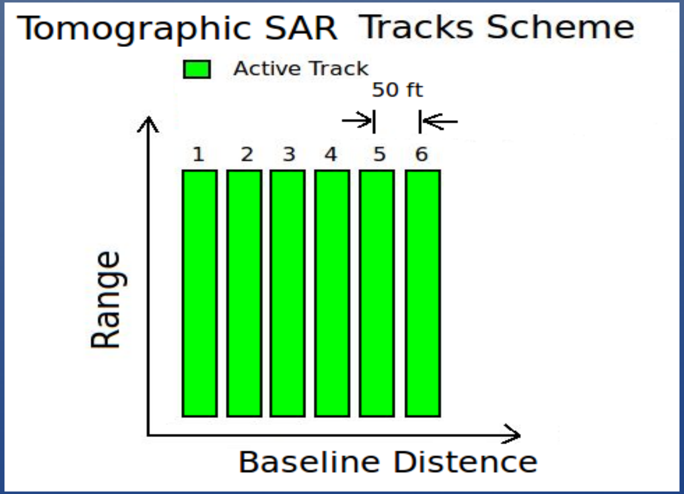
CVX – Convex Optimization

- Red line case Tomogram







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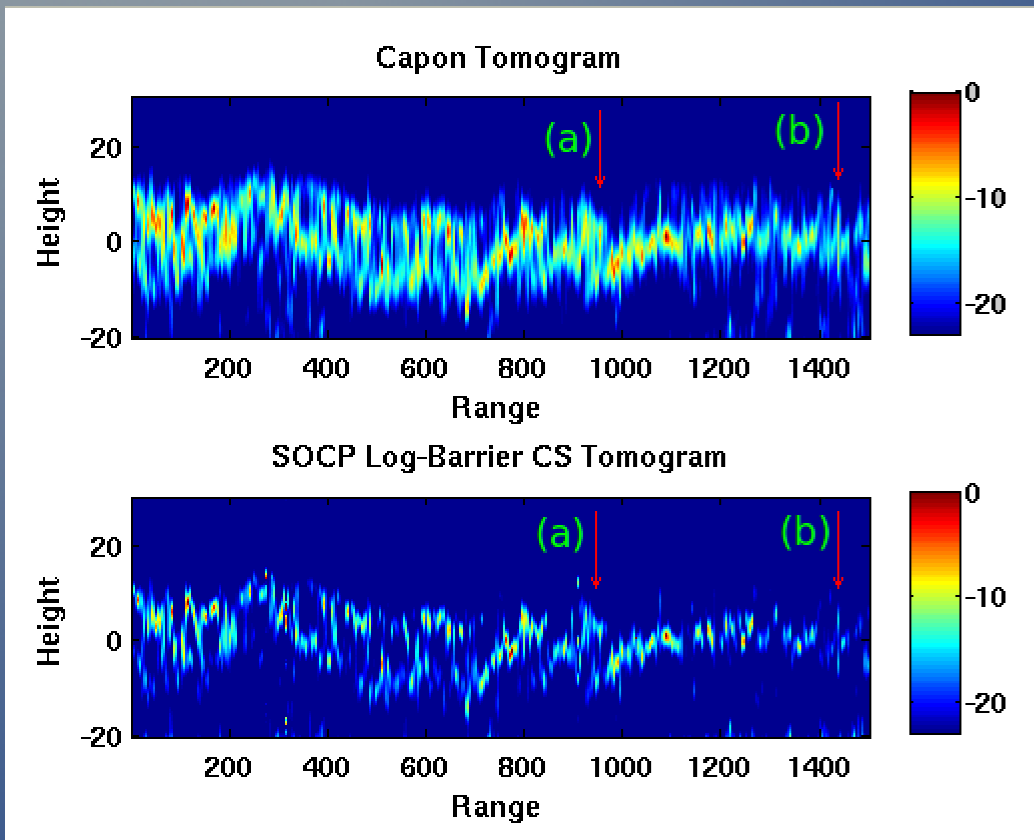
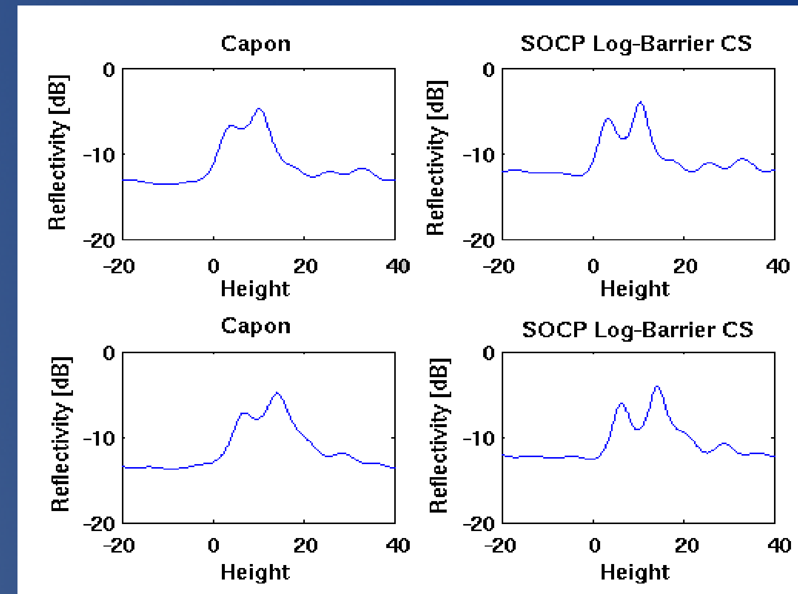
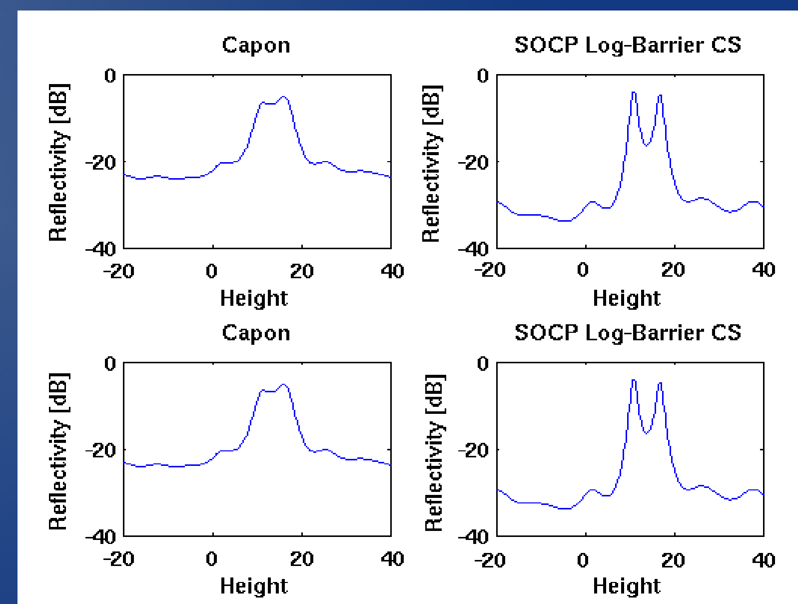


(a)-line



(b)-line



A great resolution improvement is observed

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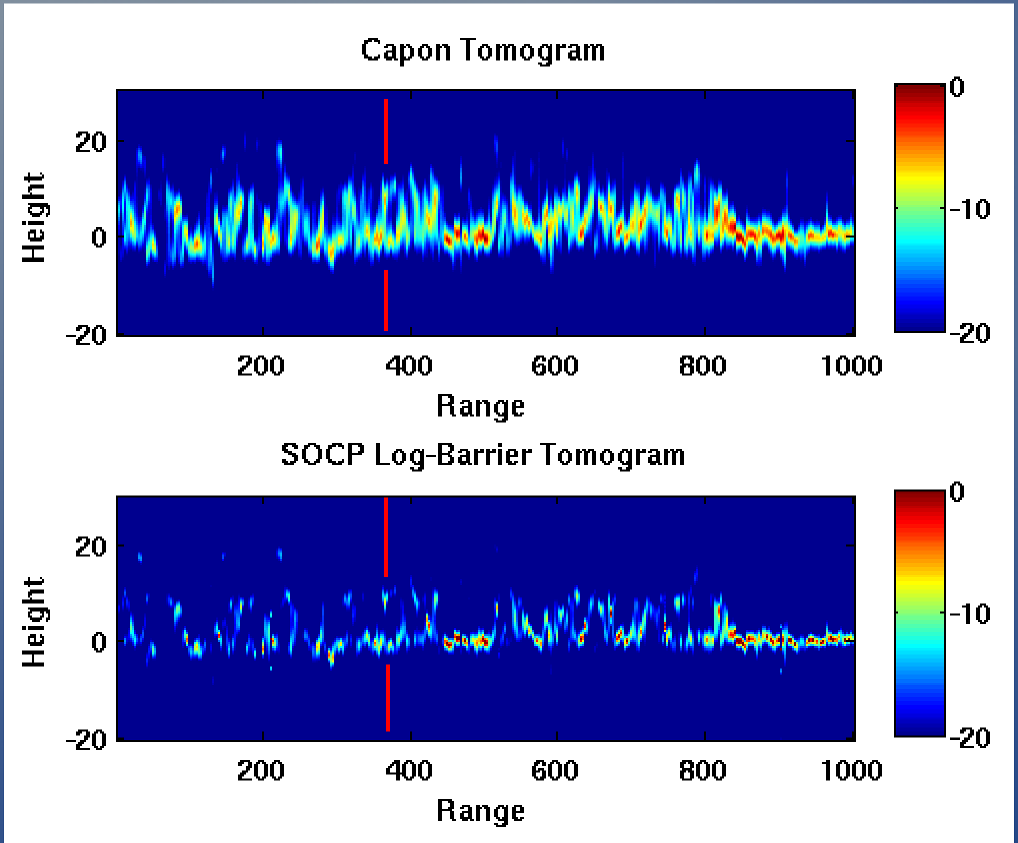
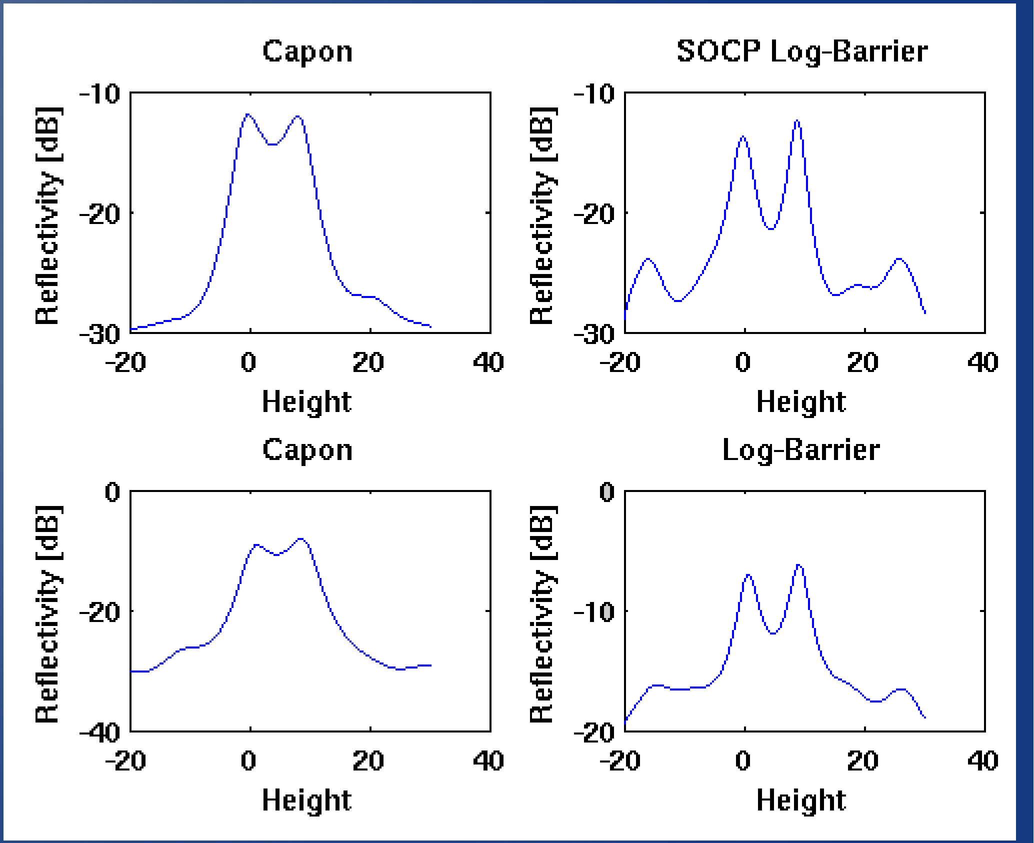
CVX – Convex Optimization





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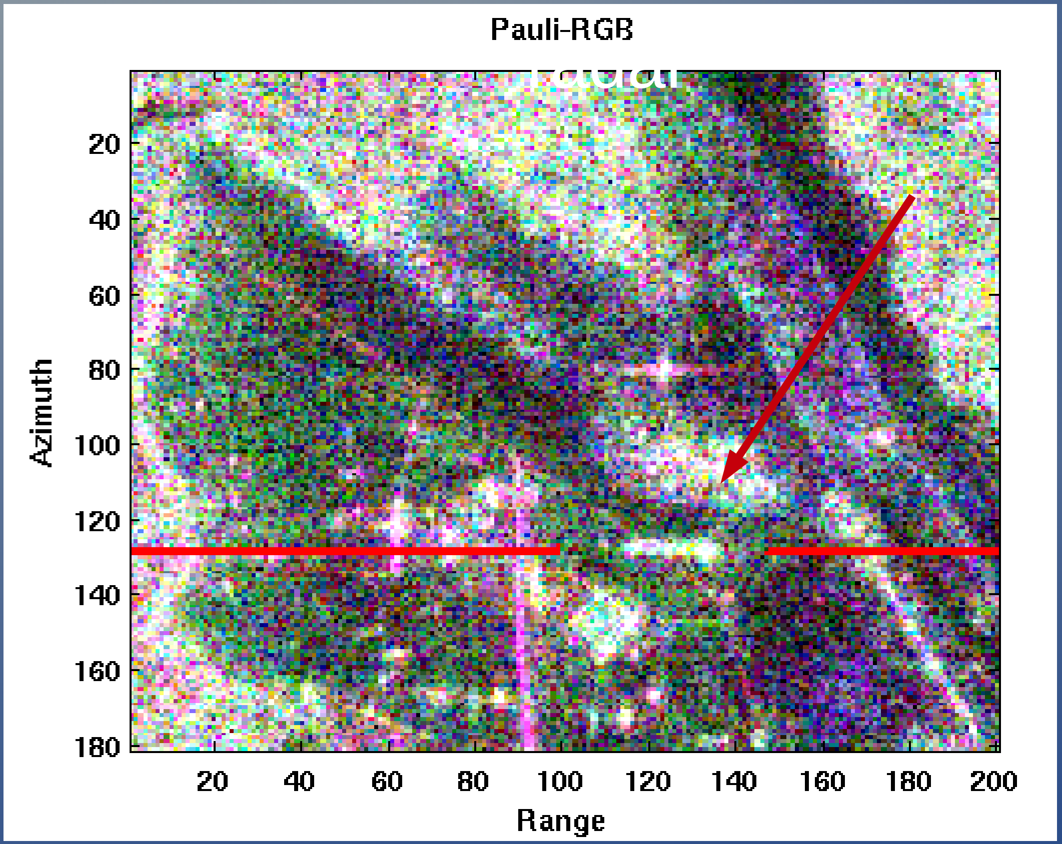
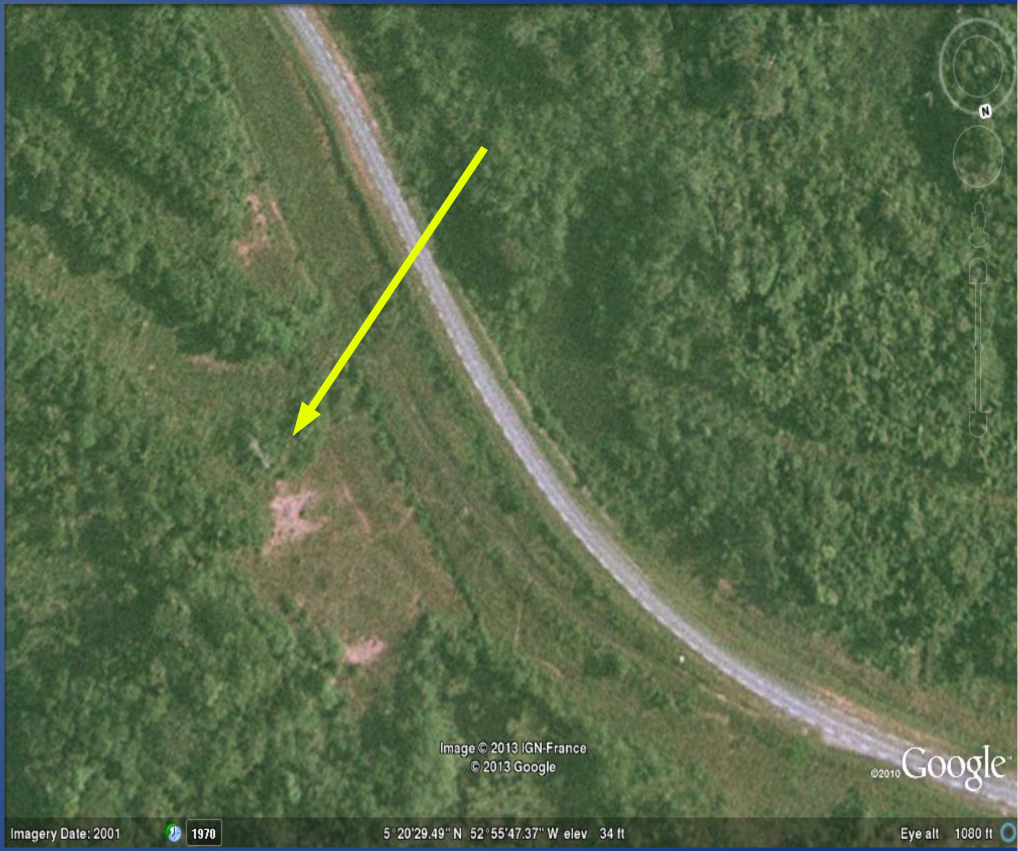
CVX – Convex Optimization

optical

radar





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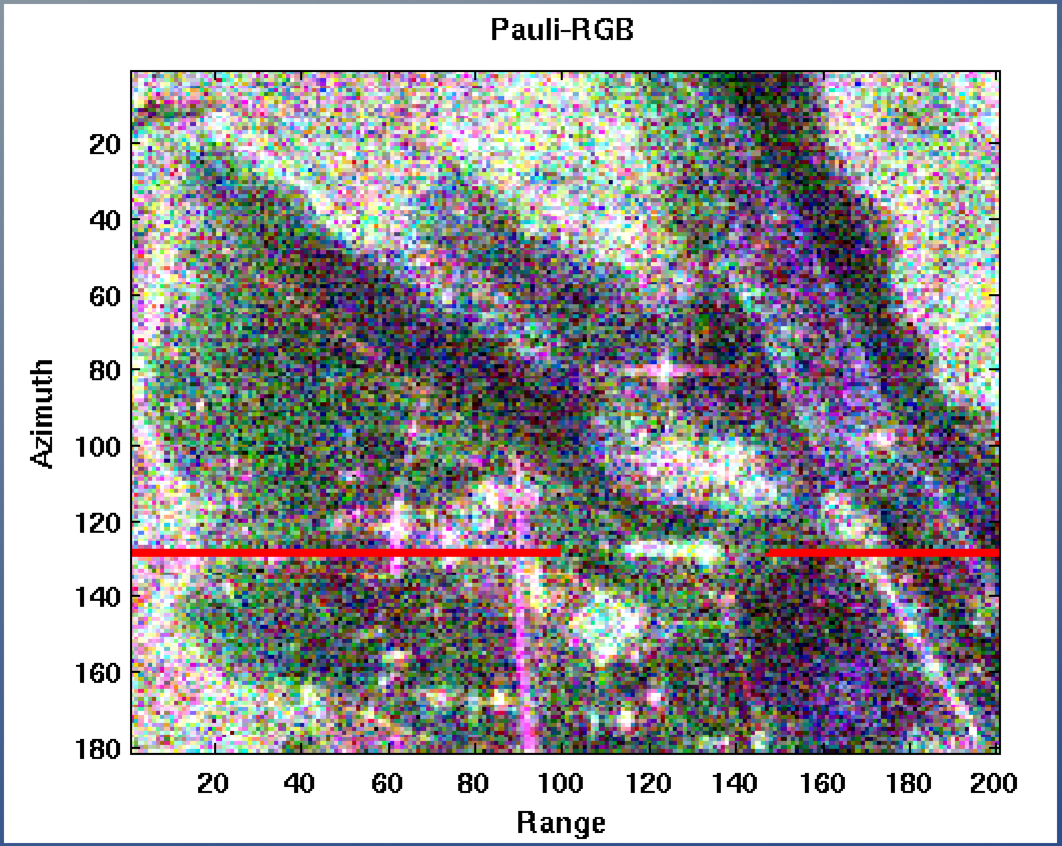

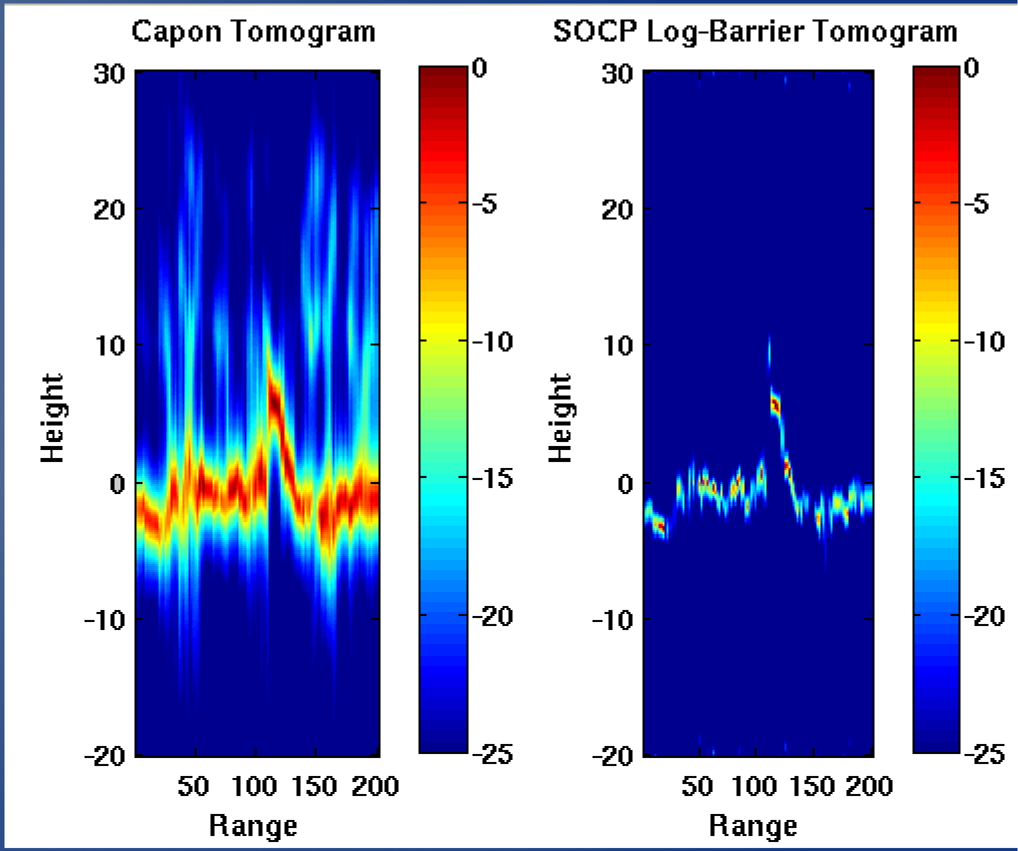

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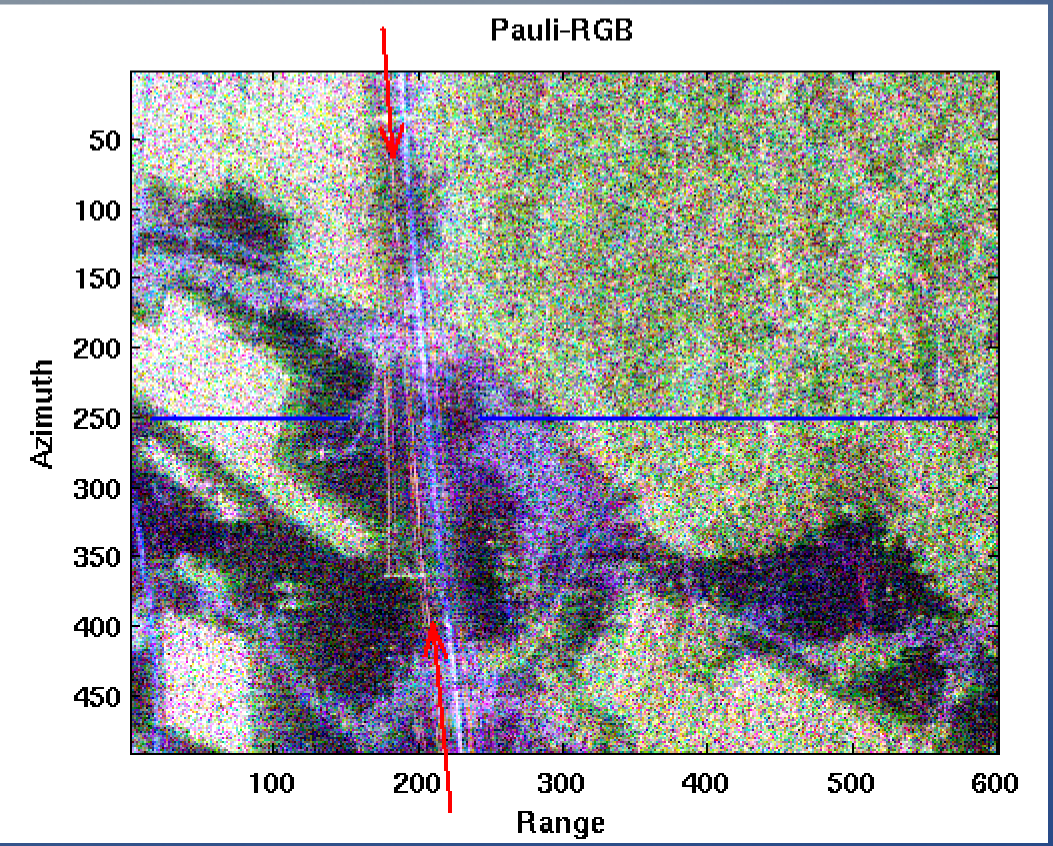

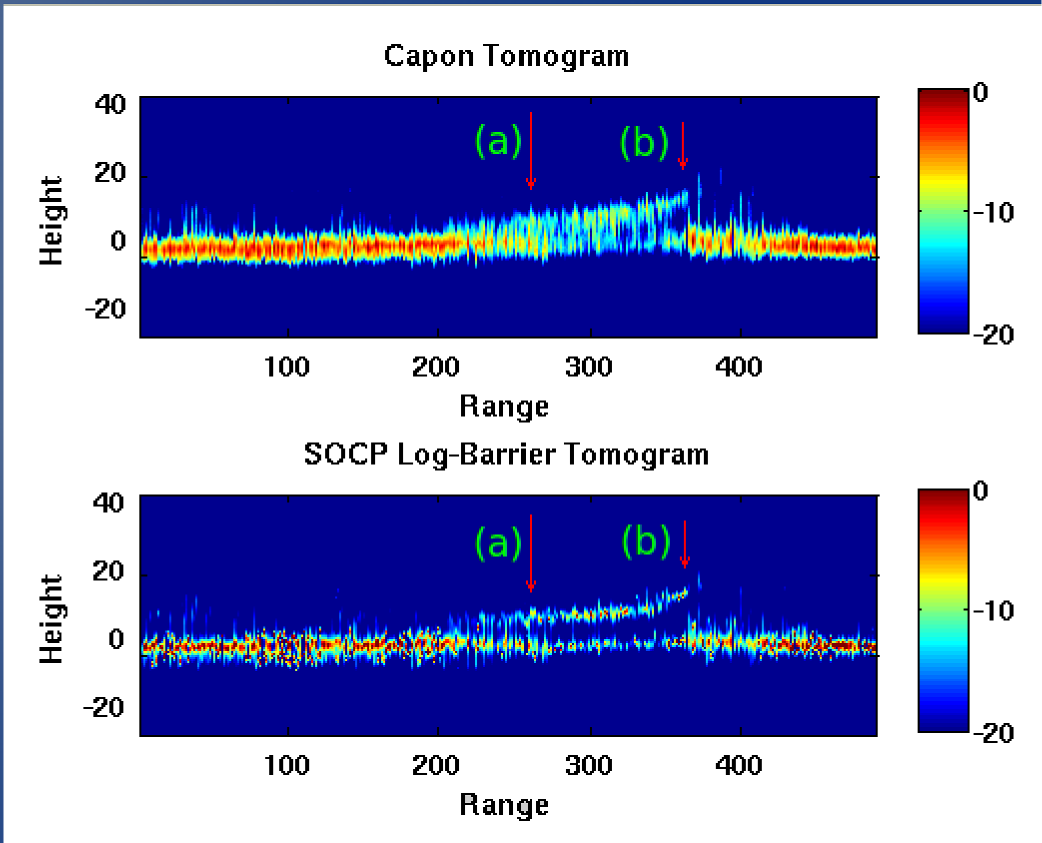

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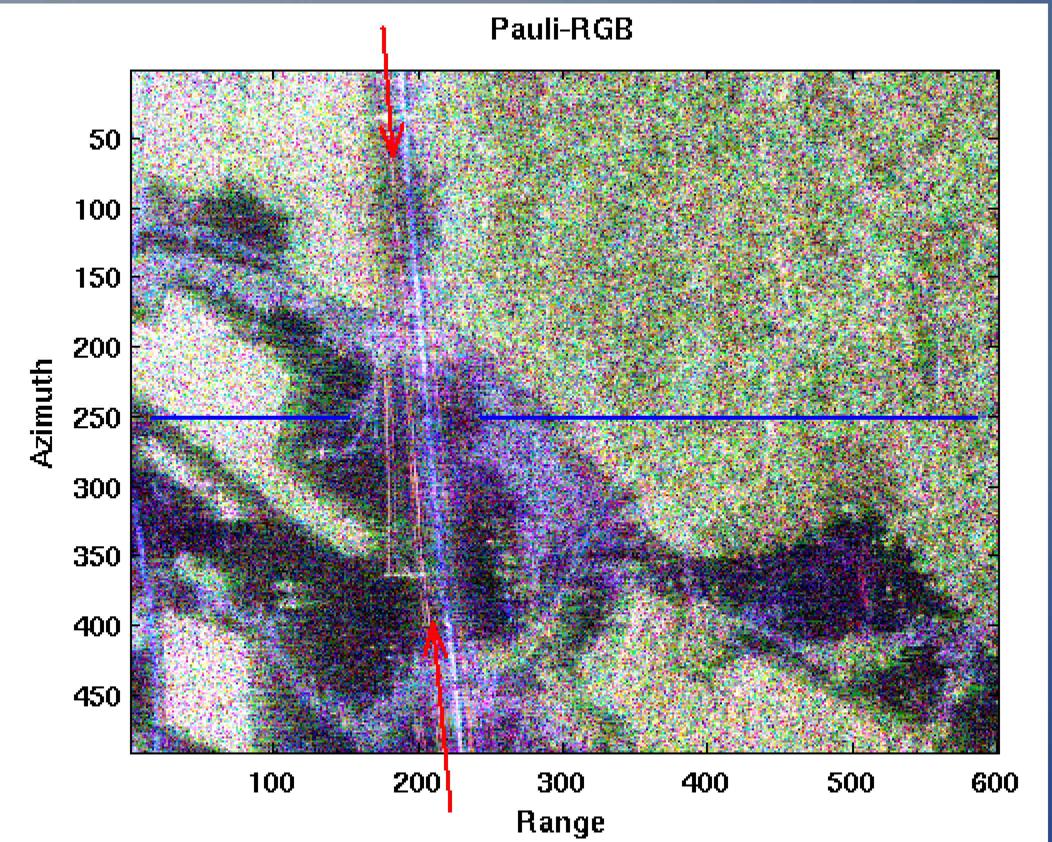
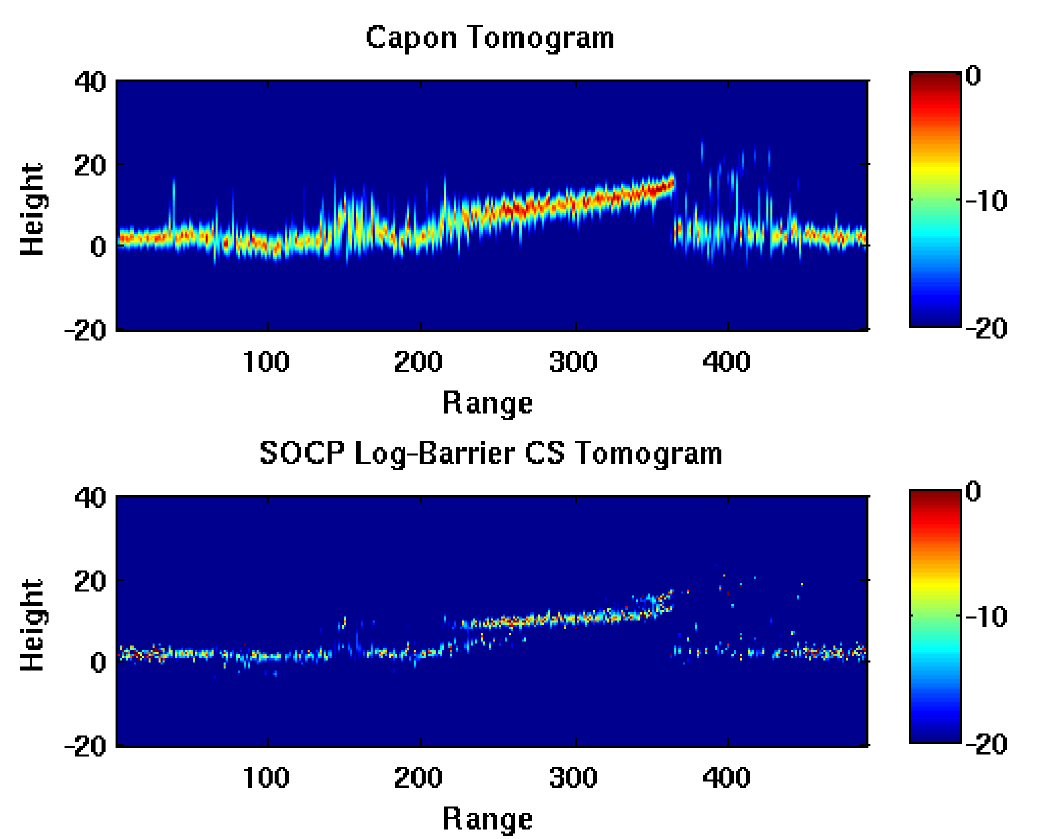
CVX – Convex Optimization





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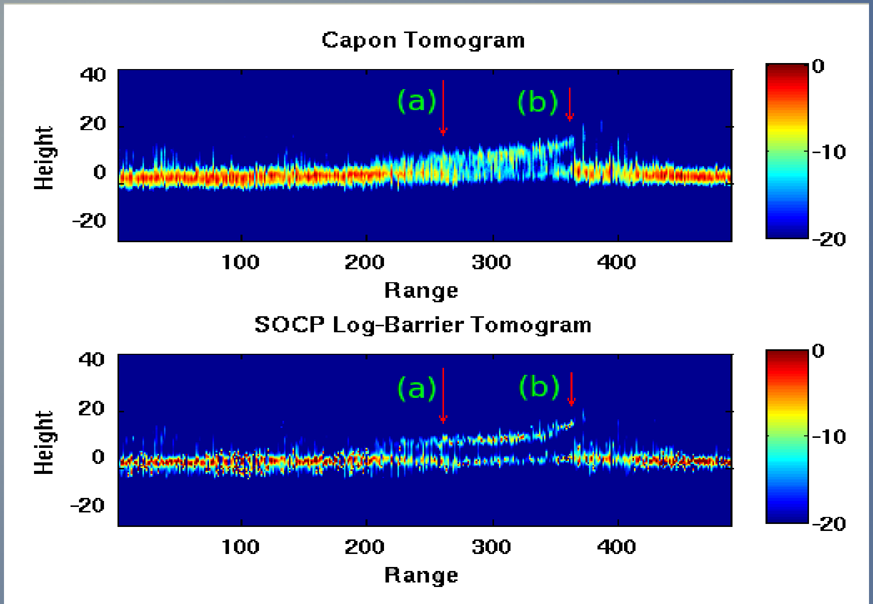
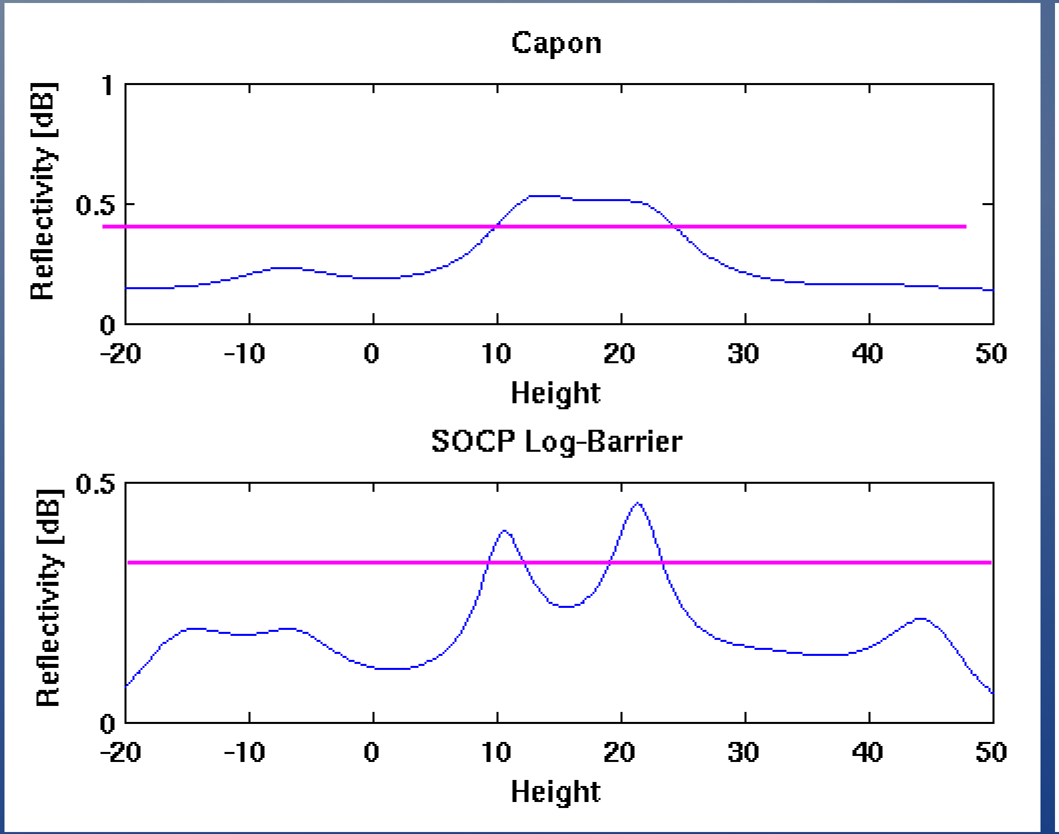
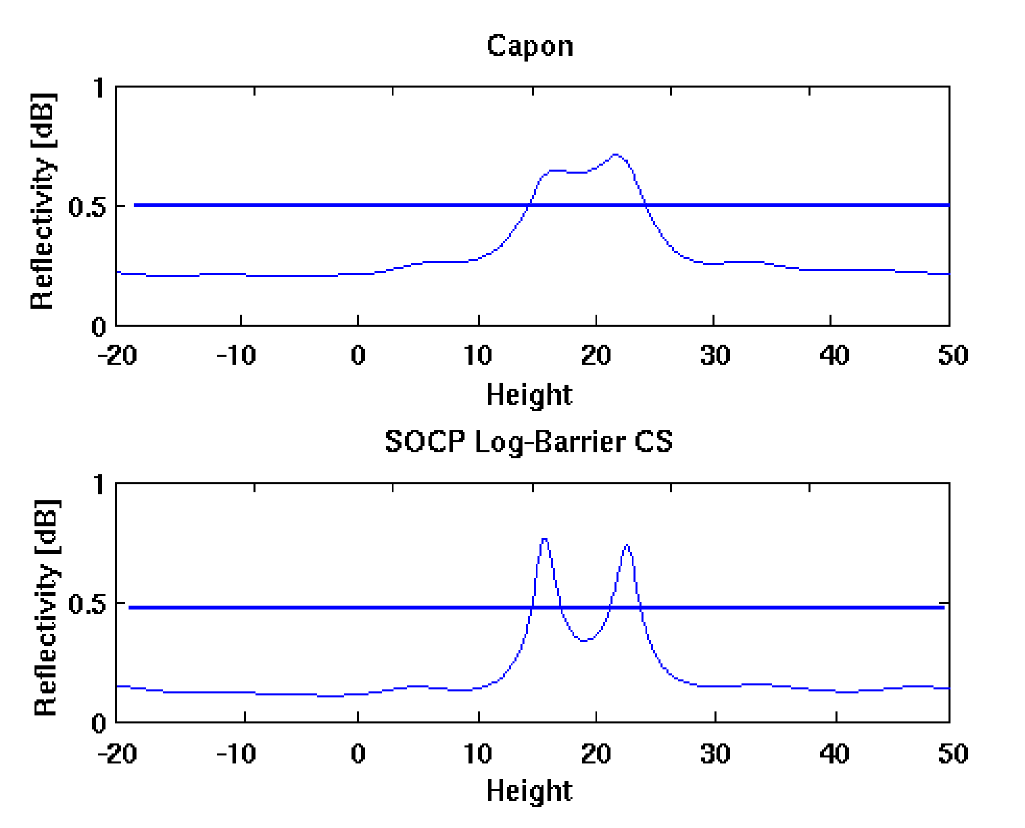
CVX
Convex Optimization



- The Resolution is doubled





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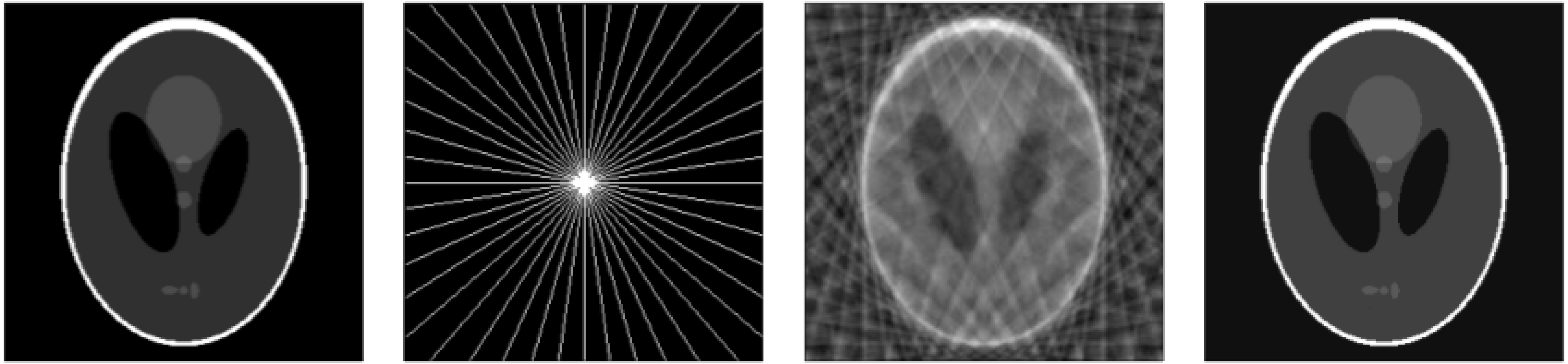
Conclusions

- Demonstration that Convex Optimization can be successfully used for SAR Tomography;
- L1 Norm minimization with quadratic constraints where a Log-Barrier algorithm with Newton iteration was implemented;
- Good resolution improvement has been observed.

Future Work

- Efficient Tomographic reconstruction based on the Total Variational (TV) minimization of estimated image;
- TV may have higher computational speed respect to Classical CS.



(a) Phantom (b) Sampling pattern (c) Min energy (d) min-TV reconstruction

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THANKS FOR YOUR ATTENTION