A COMPRESSIVE SENSING APPROACH TO THE FUSION OF PCL (and DF) SYSTEMS

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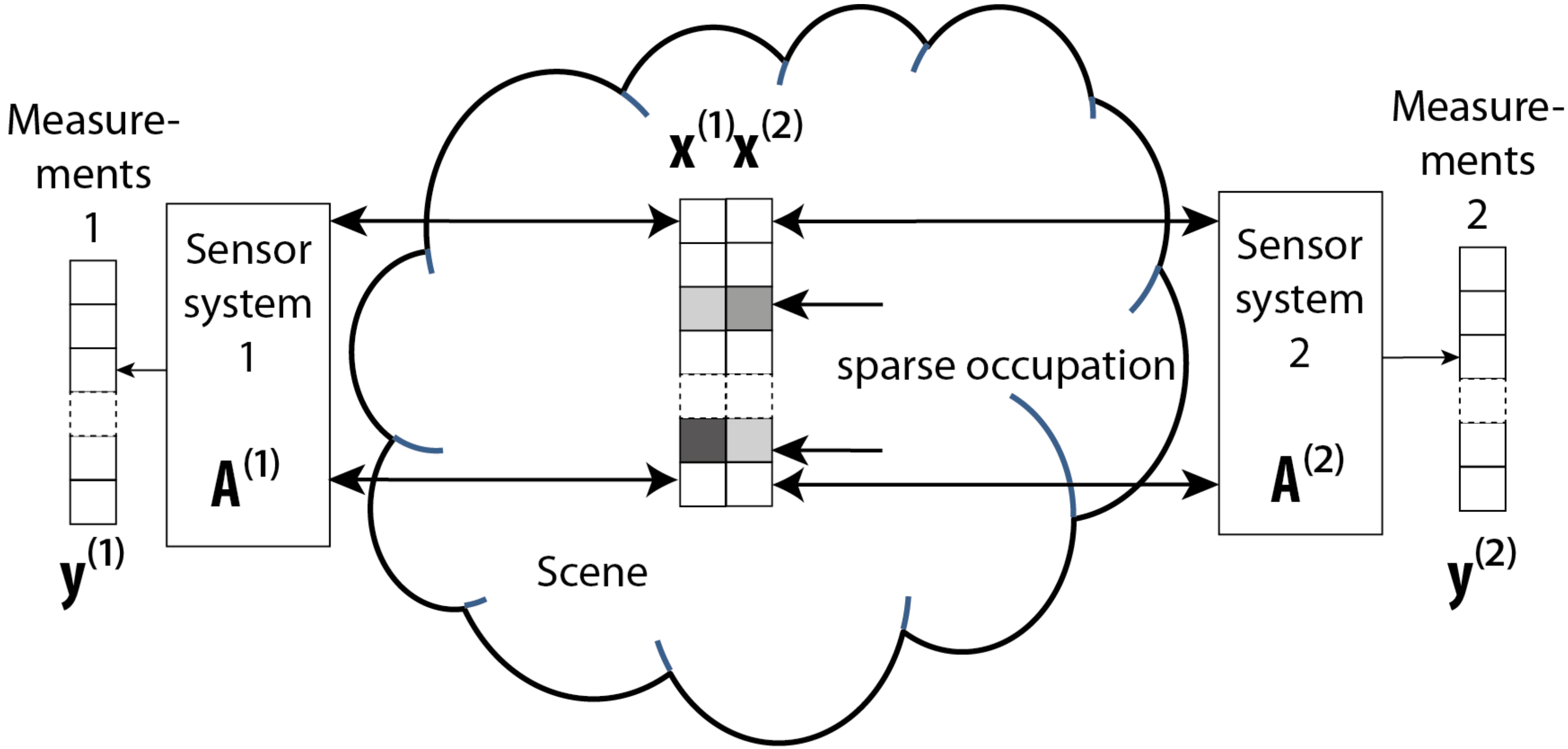
BLOCK SPARSITY

Principle for the observation of the scene by L linear sensors



DESCRIPTION OF SCENE OBSERVED BY L SENSORS

- A **scene** is characterized by a number L of amplitude distributions over **scene-points** which basically are positions in the two- or three-dimensional space, but can be extended to higher dimensions including velocities and other parameters. L is the number of elementary sensors.
- The scene is discretized to N scene points.

Amplitude matrix, L = number of sensors, N = number of scene points

$$\mathbf{X} = \begin{pmatrix} x_1^{(1)} & \dots & x_1^{(L)} \\ \dots & \dots & \dots \\ x_N^{(1)} & \dots & x_N^{(L)} \end{pmatrix} = \left(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(L)}\right) = \begin{pmatrix} \mathbf{x}[1] \\ \vdots \\ \mathbf{x}[N] \end{pmatrix}$$

Block sparse recovery:

Yonina Eldar, Patrick Kuppinger, Helmut Bölcskei: Block-sparse signals: Uncertainty relations and efficient recovery , *IEEE Transactions on Signal Processing*, Vol. 58, No. 6, pp. 3042 – 3054, June 2010

MODELLING OF SENSORS AND MEASUREMENTS

Measurements of elementary sensors

The elementary sensor (l) performes measurements which are linear superpositions of the measurements at each scene point:

$$\mathbf{y}^{(l)} = \mathbf{A}^{(l)}\mathbf{x}^{(l)} + \mathbf{n}^{(l)}.$$

The $M^{(l)} \times N$ dimensional matrix $\mathbf{A}^{(l)}$ is the *sensing matrix* related to the l-th elementary sensor. The column vectors $\mathbf{a}^{(l)}[n]$ represent the model measurement signal when a normalized amplitude is present at scene point ω_n. $\mathbf{n}^{(l)}$ names a noise, interference or remaining error vector.

ALGORITHMS FOR BLOCK-SPARSE RECOVERY

Mixed ℓ_1/ℓ_2 optimization, group-LASSO

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}[1] \\ \vdots \\ \mathbf{x}[N] \end{pmatrix} \rightarrow \xi(\mathbf{X}) = \begin{pmatrix} \|\mathbf{x}[1]\|_2 \\ \vdots \\ \|\mathbf{x}[N]\|_2 \end{pmatrix}$$

Vector of square roots of accumulated energies

Mixed ℓ_1/ℓ_2 optimization; group-LASSO

Minimize

$$\|\xi(\mathbf{X})\|_1 = \sum_{n=1}^{N} \|\mathbf{x}[n]\|_2$$

under the constraint

$$\sum_{l=1}^{L} \|\mathbf{y}^{(l)} - \mathbf{A}^{(l)}\mathbf{x}^{(l)}\|_2^2 \leq \sigma^2!$$

ALGORITHMS FOR BLOCK-SPARSE RECOVERY

Block orthogonal matching pursuit (BOMP)

Block Orthogonal Matching Pursuit (BOMP)

Input: sensing matrices $\mathbf{A}^{(\mathrm{l})}$, measurement vectors $\mathbf{y}^{(\mathrm{l})}, \mathrm{l} \in [L]$

constants S_{max}, ϵ

Initialization: $it = 0$, $\mathrm{I}^0 = \emptyset$, $\mathbf{r}^{(\mathrm{l})0} = \mathbf{y}^{(\mathrm{l})}, \mathrm{l} \in [L]$

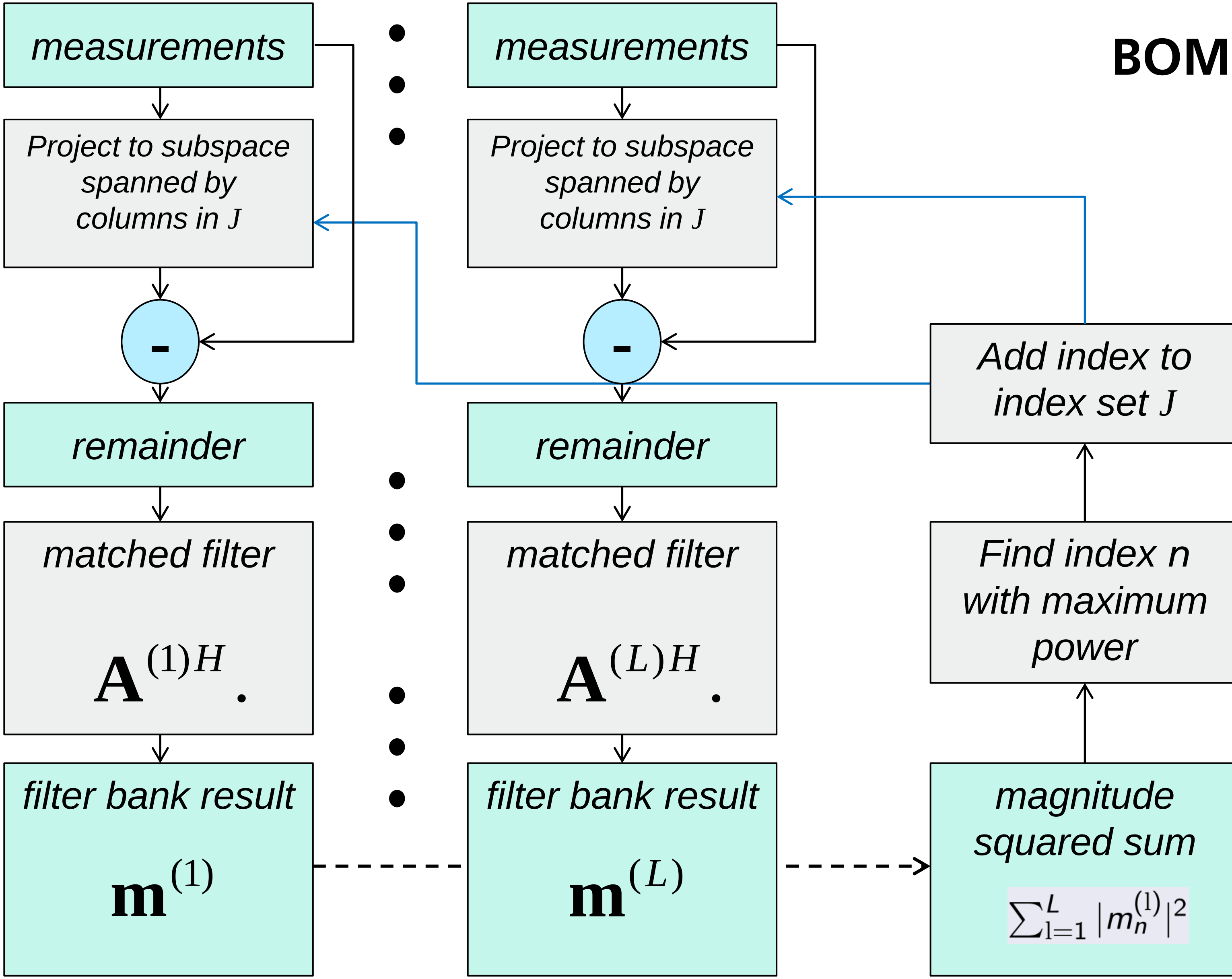
Iteration: repeat

$$
\begin{aligned}
it &= it + 1 & & \\
\mathbf{m}^{(\mathrm{l})} &= \mathbf{A}^{(\mathrm{l})H}\mathbf{r}^{(\mathrm{l})it-1}, \mathrm{l} \in [L] & & \text{(A)} \\
\mathrm{I}^{it} &= \mathrm{I}^{it-1} \cup \operatorname{argmax}\{\textstyle\sum_{\mathrm{l}=1}^{L} |m_n^{(\mathrm{l})}|^2 : n \in [N]\} & & \text{(B)} \\
\mathbf{E}^{(\mathrm{l})} &= \mathbf{A}^{(\mathrm{l})}_{\mathrm{I}^{it}}, \mathrm{l} = 1, \ldots, L & & \text{(C)} \\
\mathbf{w}^{(\mathrm{l})} &= \left(\mathbf{E}^{(\mathrm{l})H}\mathbf{E}^{(\mathrm{l})}\right)^{-1}\mathbf{E}^{(\mathrm{l})H}\mathbf{y}^{(\mathrm{l})}, \mathrm{l} \in [L] & & \text{(D)} \\
\mathbf{r}^{(\mathrm{l})it} &= \mathbf{y}^{(\mathrm{l})} - \mathbf{E}^{(\mathrm{l})}\mathbf{w}^{(\mathrm{l})}, \mathrm{l} \in [L] & & \text{(E)}
\end{aligned}
$$

until $it \geq S_{max}$ or $\sqrt{\sum_{\mathrm{l}=1}^{L} \|\mathbf{r}^{(\mathrm{l})it}\|_2^2} \leq \epsilon$

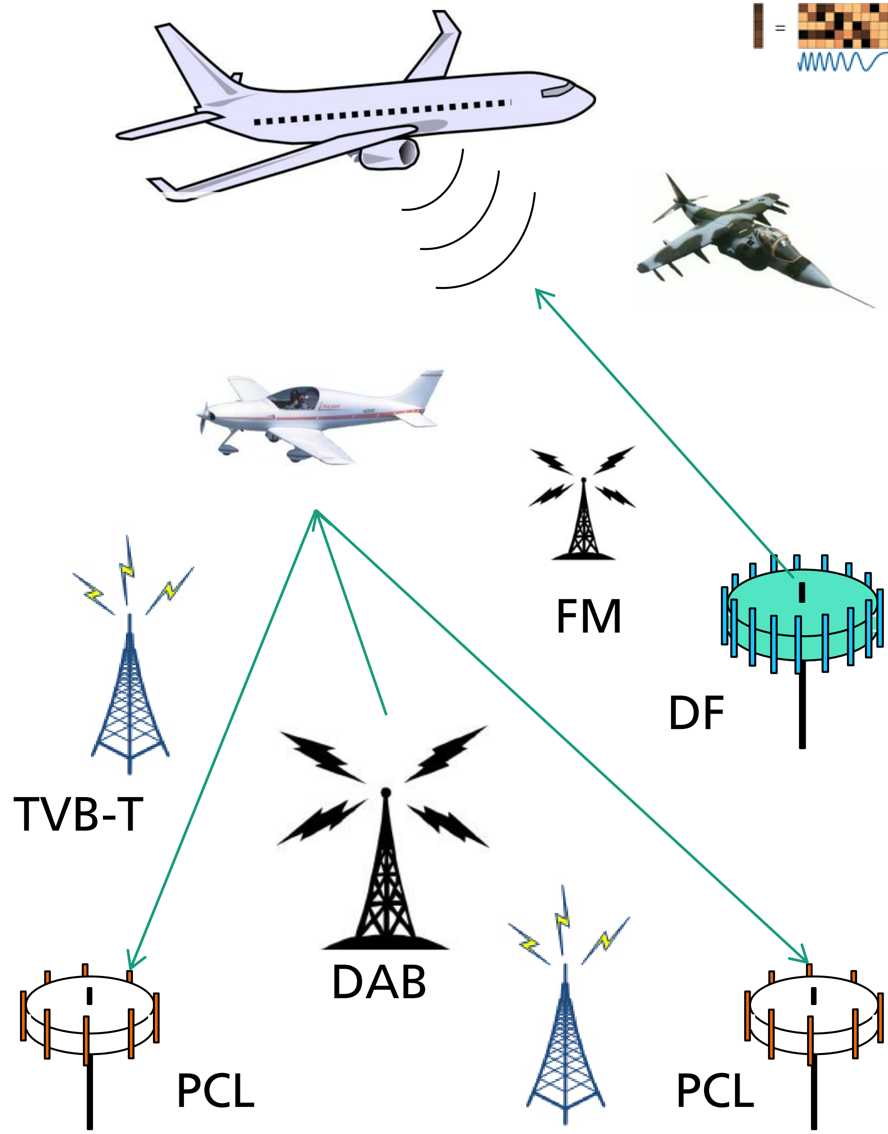
$\widehat{\mathbf{x}}^{(\mathrm{l})}_{\mathrm{I}^{it}} := \mathbf{w}^{(\mathrm{l})}$, the other coefficients are set to zero.

BOMP


measurements
Project to subspace spanned by columns in J
-
remainder
matched filter
$\mathbf{A}^{(1)H}\cdot$
filter bank result
$\mathbf{m}^{(1)}$
measurements
Project to subspace spanned by columns in J
-
remainder
matched filter
$\mathbf{A}^{(L)H}\cdot$
filter bank result
$\mathbf{m}^{(L)}$
Add index to index set J
Find index n with maximum power
magnitude squared sum
$\sum_{l=1}^{L} |m_n^{(l)}|^2$

EXAMPLE PCL + DF:
The problem

- A number of transmitters of opportunity (radio, television, …)
- A number of PCL arrays
- Some airplanes, a few of them actively transmitting
- Optionally, additional linear sensors, like a number of passive DF arrays (DF = direction finder)
- **Use the measurements of all sensors to detect and locate the targets!**



COMPRESSIVE SENSING FOR PCL (and DF)

- Sparsity of the Scene? Yes, only a few aircrafts at the sky!
- Sparsity of the measurements? Yes, a few sensors are spatially distributed.
- ➢ A case for compressive sensing!

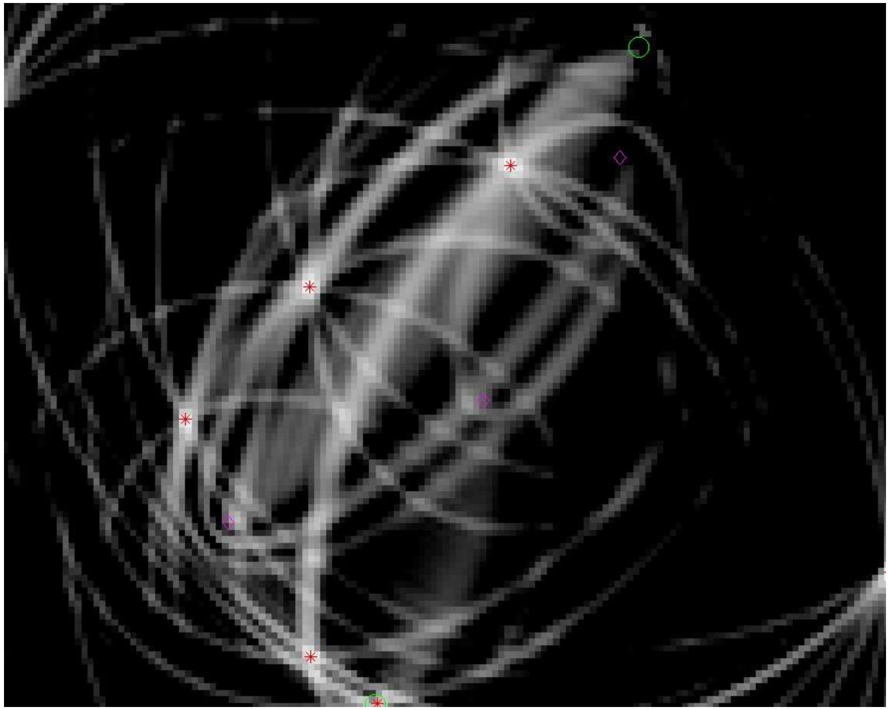
- CS for PCL has been treated in a few papers, e.g.

C. Berger, B. Demissie, J. Heckenbach, P. Willett, and S. Zhou, ``Signal processing for passive radar using OFDM waveforms, IEEE Journal of Selected Topics in Signal Processing, vol.~4, no.~1, pp. 226 --238, 2010.

P. Maechler, N. Felber, and H. Kaeslin, Compressive sensing for WIFI-based passive bistatic radar, in Proceedings of the 20th European Signal Processing Conference (EUSIPCO), 2012, pp. 1444 --1448.

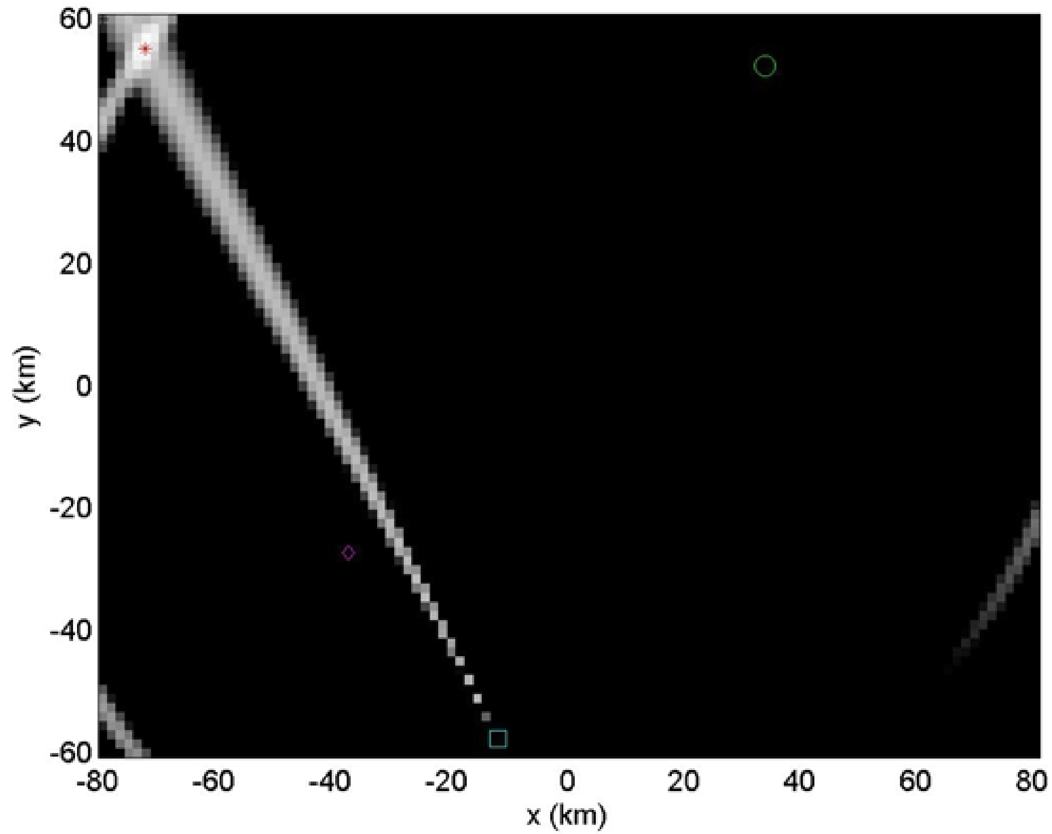
THE DE-GHOSTING PROBLEM FOR PCL

- PCL: Each individual Tx-Rx pair measures primarily the bistatic distance to an object determining its position only partially, i.e. the surface of an ellipsoid.
- Generally, there are much more intersection points then targets.
- It is difficult to discriminate ghosts from targets (count the number of ellipsoids intersecting in a point ...)



PCL AND DF SYSTEMS COMPLEMENT EACH OTHER!

- DF devices estimate the direction to an emitting target
- Beams intersecting ellipsoid surfaces reduce ambiguity problems!



THE CLASSICAL APPROACH TO SENSOR FUSION

Measurements

Sensor 1

Detection

Estimation

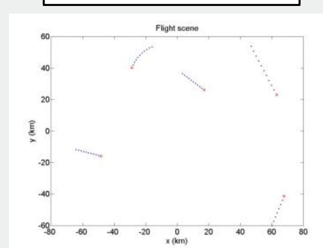
Tracking in sensor coordinates

Fusion center

tracking

De-ghosting

Improve-ment of estimates



Measurements

Sensor 2

Detection

Estimation

Tracking in sensor coordinates

One way information flow from sensors to fusion center!

Martina Daun: Deghosting in passive air surveillance systems, IRS, 2010, pp. 1 --8.

SIGNAL MODELS FOR THE SENSING MATRIX

- **Elementary sensors**:
  - PCL: transmitter (of opportunity) plus a 'co-located' array of receiving antennas
  - DF: receiving array for direction finding

PCL signal model:

Bistatic range

Pattern of Tx antenna

Position of Tx

$$s_{\mu\nu}(k;\vec{r}) = \exp\{-jkR_{\mu\nu}(\vec{r})\}\, D_{\mu}^{(Tx)}\left(\frac{\vec{r}-\vec{p}^{\,(Tx)}}{\left|\vec{r}-\vec{p}^{\,(Tx)}\right|}\right) \times D_{\nu}^{(Rx)}\left(\frac{\vec{r}-\vec{p}^{\,(Rx)}}{\left|\vec{r}-\vec{p}^{\,(Rx)}\right|}\right).$$

Wave number

Position of scene point

Pattern of Rx antenna

Position of Rx

DF signal model:

Position of scene point

Wave number

Element positions

Element indices

$$\mathbf{s}^{(l)}(k,\vec{r}) = \left[\exp\left\{jk\left\langle \vec{u}(\vec{r}),\vec{p}_{\nu}^{\,(l)}\right\rangle\right\} D_{\nu}^{(l)}(\vec{u}(\vec{r}))\right]_{\nu\in[M^{(l)}]}$$

where $\vec{u}(\vec{r})$ is the direction to the source: $\vec{u}(\vec{r}) = \left(\vec{r}-\vec{p}^{\,(l)}\right)/\left|\vec{r}-\vec{p}^{\,(l)}\right|$, $\vec{p}^{\,(l)}$ being the center of gravity of the array positions, and $D_{\nu}^{(l)}(\vec{u})$ is the individual antenna pattern.

JOINED ESTIMATION
Mixed ℓ_1/ℓ_2 optimization

Aim: Estimate jointly the deviation from the grid and other parameters

Joined estimation of a parameter vector $\vartheta \in \theta$

Parameter-dependent sensing matrices $\mathbf{A}^{(l)}(\vartheta)$. Measurements:

$$\mathbf{y}^{(l)} = \mathbf{A}^{(l)}(\vartheta)\mathbf{x}^{(l)} + \mathbf{n}^{(l)}.$$

Joined estimation by ℓ_1/ℓ_2 minimization

Minimize

$$\sum_{l=1}^{L} \|\mathbf{A}^{(l)}(\vartheta)\mathbf{x}^{(l)} - \mathbf{y}^{(l)}\|_2^2 + \lambda \sum_{n=1}^{N} \|\mathbf{x}[n]\|_2$$

simultaneously in ϑ and $\mathbf{X}$ to enhance sparsity and to minimize the error at the same time!

JOINED ESTIMATION

Minimum mean-square estimation for given support

Joined estimation by MMS

Considering an index subset J reflecting the estimated grid positions occupied by targets, we minimize

$$\sum_{l=1}^{L} \|\mathbf{A}_J^{(l)}(\vartheta)\mathbf{x}_J^{(l)} - \mathbf{y}^{(l)}\|_2^2$$

over $\mathbf{x}_J^{(l)}$ and ϑ resulting in

$$\widehat{\vartheta} = \operatorname{argmin}\left\{\sum_{l=1}^{L} \|\mathbf{P}_{\perp}^{(l)}(\vartheta)\mathbf{y}^{(l)}\|_2^2 : \vartheta \in \theta\right\}$$

with $\mathbf{P}_{\perp}^{(l)}(\vartheta)$ being the projector orthogonal to the subspace spanned by the columns of $\mathbf{A}_J^{(l)}(\vartheta)$.

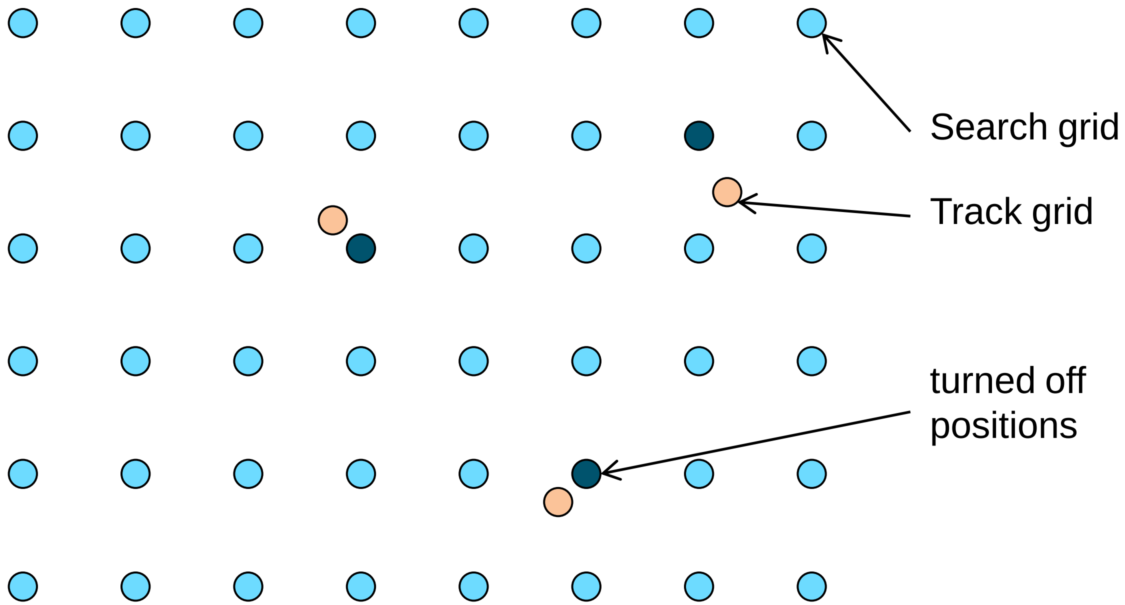
JOINED ESTIMATION

Minimum mean-square estimation

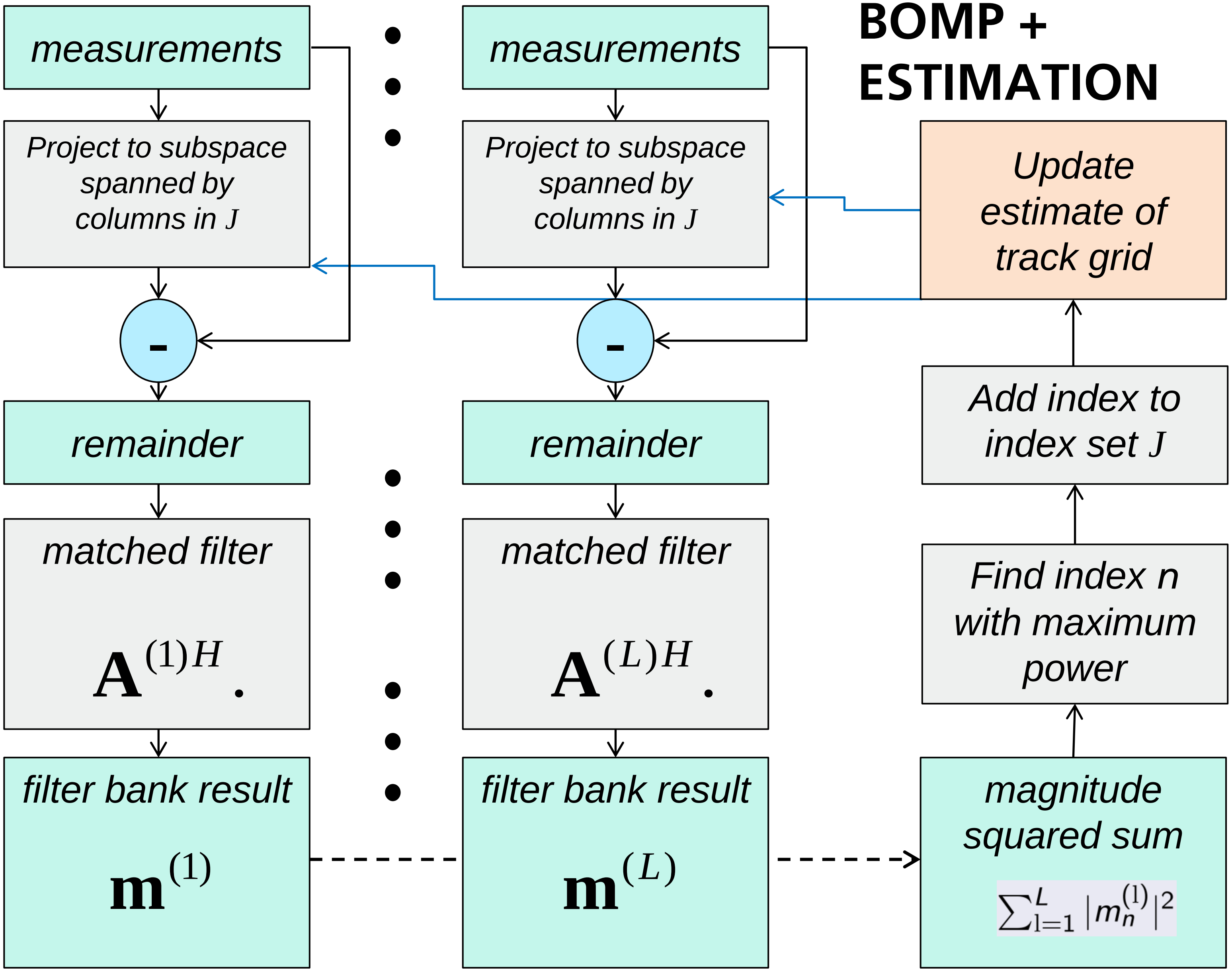
- For the simulations, the MMS estimation algorithm has been used.
- The parameter vector consists of the three-dimensional deviations from the grid point where the target was detected.
- The minimum was found by expanding the cost function into a Taylor series of order two (gradient and Hesse-matrix for each linear sensor – then summed up to the entity of sensors).
- Each parameter vector was treated separately to simplify the inversion of the Hesse-matrix.

SEARCH GRID AND TRACK GRID

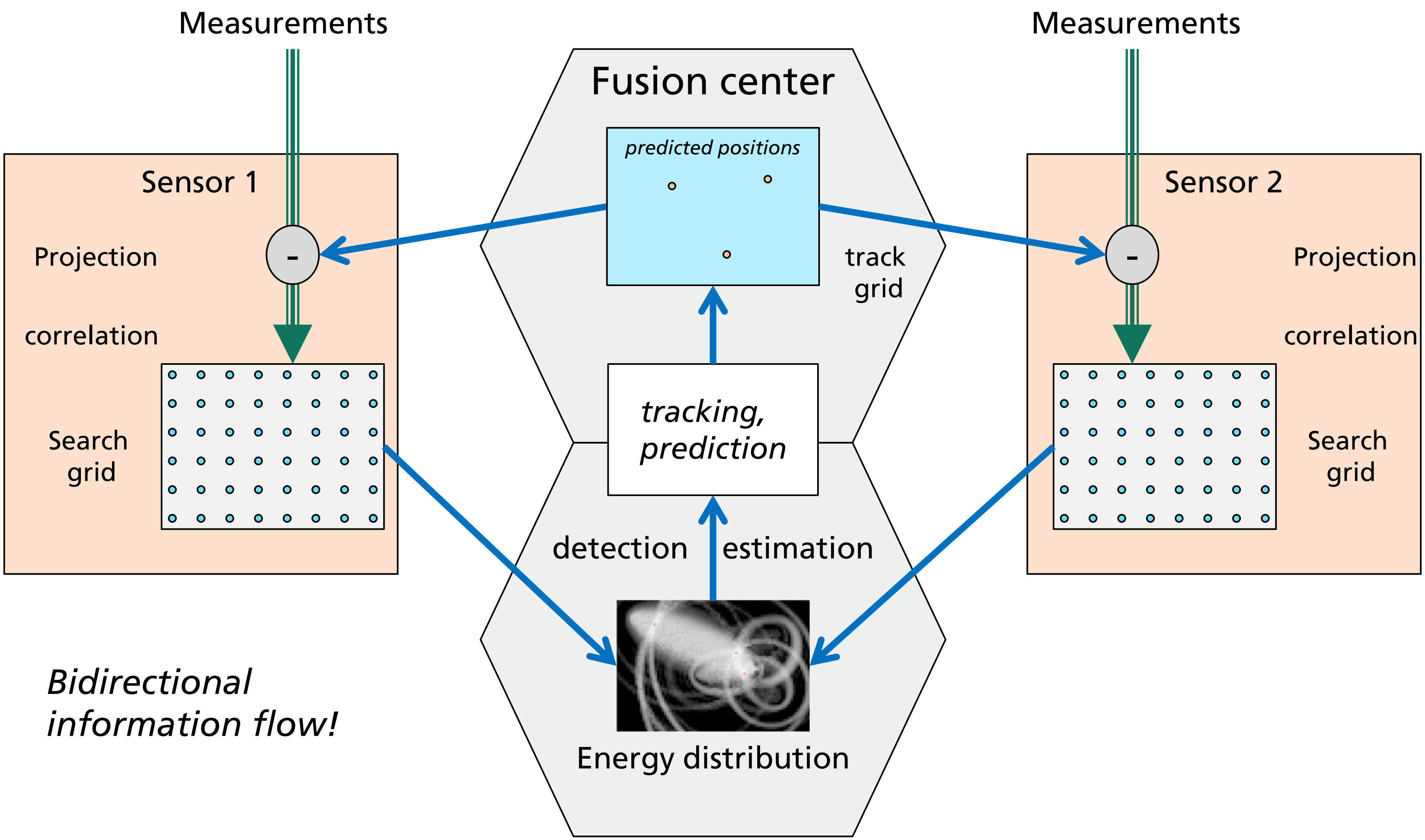
- Fixed search grid for the detection of new airplanes
- Dynamic track grid for tracked airplanes, basis for evaluating the remainders by projection
- Fine estimate of positions (here obtained by a second order Taylor approximation) can be integrated into the BOMP iteration



BOMP + ESTIMATION

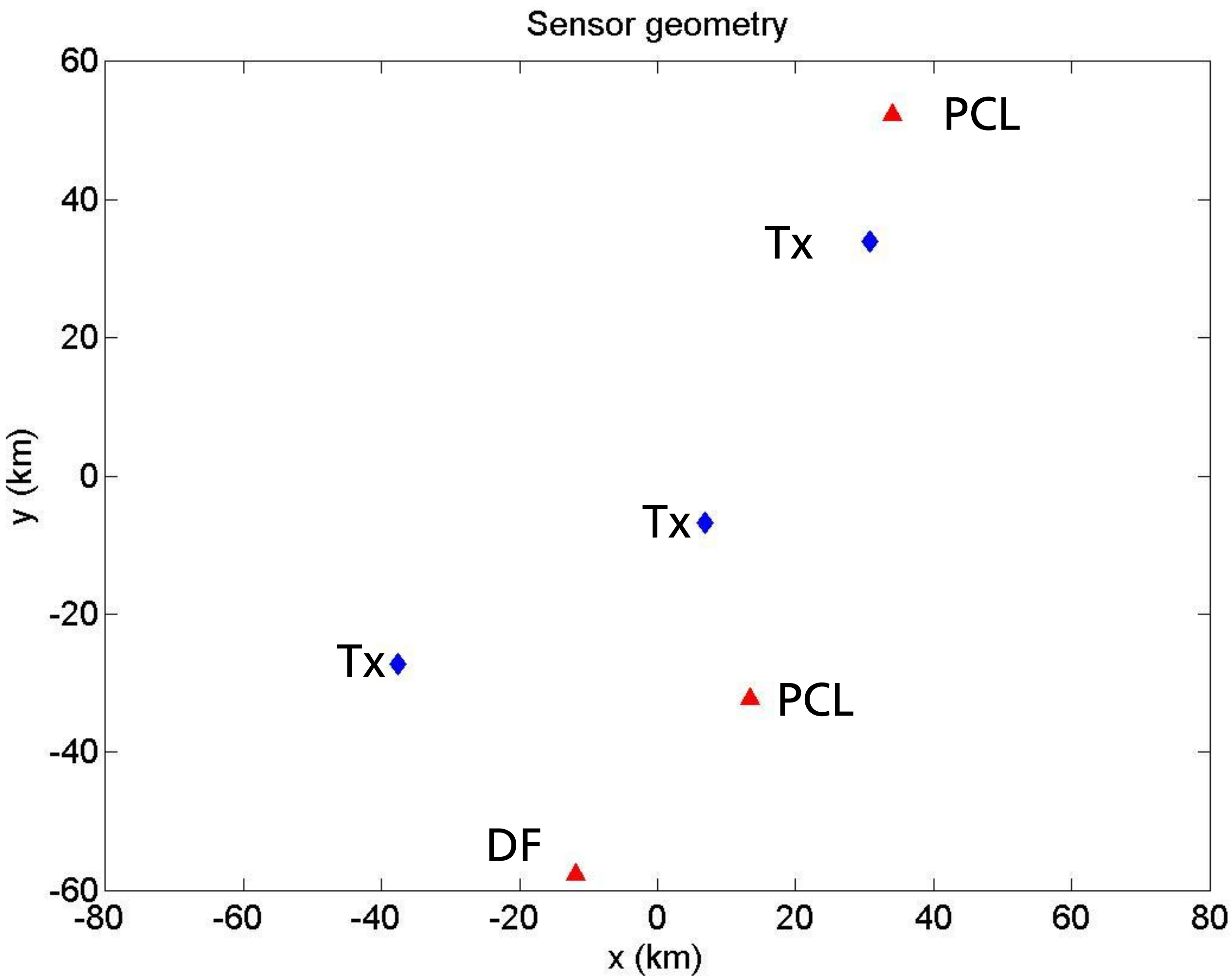


DISTRIBUTED SIGNAL PROCESSING



Bidirectional information flow!

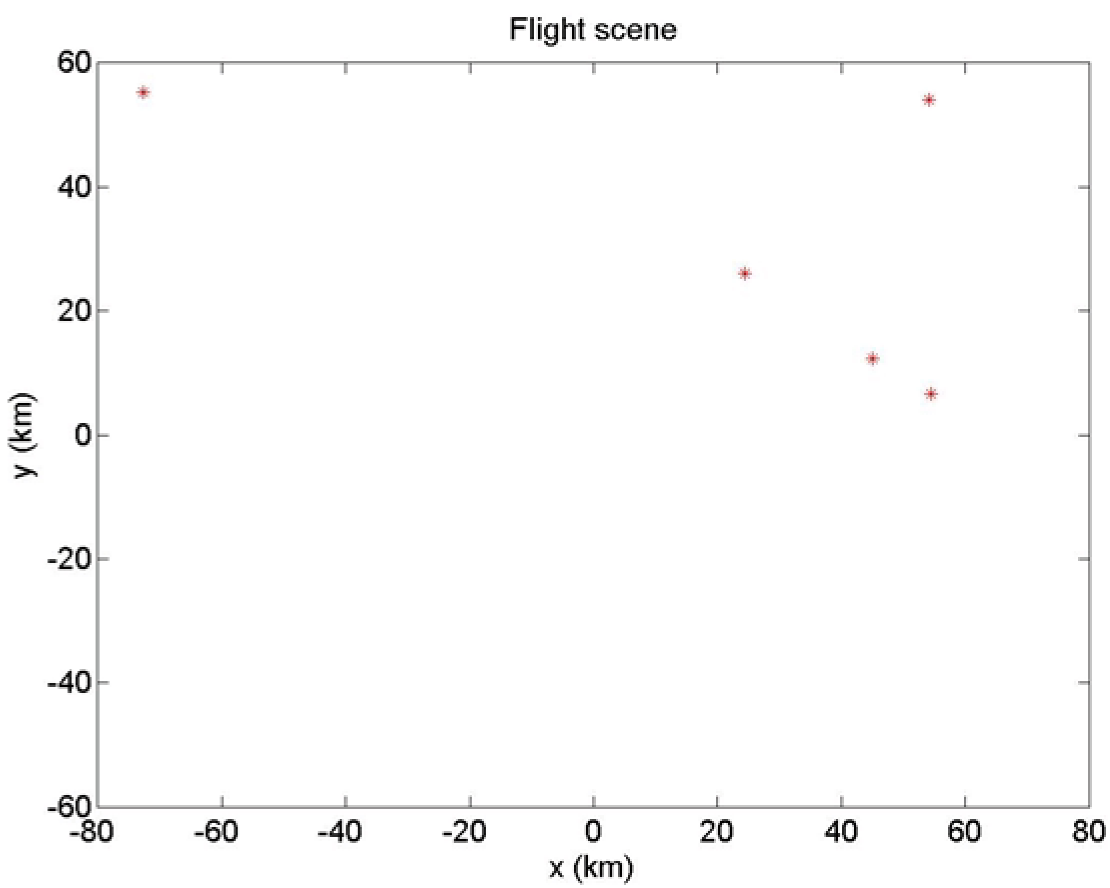
SENSOR GEOMETRY USED FOR SIMULATION



PCLs	triangular arrays
DF	60-element circular array
Scene grid	151 x 114 N = 17214
Bandwidth PCL	70 MHz

The positions of the transmitters are that of a real ensemble of DAB transmitters in Italy whereas the places of the receivers are the result of a coverage optimization performed in the EU project 'ARGUS 3D'

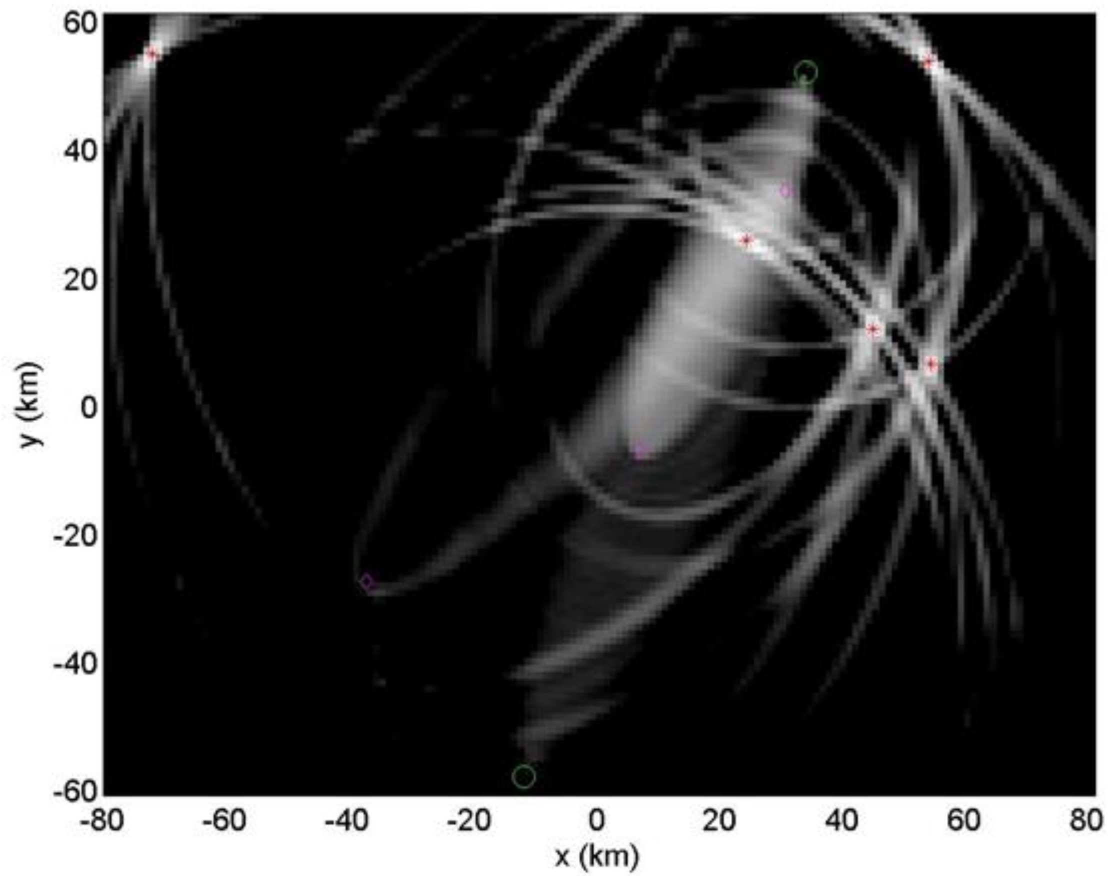
SIMULATED FLIGHT PATHES



- Velocities between 100 and 500 m/s
- Straight and curved trajectory segments
- 500 points of time at a spacing of 2 seconds
- Simulation of signals:
  No Doppler
  no direct signal
  no clutter
  inverse filtering
  no tracker
  DF always active

MATCHED FILTER RESULTS FOR 2 ELEMENTARY SENSORS 3 Tx for 2 PCL

True positions of airplanes marked by red stars

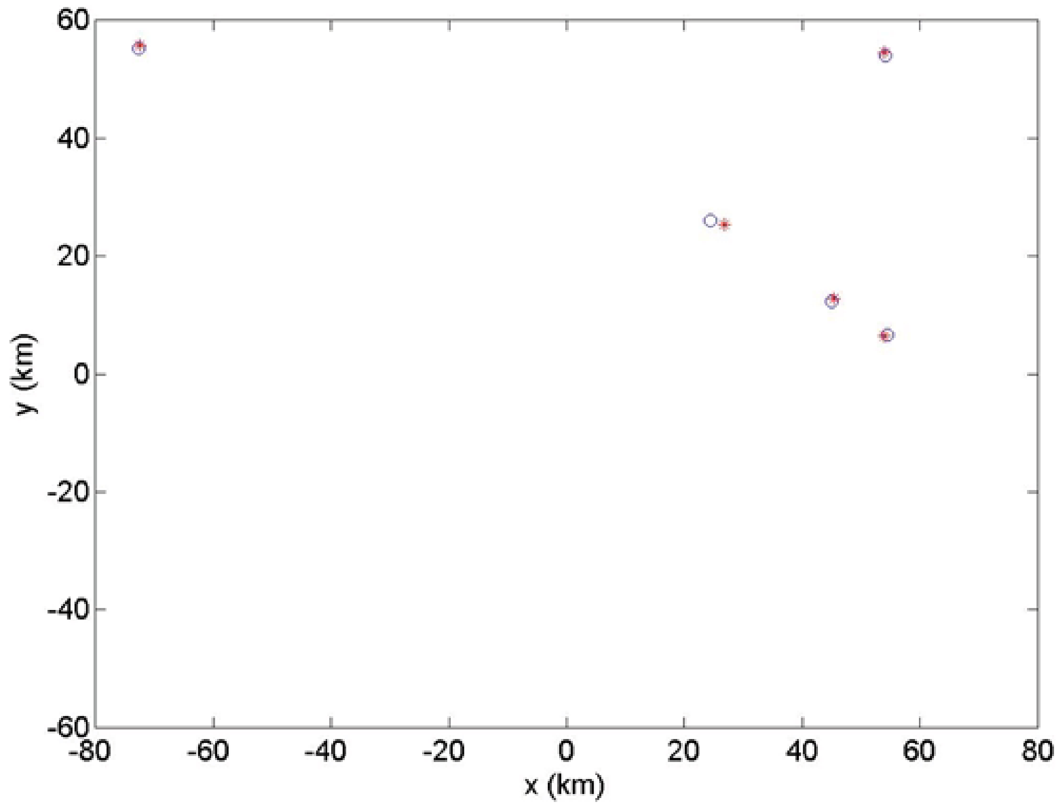


f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	3	
NRx	6	
Narray	2	
L (number sensors)	6	
Nk	94	
Mtotal	1692	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

BOMP RECONSTRUCTION FOR 2 ELEMENTARY SENSORS

3 Tx for 2 PCL

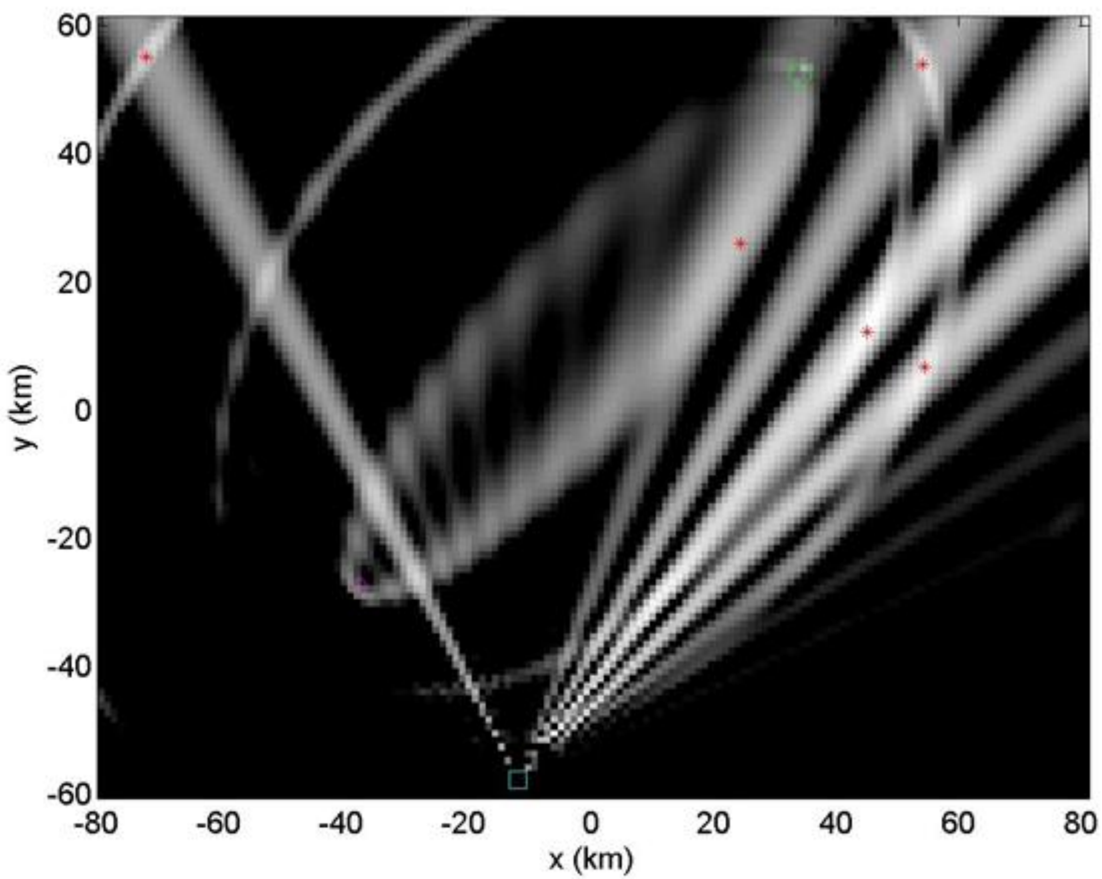
True positions of airplanes marked by blue circles



f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	3	
NRx	6	
Narray	2	
L (number sensors)	6	
Nk	94	
Mtotal	1692	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

MATCHED FILTER RESULTS FOR 2 ELEMENTARY SENSORS 1 Tx for 1 PCL, 1 DF

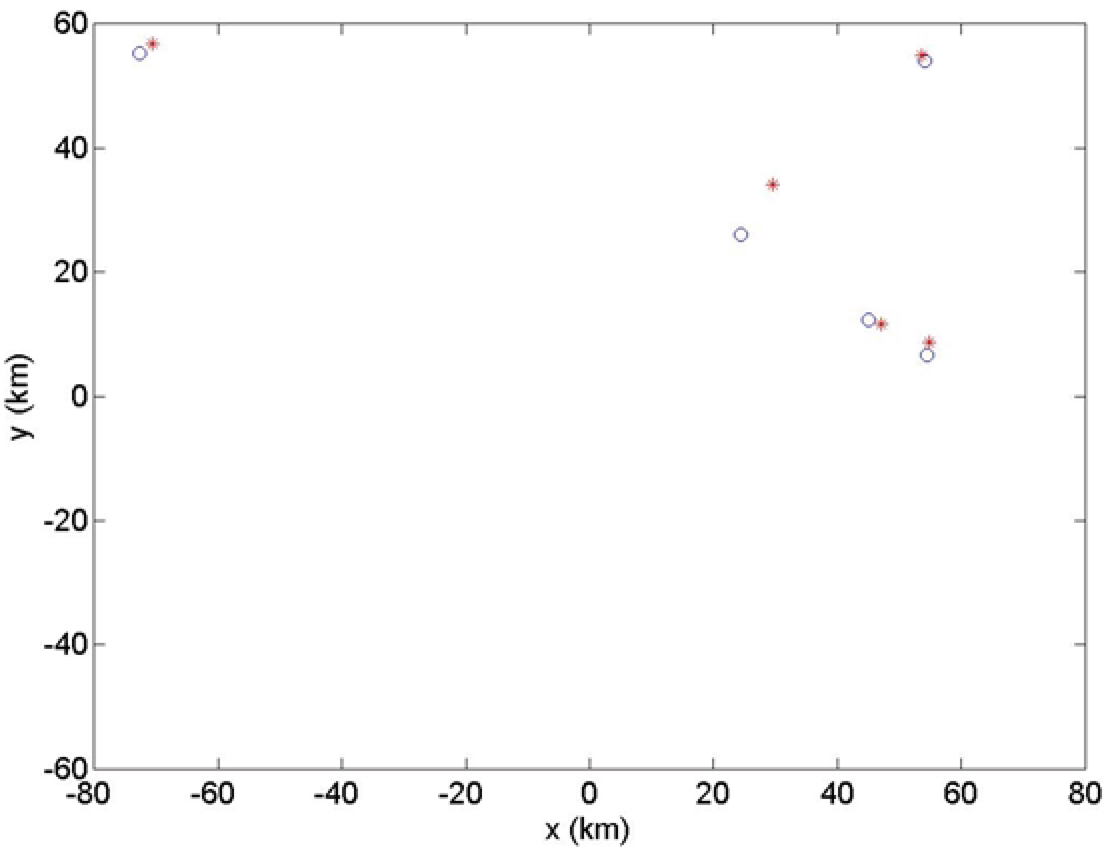
True positions of airplanes marked by red stars



f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	1	
NRx	63	
Narray	2	
L (number sensors)	2	
Nk	94	
Mtotal	342	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

BOMP RECONSTRUCTION FOR 2 ELEMENTARY SENSORS 1 Tx for 1 PCL, 1 DF

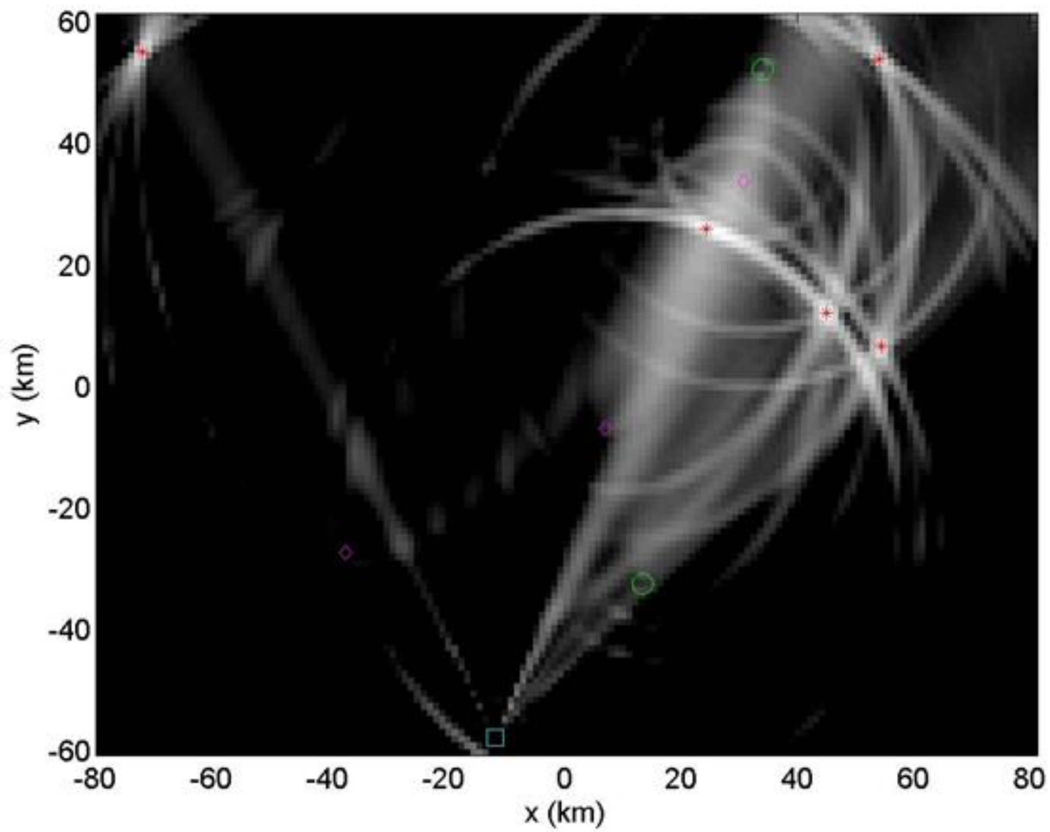
True positions of airplanes marked by blue circles



f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	1	
NRx	63	
Narray	2	
L (number sensors)	2	
Nk	94	
Mtotal	342	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

MF RESULTS FOR 7 ELEMENTARY SENSORS
3 Tx for 2 PCL, 1 DF

True positions of airplanes marked by red stars

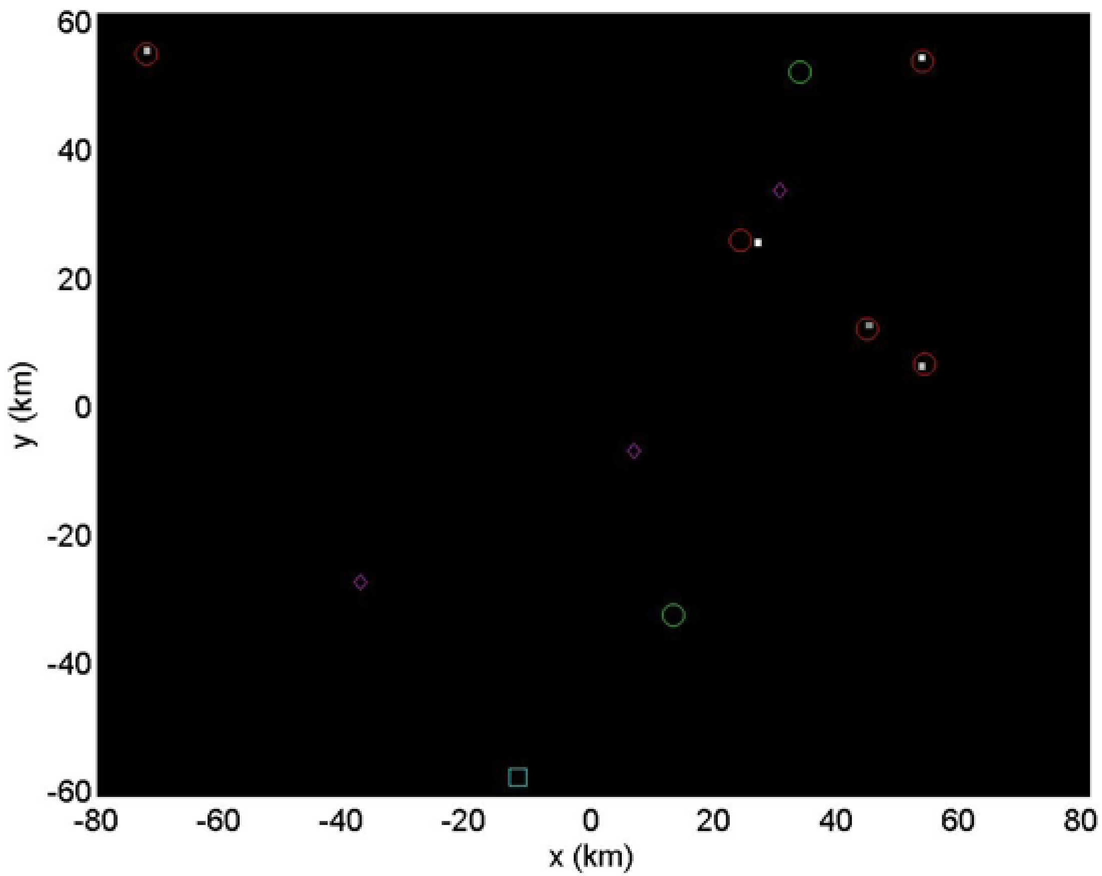


f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	3	
NRx	46	
Narray	3	
L (number sensors)	7	
Nk	94	
Mtotal	1732	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

BOMP IMAGES FOR 7 ELEMENTARY SENSORS

3 Tx for 2 PCL, 1 DF

True positions of airplanes marked by red circles



f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	3	
NRx	46	
Narray	3	
L (number sensors)	7	
Nk	94	
Mtotal	1732	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

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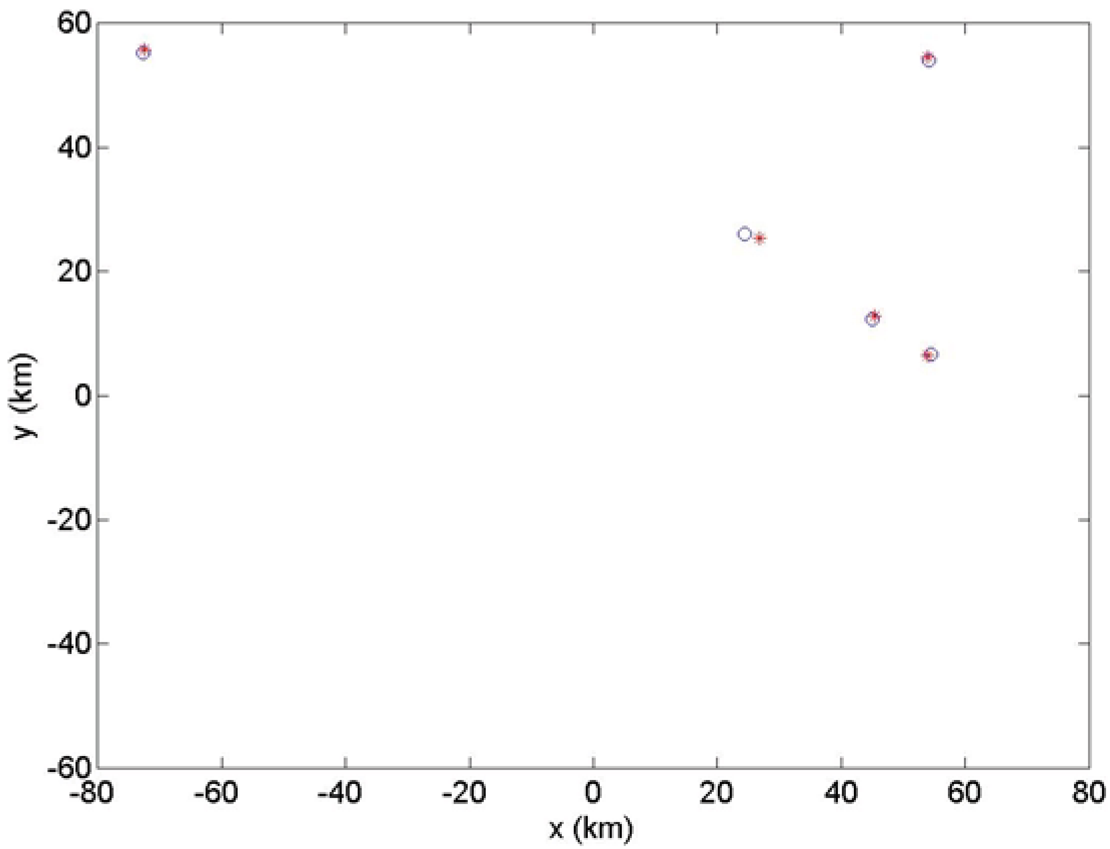
BOMP RESULTS FOR 7 ELEMENTARY SENSORS

3 Tx for 2 PCL, 1 DF

True positions of airplanes marked by red stars



f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	3	
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Narray	3	
L (number sensors)	7	
Nk	94	
Mtotal	1732	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

CONCLUSIONS AND OUTLOOK

Aim: Attract attention to the potentials of compressive sensing for block sparse scenes in the application to sensor fusion problems.

- Alternative approach for the fusion of PCL and DF sensors (SFBSR), based on block sparse reconstruction techniques and a joined fine estimation of the target positions.
- Explain all measurements by a few target positions.
- Bi-directional transfer of data from and to the fusion center.
- Joined estimation of target parameters
- Fixed search grid for the detection of new incoming targets and a track grid for the already known targets.
- ‘De-ghosting before detect' property.

CONCLUSIONS AND OUTLOOK

Much additional work is necessary

- Evaluation of P_A and P_D
- Systematic estimation error analysis, comparison to Cramer-Rao bounds.
- Comparison of mixed ℓ_1/ℓ_2 optimization and BOMP
- Comparison to established methods for PCL fusion
- Implementation of a track component
- Regarding more realistic situations taking into account clutter, the direct signal, multi-path effects and the handling of Doppler filtering
- Validation for real data

Joachim H.G. Ender: "A compressive sensing approach to the fusion of PCL sensors" 21th European Signal Processing Conference EUSIPCO, Marrakech, Morocco, 9-13 September