On-Grid and Off-Grid Compressive MIMO Radar

(Part of Ongoing Work)

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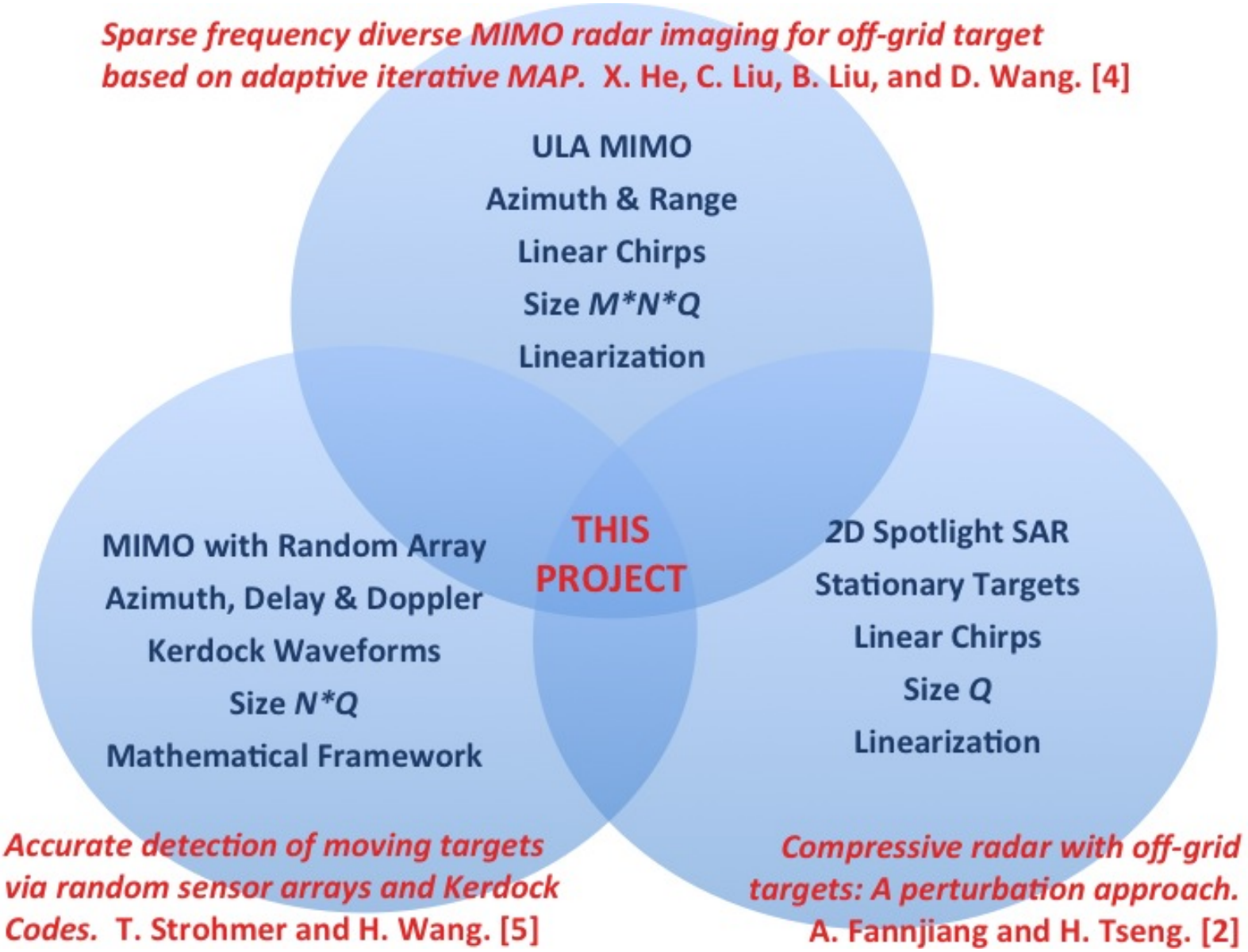
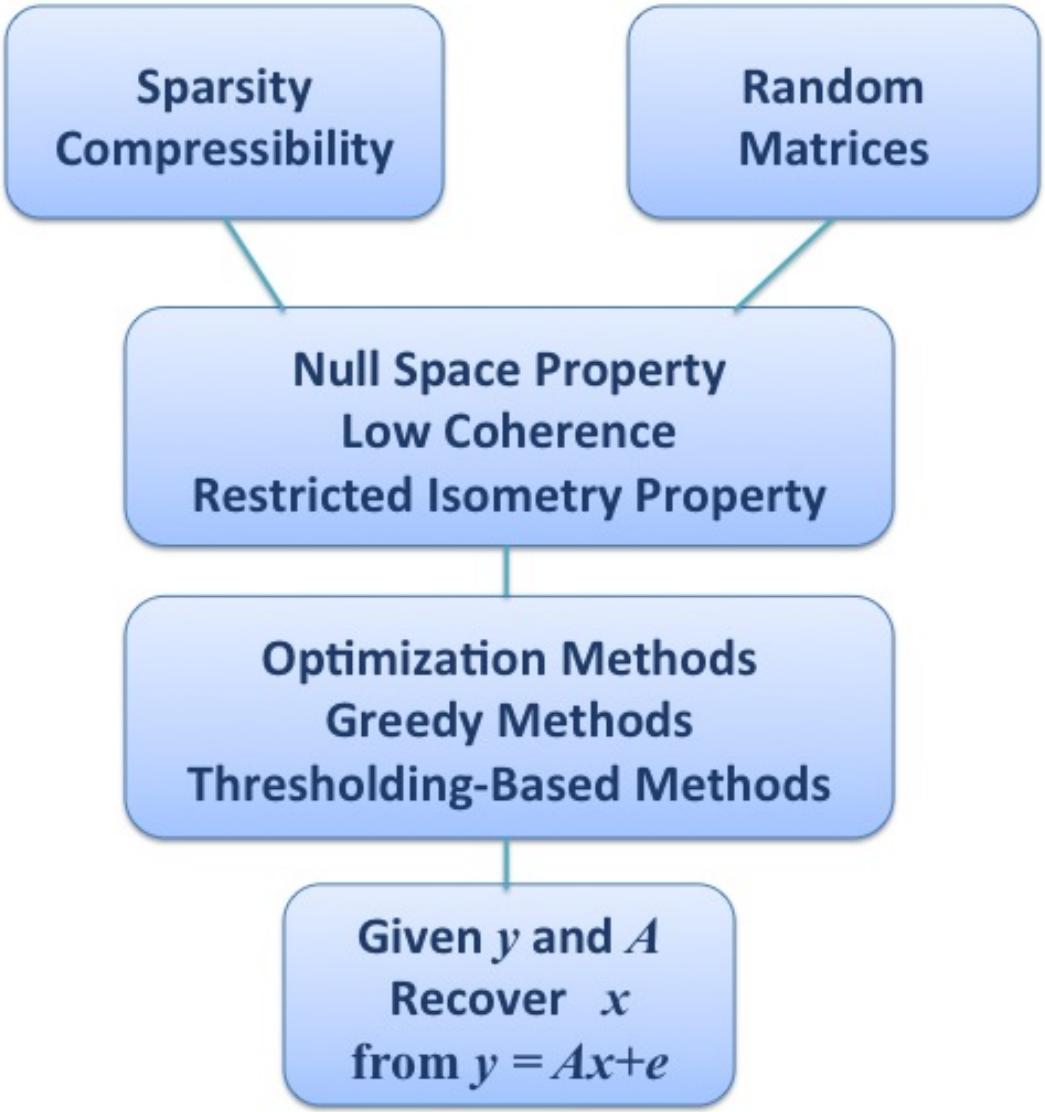
Introduction

4th International Summer School on Radar/SAR

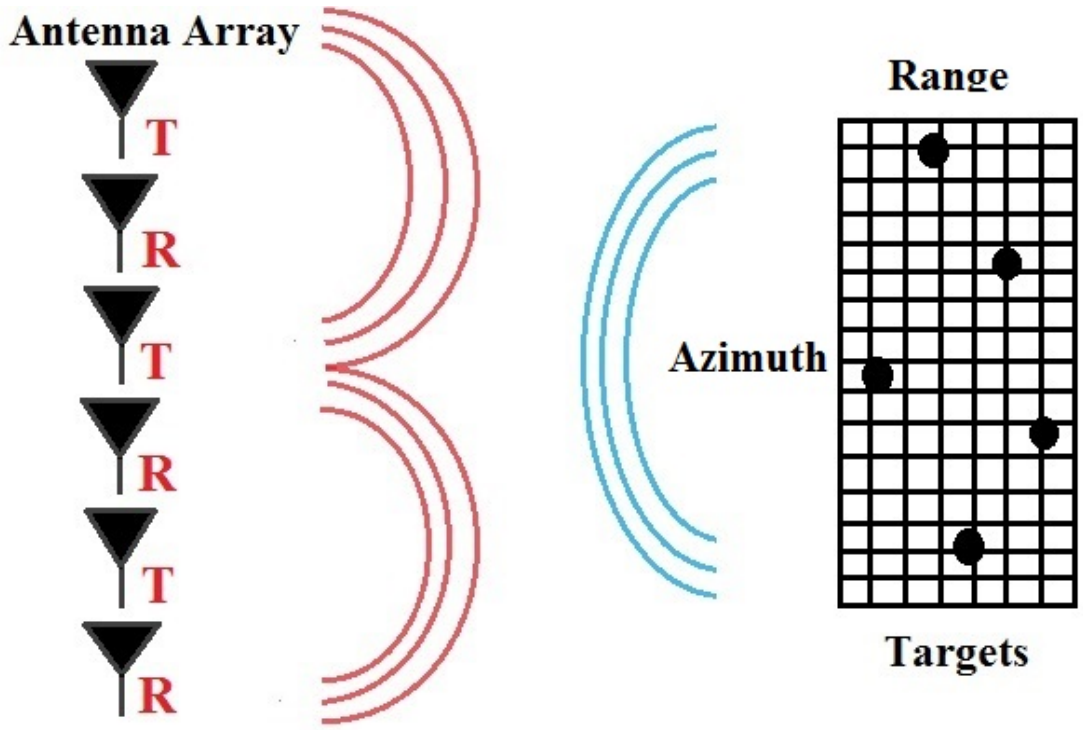
Relation to Recent Work in the Area



Compressive Sensing



MIMO Radar



M Transmitters
Signals $s_m(t)$
N Receivers

P Point Targets
Far-Field
Small Scene

Discretizations

- Discretize the scene with U angle bins, V time delay bins and W Doppler bins and associated discretization steps $\Delta_\theta, \Delta_\tau, \Delta_\upsilon$.
- Grid locations: $(\theta_k, \tau_k, \upsilon_k) = (\theta_{\text{ref}} + \mathsf{u}_{(k)}\Delta_\theta, \tau_{\text{ref}} + \mathsf{v}_{(k)}\Delta_\tau, \upsilon_{\text{ref}} + \mathsf{w}_{(k)}\Delta_\upsilon)$, where $k = 1, ..., K$, with $K = UVW$.
- These point targets possess complex reflectivity coefficients ρ_k and are assumed to be (slow) moving with constant velocity towards the antennas.

Setup

Array Manifolds:

$$\mathbf{a}(\theta_k) = \left[1, e^{i\frac{2\pi}{\lambda}dt_2\theta_k}, ..., e^{i\frac{2\pi}{\lambda}dt_M\theta_k}\right]^T$$

$$\mathbf{b}(\theta_k) = \left[1, e^{i\frac{2\pi}{\lambda}dr_2\theta_k}, ..., e^{i\frac{2\pi}{\lambda}dr_N\theta_k}\right]^T$$

Received Signal:

$$z_{mn}(t) = \sum_{k=1}^{P} \rho_k b_n(\theta_k) a_m(\theta_k) s_m(t - \tau_k) \exp[-i2\pi\upsilon_k t] + w_{mn}(t)$$

Transmitted Signals

Linear Frequency Modulated (LFM) chirps:

$$s_m(t) = \exp\left[i2\pi\left(\frac{\alpha}{2}t^2 + f_m t\right)\right] I_T(t)$$

Here $f_m = f_0 + m\alpha T$ is the carrier frequency.

Dechirping gives the following:

$$s_m(t-\tau_k)s_m^*(t-\tau_{\text{ref}}) = I_T(t')\exp\left[-i2\pi(f_m + \alpha t')(\tau_k - \tau_{\text{ref}})\right] \\ \exp\left[i\pi\alpha(\tau_k - \tau_{\text{ref}})^2\right],$$

where $t' = t - \tau_{\text{ref}}$.
Remove the Residual Video Phase term [1].

Measurements

After dechirping and sampling at times t'_q for $q = 1, ..., Q$:

$$z_{mn}(t'_q) = \sum_{k=1}^{K} \rho_k \exp\left[\frac{i2\pi}{\lambda}(dt_m + dr_n)\theta_k\right]$$
$$\exp\left[-i2\pi(f_m + \alpha t'_q)(\tau_k - \tau_{\text{ref}})\right]$$
$$\exp[-i2\pi\upsilon_k(t'_q + \tau_{\text{ref}})] + w_{mn}(t'_q).$$

Goal: Recover $\{\rho_k, \theta_k, \tau_k, \upsilon_k\}_{k=1}^{K}$ from the set of $\{z_{mn}(t'_q)\}$.

Compressive Sensing for On-Grid Targets

Vectorize: $\mathbf{z} := \mathrm{vec}\left(z_{mn}(t'_q)\right)$, $\mathbf{w} := \mathrm{vec}\left(w_{mn}(t'_q)\right)$ and

$$\mathbf{h}_k := \mathrm{vec}\Big(\exp\left[\frac{i2\pi}{\lambda}(dt_m + dr_n)\theta_k\right] \exp[-i2\pi\upsilon_k(t'_q + \tau_{\mathrm{ref}})] \\ \exp\left[-i2\pi(f_m + \alpha t'_q)(\tau_k - \tau_{\mathrm{ref}})\right] \Big),$$

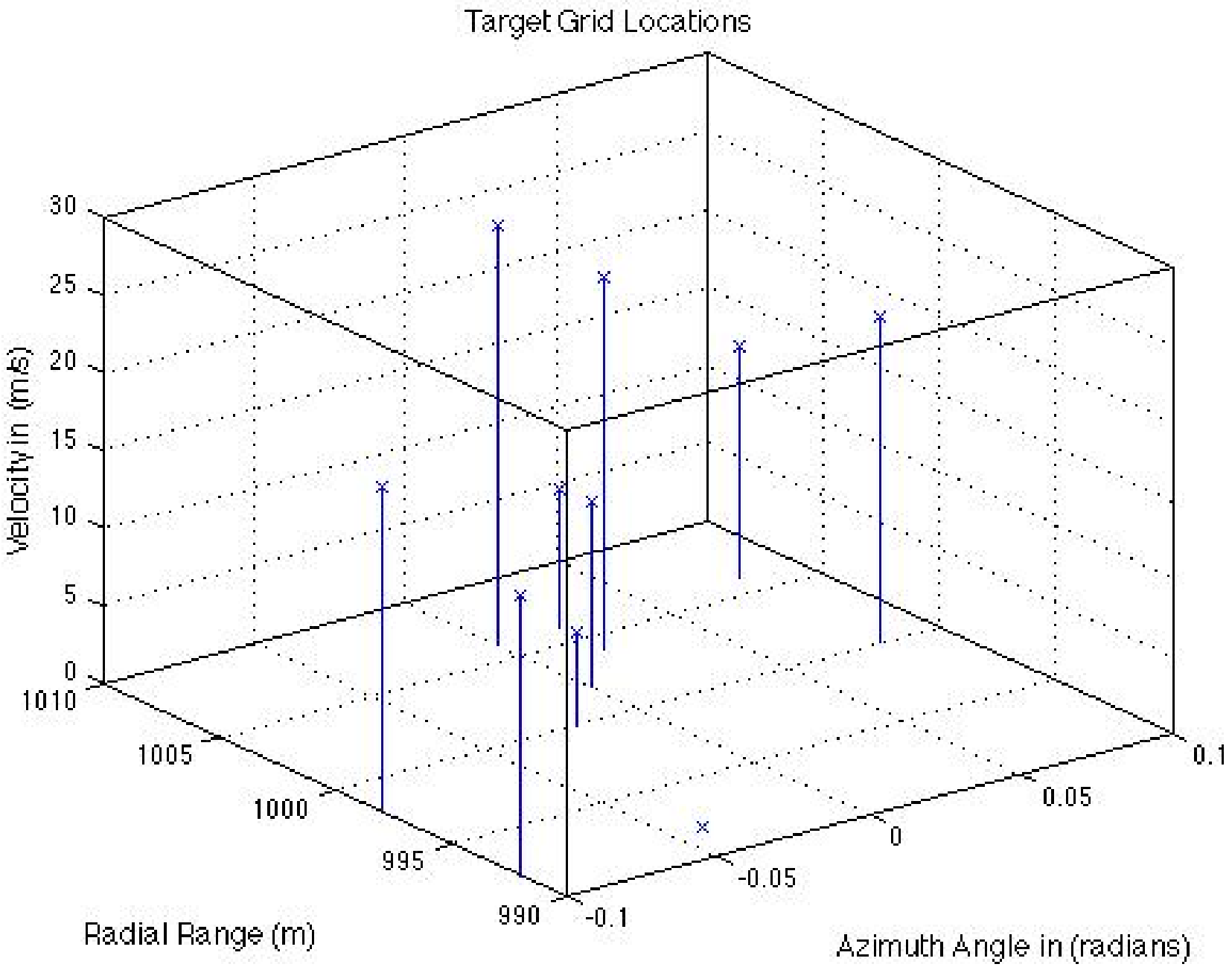
which are all vectors of size MNQ. We store the $\mathbf{h}_k$'s via:

$$\mathbf{H} = \begin{pmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_\mathbf{K} \end{pmatrix}.$$

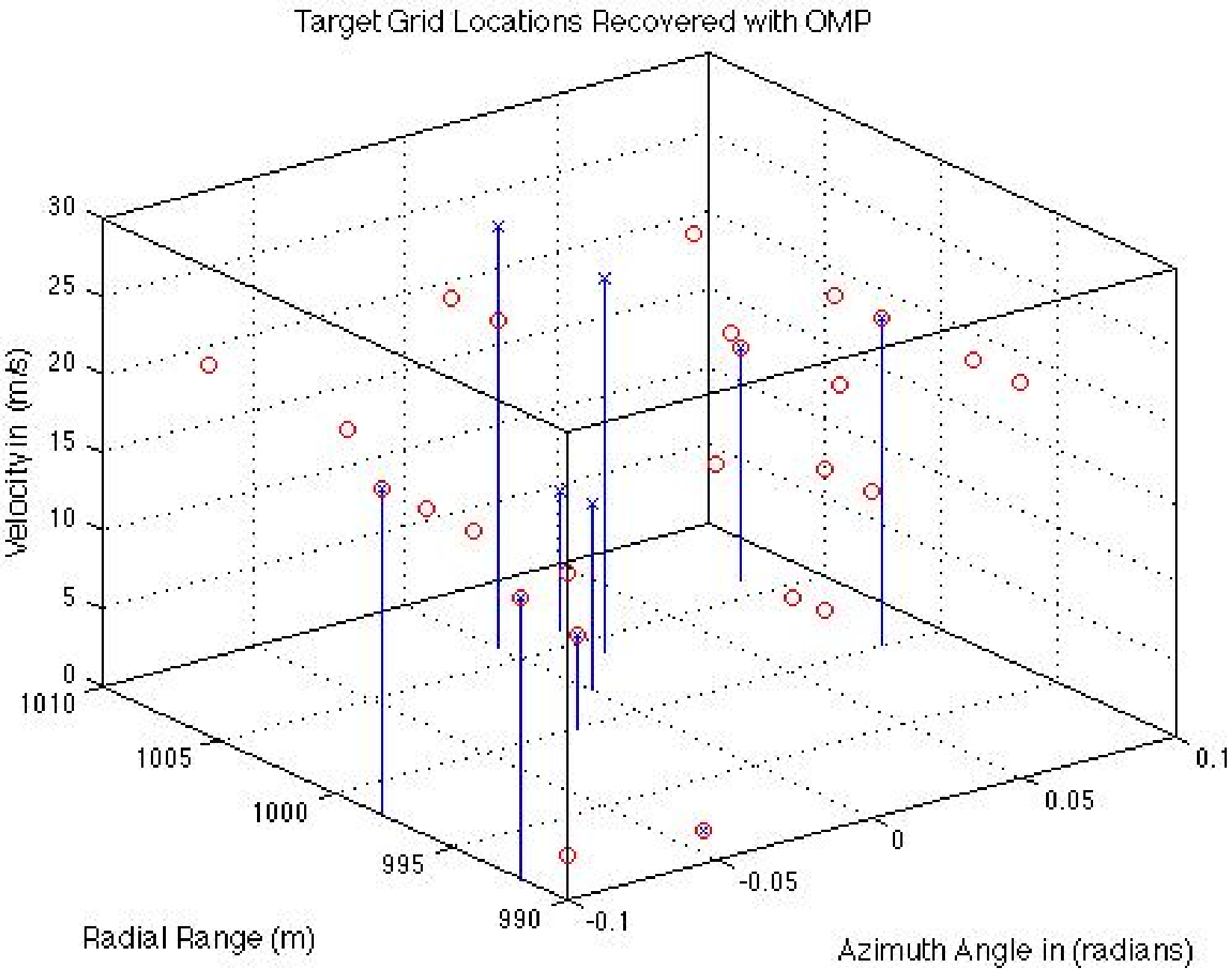
Letting $\boldsymbol{\rho} = \begin{bmatrix} \rho_1 & \cdots & \rho_K \end{bmatrix}^T$, which is P-sparse, we arrive at

$$\mathbf{z} = \mathbf{H}\boldsymbol{\rho} + \mathbf{w}.$$

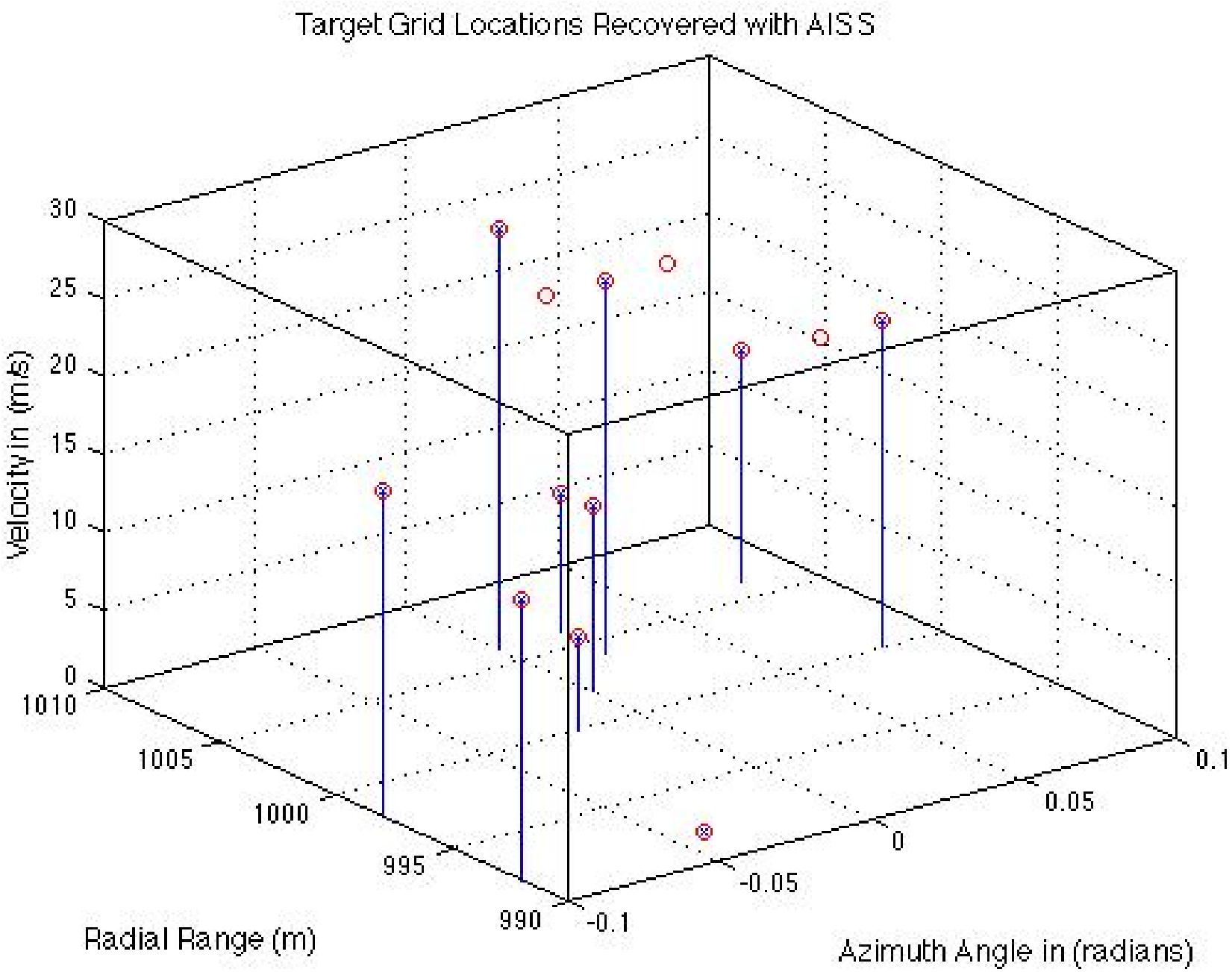
On-Grid Target Locations



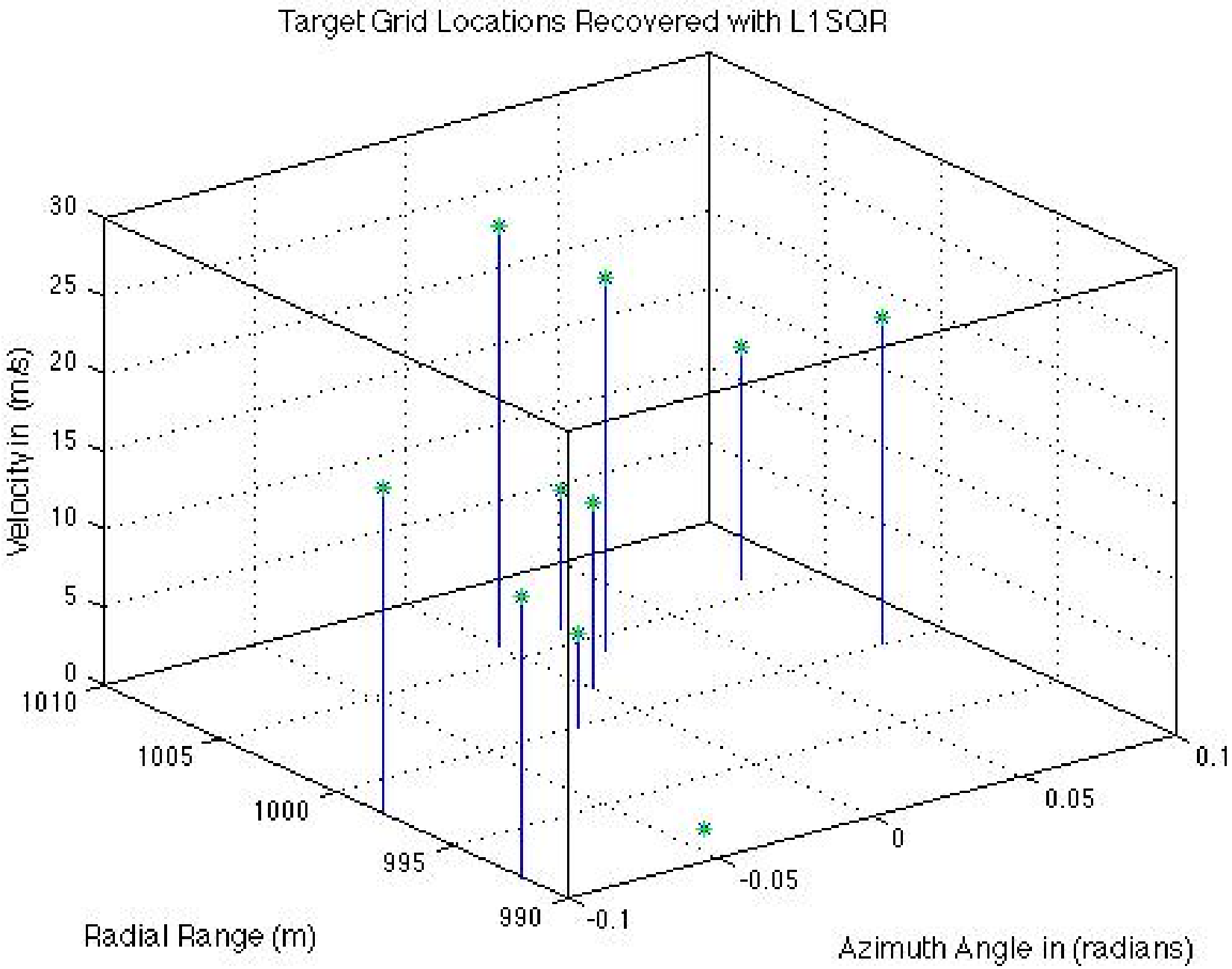
Orthogonal Matching Pursuit



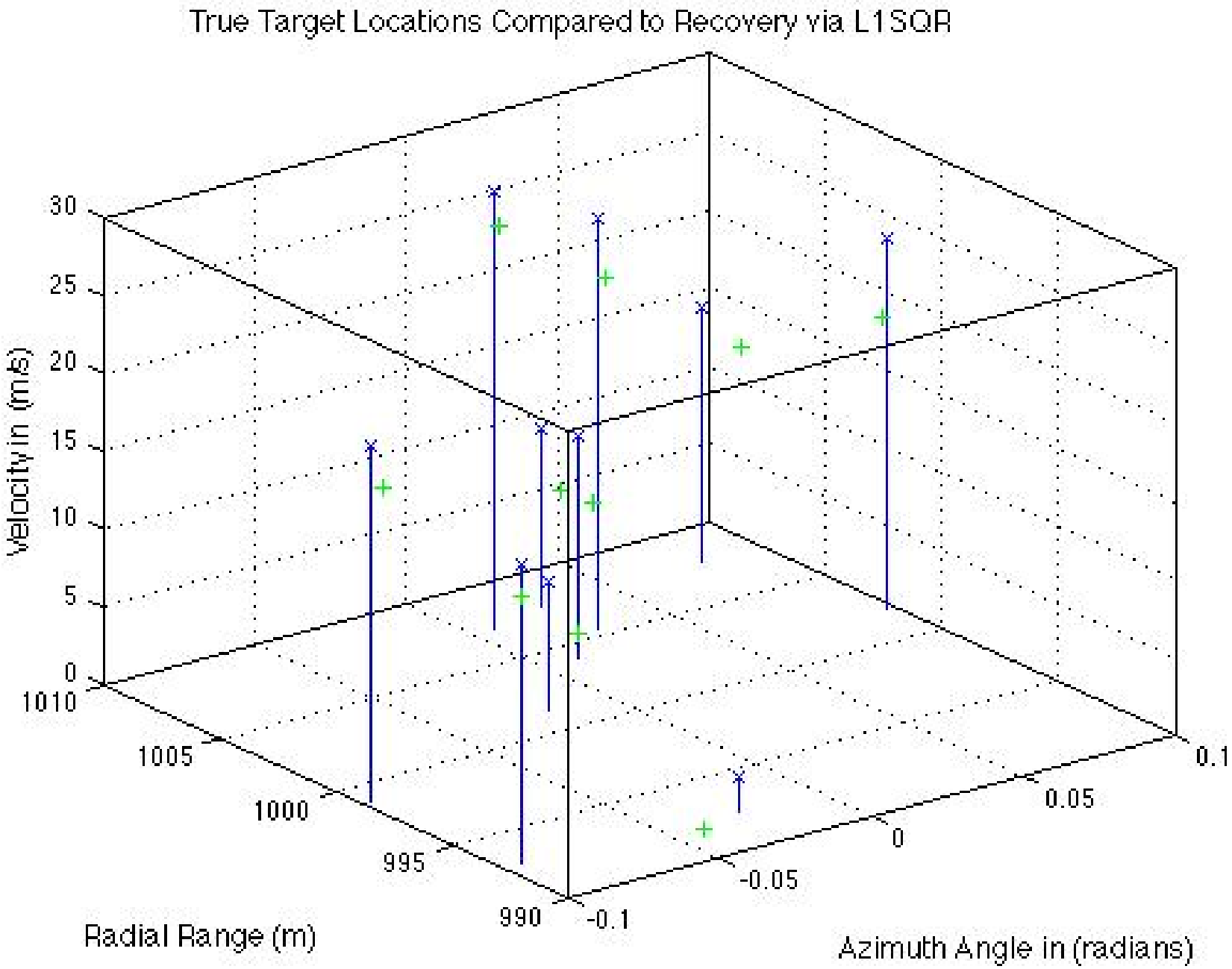
Adaptive Inverse Scale Space [M. Burger, et al. 2012]



ℓ_1^2 Regularization [S. Foucart, D. Koslicki, 2013]



True Target Locations



Perturbations

True Locations:

$$(\tilde{\theta}_k, \tilde{\tau}_k, \tilde{\upsilon}_k) = (\theta_k + \phi_k \Delta_\theta, \tau_k + \xi_k \Delta_\tau, \upsilon_k + \nu_k \Delta_\upsilon),$$

with ϕ_k, ξ_k and $\nu_k \in [0, 1)$.

$$\begin{aligned} z_{mn}(t'_q) = \sum_{k=1}^{K} \rho_k \exp\left[\frac{i2\pi}{\lambda}(dt_m + dr_n)(\theta_k + \phi_k \Delta_\theta)\right] \\ \exp\left[-i2\pi(f_m + \alpha t'_q)(\tau_k + \xi_k \Delta_\tau - \tau_{\text{ref}})\right] \\ \exp[-i2\pi(t'_q + \tau_{\text{ref}})(\upsilon_k + \nu_k \Delta_\upsilon)] + w_{mn}(t'_q) \end{aligned}$$

Goal: Recover $\{\rho_k, \theta_k, \tau_k, \upsilon_k, \phi_k, \xi_k, \nu_k\}_{k=1}^{K}$ from the set of measurements $\{z_{mn}(t'_q)\}$.

Approximation

How to accommodate these perturbations?
Linearization:

$$\exp\left[\frac{i2\pi}{\lambda}(dt_m + dr_n)\phi_k\Delta_\theta\right] \approx 1 + \frac{i2\pi}{\lambda}(dt_m + dr_n)\phi_k\Delta_\theta$$
$$\exp\left[-i2\pi(f_m + \alpha t'_q)\xi_k\Delta_\tau\right] \approx 1 - i2\pi(f_m + \alpha t'_q)\xi_k\Delta_\tau$$
$$\exp[-i2\pi(t'_q + \tau_{\text{ref}})\nu_k\Delta_\upsilon] \approx 1 - i2\pi(t'_q + \tau_{\text{ref}})\nu_k\Delta_\upsilon.$$

Ignore cross terms in the multiplication.

Off-Grid Measurements

We introduce the matrix $\mathbf{G}$, of size $MNQ \times 4K$, where

$$\mathbf{G} = \begin{pmatrix} \mathbf{H} & \mathbf{H}_1 & \mathbf{H}_2 & \mathbf{H}_3 \end{pmatrix}.$$

Here $\mathbf{H}$ is the same matrix as in the on-grid scenario, while $\mathbf{H}_1, \mathbf{H}_2$ and $\mathbf{H}_3$ are all matrices of size $MNQ \times K$, whose columns are given by

$$\begin{aligned}
\mathbf{H}_{1k} &:= \mathbf{h}_k \odot \mathrm{vec}\left(\frac{i2\pi}{\lambda}(dt_m + dr_n)\Delta_\theta\right), \\
\mathbf{H}_{2k} &:= \mathbf{h}_k \odot \mathrm{vec}\left(-i2\pi(f_m + \alpha t'_q)\Delta_\tau\right), \\
\mathbf{H}_{3k} &:= \mathbf{h}_k \odot \mathrm{vec}\left(-i2\pi(t'_q + \tau_{\mathrm{ref}})\Delta_\upsilon\right).
\end{aligned}$$

Off-Grid Compressive Sensing MIMO Problem

We introduce the vector $\boldsymbol{\sigma}$, of size $4K$ and $4P$-sparse, to represent the target scene:

$$\boldsymbol{\sigma} = \left(\boldsymbol{\rho}^T | \quad \boldsymbol{\rho}_1^T | \quad \boldsymbol{\rho}_2^T | \quad \boldsymbol{\rho}_3^T\right)^T,$$

here $\boldsymbol{\rho}_{1k} := \phi_k, \;\; \boldsymbol{\rho}_{2k} := \xi_k, \;\; \boldsymbol{\rho}_{3k} := \nu_k$. Hence, the measurement process is approximated by

$$\mathbf{y} = \mathbf{G}\boldsymbol{\sigma} + \mathbf{w}.$$

Goal: Recover $\boldsymbol{\sigma}$ from $\mathbf{y}$ and $\mathbf{G}$ and in turn estimate $\theta_k, \tau_k, \upsilon_k, \phi_k, \xi_k$ and ν_k associated with each nonzero ρ_k.

Ongoing Work

- Currently, this approach does not yield favorable results with standard methods.
- The on-grid locations are correctly identified but the perturbation terms are lost.
- Problems, Questions & Future Work...

References

[1] W. Carrara, R. Goodman, and R. Majewski.
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[4] X. He, C. Liu, B. Liu, and D. Wang.
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[5] T. Strohmer and H. Wang.
Accurate detection of moving targets via random sensor arrays and Kerdock codes.
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Thank You!!