Oceanographic Data Transmission: A Compressed Sensing-based Approach

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Abstract — **In this paper, a new application of the Compressed Sensing (CS) theory to the data transmission problem in oceanographic large-scale monitoring missions is proposed. The amount of the data (temperature, salinity and so on) collected during this mission can be huge and the transmission process could become prohibitively expensive in terms of both battery consumption and monetary cost of the satellite link. A new CS-based algorithm is thus developed in order to attain a cost-preserving and energy-efficient data transmission. Moreover, the performance of the proposed CS algorithm is investigated with various parameter settings (different sensing and representation matrices, classical ℓ_1 and fast ℓ_0 minimization algorithms) using real oceanographic data.**

Keywords - Sea measurements, underwater vehicles, sampling methods, signal reconstruction, acoustic velocity.

I. INTRODUCTION

Compressive Sampling, also known as Compressed Sensing (CS), is a novel sensing/sampling paradigm that goes against conventional wisdom in data acquisition [1]. CS theory asserts that one can recover certain signals and images from far fewer samples or measurements than traditional methods require. To make this possible, CS relies on two principles: sparsity, which pertains to the signals of interest, and incoherence, which pertains to the sensing modality.

In the last few years, CS theory has been successfully used in a wide variety of practical applications (see, for example, [2] and references therein). This paper concerns ocean sampling applications, involving autonomous underwater gliders to map physical properties of seawater. An underwater glider is essentially an autonomous vehicle that profiles vertically by controlling its buoyancy and translating some of its vertical motion into horizontal motion by virtue of its wings. In oceanographic applications, the domain to be sampled is very large and multiple gliders are required to speed up the gathering of the data and to have sufficient characterization to get a synoptic picture. In the literature, some recent papers ([3], [4], and [5]) apply the CS theory to this large-scale monitoring problem.

In this paper, a different application of CS in the oceanographic field is proposed. In all the data acquisition missions for gliders, in addition to the sampling problem, there is another fundamental practical issue: the transmission of the collected data. As stated previously, the measured values of the ocean fields of interest (temperature, pressure, salinity and so on) are collected using a certain number of gliders. Generally a glider moves through a 3D space following a saw tooth shape trajectory (see Fig. 1). The trajectory is composed of a certain number of dive/climb cycles in the interval between two surfacings of the glider. The data, collected during each dive or climb cycle, are stored and finally transmitted during a surfacing of the glider. Since the data transmission process is an expensive procedure in terms of both battery consumption for the data uploading and monetary cost of the satellite link, and in order to minimize the time on surface to reduce the risk of collision with vessels, it would be useful to find an algorithm that is able to reconstruct the ocean field of interest using a small part of the acquired data. A possible solution is to use a CS based compression algorithm providing that the sparsity assumption is fulfilled by the ocean field at hand. More precisely, one could use the CS framework to select a certain number of linear CS measurements of the acquired data to be transmitted to the receiver for the field reconstruction. The receiver could then reconstruct the original data by performing a sparse minimization algorithm.

In order to exploit CS techniques at the maximum extent, the signal of interest should be sampled by using CS hardware devices operating directly at sub-Nyquist rate [2]. This work is a preliminary investigation to assess the performance of CS reconstruction algorithms regardless of the physical implementation of the CS sampling process. The sampling is just simulated by applying the so-called CS measurement matrix to the data acquired at the Nyquist (or a greater) rate. The sampling matrices (like the Gaussian one), that cannot be implemented as a physical CS device on board a glider due to kinematic constraints of the platform, are only taken as a reference to compare the reconstruction performance with respect to different sampling matrices. The implementation of a CS device on board a glider is envisioned as part of a future research effort to transform the vehicle in a "CS platform" so as compressed measurements could directly be stored and transmitted to the receiver for signal reconstruction.

The paper is organized as follow. In Section II, an overview of the CS theory is provided, in order to make the manuscript as self-contained as possible. In Section III, the proposed CS-based transmission algorithm is described. Numerical results, relative to the proposed algorithm, are provided in Section IV. Our conclusions are finally collected in Section V.

II. BRIEF OVERVIEW OF THE CS THEORY

A. Representation basis and measurement model

Let $\mathbf{f} \in \mathbb{R}^n$ be a discrete time signal and let $\mathbf{\Psi} \in \mathbb{R}^{n\times n}$ be an orthonormal basis in which the signal **f** has a sparse representation. Suppose now that $\mathbf{f}=\mathbf{\Psi x}$, where $\mathbf{x} \in \mathbb{R}^n$ is the coefficient vector. Formally, **f** admits an *s*-sparse representation through $\mathbf{\Psi}$ (the so-called representation basis), if its coefficient vector **x** has at most *s* non-zero elements. Given a set of vectors, $\{\boldsymbol{\varphi}_k\}_{k=1}^m$, the CS measurements are collected by means of linear functionals of the signal **f** [6]:

$$y_k = \boldsymbol{\varphi}_k^T \mathbf{f}, \quad k = 1,\ldots,m. \tag{1}$$

The interest is in *undersampled* situations in which the number *m* of available CS measurements is much smaller than the dimension *n* of the signal **f**. Letting $\mathbf{\Phi}$ denote the $m\times n$ sensing matrix with the vector $\boldsymbol{\varphi}_1,\ldots,\boldsymbol{\varphi}_m$ as rows, the process of recovering **f** from the measurement vector $\mathbf{y} = \mathbf{\Phi f} \in \mathbb{R}^m$ is, in general, ill-posed when $m<n$. However, if two fundamental requirements are satisfied, i.e. the sparsity and the incoherence, it is possible to reconstruct the signal of interest with $m \ll n$ CS measurements. The incoherence captures how dissimilar a pair of representation and sensing bases are. The idea is that local information in one basis will be spread out in the other basis. Incoherence is defined as [1]:

$$\mu(\mathbf{\Phi},\mathbf{\Psi}) = \sqrt{n} \max_{1\le k,j\le n} \left|\left\langle \boldsymbol{\varphi}_k, \boldsymbol{\psi}_j \right\rangle\right|. \tag{2}$$

B. The reconstruction algorithm

Given the CS measurement vector **y** and the knowledge that the signal **f** admits a sparse representation **x**, it is natural to attempt to recover $\mathbf{f}=\mathbf{\Psi x}$ by solving the following optimization problem [7]:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^n} \|\mathbf{x}\|_0 \quad \text{s. t. } \mathbf{\Phi\Psi x} = \mathbf{y} \tag{3}$$

where the ℓ_0-norm of a vector is defined as the cardinality of the support of the vector. The main problem of this approach is that the ℓ_0-norm is a discrete and non convex function and hence it is potentially very difficult to solve the optimization problem in eq. (3). A way to reformulate this problem into something more tractable is to replace the ℓ_0-norm with a certain non-negative and continuous function *F*, that is:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathbb{R}^n} F(\mathbf{x}) \quad \text{s. t. } \mathbf{\Phi\Psi x} = \mathbf{y}. \tag{4}$$

The most important case is the ℓ_1-minimization (the Basis Pursuit problem) where $F(\mathbf{x}) = \|\mathbf{x}\|_1 \triangleq \sum_i |x_i|$. In fact, a key theorem of CS states that if the coefficient vector **x** of **f** is *s*-sparse in the basis $\mathbf{\Psi}$, then the reconstruction in ℓ_1-norm is exact with high probability if $m \ge C\mu^2(\mathbf{\Phi},\mathbf{\Psi}) s \ln n$ for some positive constant *C* [7]. Although the ℓ_1-minimization is a convex optimization and can be solved efficiently, in some cases ([8], [9]) it can be suboptimal. In this paper, as well as the Basis Pursuit problem, a different approximation of the ℓ_0-norm, proposed in [10], is considered.

C. Robust CS and Restricted Isometry Property (RIP)

In order to be really useful, CS needs to be able to deal with compressible signals, i.e. signals that have an exponential decay of the entries of the coefficient vector, but are not necessary sparse. In the CS literature, some bounds on the loss in the reconstruction accuracy of a compressible signal are provided. However, all the results on the robust CS hold true if and only if another requirement on the sensing matrix $\mathbf{\Phi}$, in addition to the incoherency property, is satisfied. In particular, $\mathbf{\Phi}$ must satisfy the so-called restricted isometry property (RIP) [11]. For the application discussed in this paper, it is sufficient to observe that sensing matrices satisfying the RIP can be sampled from two families of random matrices [12]:

1) form $\mathbf{\Phi}$ by sampling i.i.d. entries from the normal distribution with zero mean and variance $1/m$,
2) form $\mathbf{\Phi}$ by sampling i.i.d. entries from symmetric Bernoulli distributions.

We note, in passing, that the identity matrix **I** does not satisfy the RIP.

III. CS AND DATA TRANSMISSION

In this section, the proposed CS-based transmission algorithm is described. In the application at hand, the signal **f** is represented by the data stored in the memory of the glider, while the CS measurement vector **y** represents the amount of data to be sent to the receiver. Then, in the processing center of the receiver, where processing power and battery consumption is not a problem, the original field data are reconstructed using the sparse minimization algorithm in eq. (4). It must be noted that the sensing matrix $\mathbf{\Phi}$, used in the glider to set the CS measurement vector **y** to be transmitted, has to be known to the receiver. This fact could be a problem if random sensing matrices are used. A possible solution would be to exchange between the glider and the receiver the initial seed of the random number generator, instead of transmitting the full-size sensing matrix. At this point, an important consideration about the sampling strategy of the glider must be done. In order to avoid a continuous sampling of the seawater and the resulting storage of all the acquired data **f**, the previously-described CS transmission scheme could also be implemented as a random sampling strategy. While the glider transverses the water column on its undulating trajectory, it could directly create the measurement vector **y** by merely collecting *m* samples at random locations, resulting in an additional energy saving. This approach is mathematically equivalent to the one discussed before, where the measurement matrix $\mathbf{\Phi}$ is the identity matrix **I**. However, as discussed before, the identity matrix does not satisfy the RIP property and this could lead to a performance degradation of the reconstruction algorithm. This possibility is investigated in the next section.

A. Data format, representation bases and evaluation metrics

The matrix data structure

The first question is: “Which is the best way to rearrange the collected data, in order to obtain the minimal information loss in the reconstruction phase?”

As depicted in Fig. 1, the trajectory covered in each dive cycle is composed of a certain number of diving-climbing paths. During a typical dive cycle, the glider moves between two levels of depth, namely *a* and *b*. Given the strong correlation between the data collected during two adjacent diving-climbing paths, it would be useful to rearrange the data as a matrix obtained by interpolating the measured field values during each trajectory path in a regular depth grid of *p* points between *a* and *b*. More precisely, the i^{th} row of the data matrix will contain the interpolated field value collected by the glider during the i^{th} trajectory path. The main advantage of using a "data matrix" instead of a simpler "data vector" is in the possibility of using a 2D representation basis to fully exploit the strong correlation between the data collected during two adjacent diving-climbing paths and increase the sparsity of the representation coefficient vector. For this reason, in all the simulations, the collected data are always recast in a matrix structure.

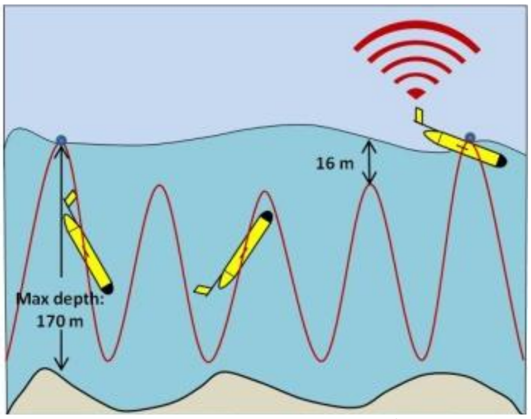

Figure 1 - Saw tooth shape trajectory of a glider.

To assess the performance of the proposed CS-based algorithm, a real data set collected during the Recognized Environmental Picture 2012 (REP12) campaign in the Mediterranean Sea is used. The glider used in this mission sampled the water column between 16 and 170 meters of depth. In order to set up the data matrix, 16 climbing-diving paths are collected and the values of the fields are interpolated on a regular depth grid of 128 points. If no compression algorithm is used, the total amount of data to be transmitted is of 16×128 (=2048) samples. In Fig. 2, a temperature data matrix, obtained from the samples gathered by the glider, is shown.

Representation bases

In this study, two representation bases have been used: the 2D Discrete Cosine Transform (DCT), that is strictly related to the Discrete Fourier Transform (DFT), and the 2D Wavelet Transform (WT), with the Duabechies 1, (db1 or Haar) wavelet function [13].

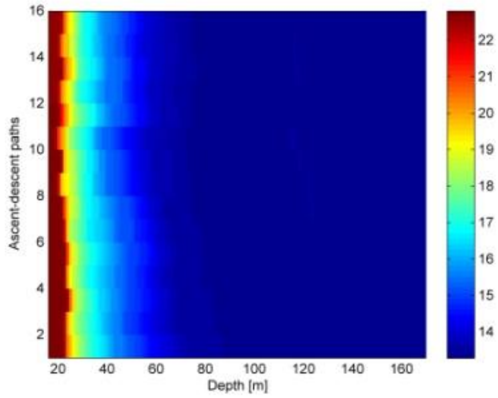

Figure 2 - Temperature data matrix.

Performance metric

The performance of the proposed algorithm have been evaluated in terms of mean value of the Root Mean Square Error (RMSE). More precisely, let *n* be the number of samples in the collected dataset $\mathbf{f}$ and let $\hat{\mathbf{f}} = \mathbf{\Psi}\hat{\mathbf{x}}$ be the reconstructed (or estimated) dataset. The RMSE($\mathbf{\Phi}$) is defined as:

$$\mathrm{RMSE}(\mathbf{\Phi}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\mathbf{f}_i - \hat{\mathbf{f}}_i(\mathbf{\Phi})\right)^2}\,. \tag{5}$$

The RMSE($\mathbf{\Phi}$) in eq. (5) is a function of the particular realization of the sensing matrix $\mathbf{\Phi}$. More precisely, RMSE($\mathbf{\Phi}$) is itself a random variable whose mean value μ is given by $\mu = E\{\mathrm{RMSE}(\mathbf{\Phi})\}$. The mean value μ represents the reconstruction RMSE, averaged over all the possible realizations of the measurement matrix $\mathbf{\Phi}$. The average RMSE will be evaluated for different values of the compression ratio, i.e. the ratio between the number of transmitted CS measurement *m* and the total amount of samples *n* in the original data.

IV. TEMPERATURE DATA ANALYSIS

The ocean field considered in this paper is the temperature field. During the REP12 campaign, the data relative to other ocean fields, such as salinity and density, were also collected. For brevity, only the numerical results relative to the temperature field are reported here. It is easy to show, however, that similar results hold for the other ocean fields.

A. RMSE of the reconstruction error for three different types of sensing matrices

In this subsection, the impact of the sensing matrix on the performance of the proposed CS algorithm for data transmission is evaluated.

The representation basis is a 2D-DCT basis and the three sensing matrices are the identity matrix, the Gaussian matrix, and the Generalized Bernoulli matrix, respectively. In this first case, the classical ℓ_1-minimization is used; the application of the fast ℓ_0-minimization is investigated later on. The performance of the CS algorithm is assessed in terms of mean value of the RMSE, μ, as a function of the compression ratio.

As shown in Fig. 3, all the three considered sensing matrices guarantee similar reconstruction performance. In particular, using a compression ratio of 60%, the temperature field can be reconstructed with a RMSE of about 0.1°C. It can be noted that, even if the identity matrix does not satisfy the RIP, it guarantees the same performance as the Gaussian and the Generalized Bernoulli matrices. This fact allows one to use a random sampling scheme of the sea column, providing that the 2D-DCT representation basis is used, without performance loss.

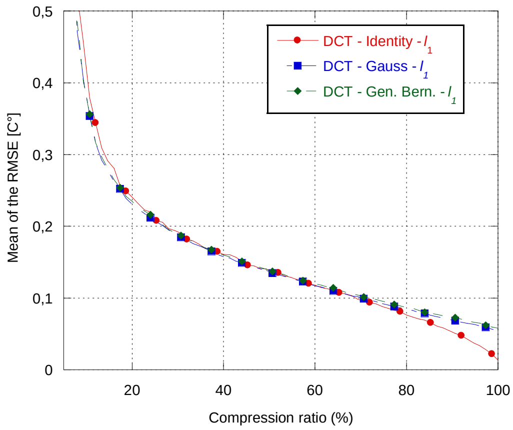

Figure 3 - Mean value of the RMSE vs the compression ratio for different sensing matrices.

B. *DCT representation basis vs WT representation basis*

In the previous section, the performance of the reconstruction algorithm is assessed against different sensing matrices. However, the choice of the representation basis also plays a crucial role. Figs. 4 and 5 show the comparison between the performances obtained by using the two considered representation basis: the 2D-DCT and the 2D-WT with a db 1 wavelet. The results in Fig. 4 concern the use of a Gaussian sensing matrix, while the ones in Fig. 5 concern the use of the identity sensing matrix. Using a Gaussian sensing matrix and having to transmit a small number of compressed measurements, better performance can be achieved by using a DCT representation basis (μ in this case is between 0.2 C° and 0.1 C° for a compression ratio between 20% and 40%). As the compression ratio increases, the WT basis provides better performance than the DCT basis. On the other hand, the WT basis is not suitable for an identity sensing matrix. In fact, as shown in Fig. 5, the reconstruction performance is very low. This means that, if a random sampling strategy is implemented, the 2D-WT cannot be used as representation basis.

C. *Classical ℓ_1-minimization vs fast ℓ_0-minimization*

In this subsection, the performances of the classical ℓ_1-minimization and the fast ℓ_0-minimization, proposed in [14], are investigated. The comparison is performed by using a Gaussian sensing matrix and the two representation bases, i.e. the 2D-DCT and the 2D-WT.

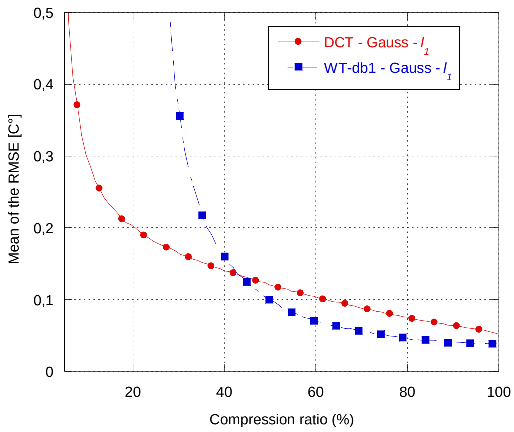

Figure 4 - Mean value of the RMSE vs the compression ratio for different representation bases (Gaussian sensing matrix).

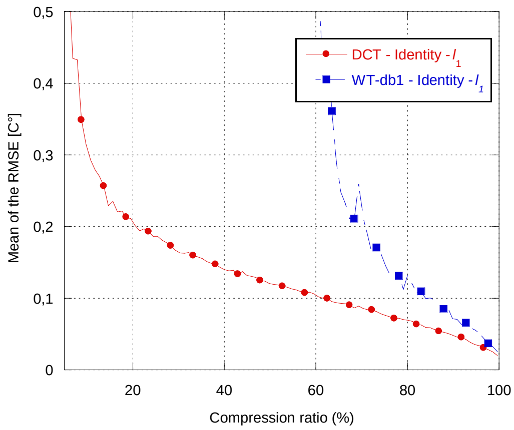

Figure 5 - Mean value of the RMSE vs the compression ratio for different representation bases (identity sensing matrix).

When the 2D-DCT is used, the performances of the two minimization algorithms are similar (see Fig. 6). On the other hand, by using the 2D-WT as representation basis, the ℓ_0-minimization has much better performance than the classical ℓ_1-minimization. Using a compression ratio of 50%, the ℓ_0-minimization is able to reconstruct the original temperature data with an error of about 0.03 °C, while the ℓ_1-minimization has a reconstruction error of about 0.1°C (see Fig. 6).

Moreover, the ℓ_0-minimization algorithm is much faster than the classical ℓ_1-minimization (i.e. less computational time). There is no clear theoretical justification for this surprisingly good behavior of the algorithm based on ℓ_0-minimization, which seems to be strongly data-dependent and non-general at all.

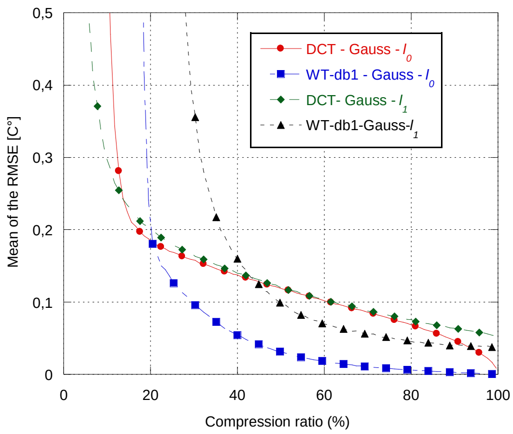

Figure 6 - Mean value of the RMSE vs the compression ratio for different minimization algorithms.

V. Conclusions

In this paper, an oceanographic application of the CS theory has been investigated: pre-processing prior to transmission of the measured data using a limited bandwidth satellite link and saving the battery energy. In particular, the possibility to transmit only a certain number of CS measurements, instead of all the collected data, and then to implement a reconstruction algorithm at the receiver was investigated. The aim of this investigation was to evaluate the minimum number of CS measurements of the ocean field of interest (for example the temperature), that needs to be transmitted in order to reconstruct the field with a given accuracy. The performance of the proposed CS algorithm has been assessed in terms of mean value of the RMSE as function of the compression ratio (i.e. the ratio between the number of transmitted CS measurements and the total amount of data samples), for different sensing matrices (identity, Gaussian, and Bernoulli matrices), and for different representation bases (2D-DCT and 2D-WT). The performance and the processing time of the classical ℓ_1-minimization algorithm have been compared with that of the ℓ_0-minimization algorithm. The simulations have shown that the ℓ_0-minimization algorithm provides equal or even better performance than the ℓ_1-minimization algorithm. Moreover, the ℓ_0-minimization algorithm is much faster than the other algorithm. However, performance of the ℓ_0-minimization is quite surprising and needs to be further investigated from both a theoretical and practical point of view.

Future works will explore the possibility to use other representation bases, in order to improve the sparsity of the considered ocean fields and transmit a lower number of samples to achieve a given accuracy. Moreover, the robustness of the proposed CS transmission algorithm will be assessed using different datasets of measured ocean fields.

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