Compressed Sensing algorithms performance with superresolution in a passive radar

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***Abstract*—The paper presents a study of Compressed Sensing application in a passive radar, where the range resolution is limited by the bandwidth of signal used. The application of Compressed Sensing allows to obtain superresolution in a presence of a point target, which is useful e.g. when exploiting multipath information for estimating the target elevation. However, in such setup, Compressed Sensing algorithms tend to fail. This paper aims to study the applicability of different algorithms and to propose some modifications that improve their performance.**

I. Introduction

Compressed Sensing paradigm was primarily conceived in connection to image processing. There exist many examples of successful application of Compressed Sensing in optical [1] or medical [3]image reconstruction.

Typical radar problems are significantly different from the examples cited above. First, the basis (or frame) elements are usually complex functions, and coefficients in sparse decompositions are also complex, as well as the measured samples. This makes some adjustments in algorithms necessary, as typically algorithms were devised with real-valued data and coefficients in mind. An example may be AMP and its complex modification under name of CAMP [4].

A second difference lies in different structure of radar signal and different setups in typical problems. Also, unknown (and not well-bounded) amplitude of the signal and noise can be a source of errors.

Very specific problems arise when Compressed Sensing is used to enable superresolution in radar, i.e. recovery of very small relative delays. The measurement matrix in this case may exhibit high coherence, which causes failures of some algorithms, however one can still recover the delay values with a brute-force algorithm.

In the following, the performance of several known algorithms will be studied with respect to the application in a passive (PCL) radar with superresolution in range.

The authors will show how different Compressed Sensing algorithms perform in such "superresolution" setup. From authors' research it is clear that the majority of errors is due to selecting wrong support subset and amplifying the wrong way in subsequent iterations. Thus, a method for encouraging the way out by modification of selected algorithms will be proposed.

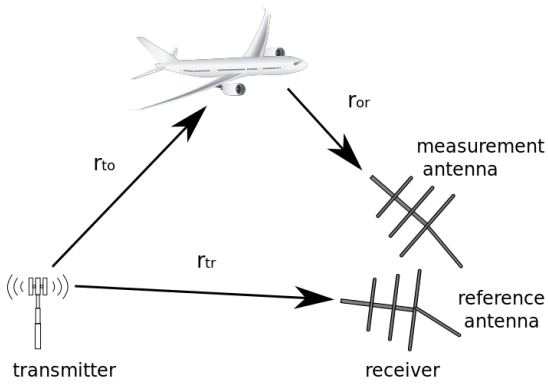

Fig. 1. Schematic PCL radar setup

II. Range superresolution in PCL radar

A passive (PCL) radar detects objects using signals from non-cooperative transmitters already present in the environment (so called "illuminators of opportunity"). Typically, transmitters such as radio or TV broadcasting, GSM Base Station or WiFi/WiMax are used. As the illuminating signal is not known a'priori, the detection is done by cross-correlating the signal gathered from observing scene (surveillance signal) with the signal obtained from antenna facing the illuminator (reference signal). The time delay between these signals for a single detection determines the ellipsoid on which target is present.

A typical PCL radar working with FM or DVB-T signal has very good Doppler frequency resolution due to long integration time. However, the range resolution cell size is limited by the signal bandwidth to >1 km for FM and 40 m for DVB-T. With sufficient signal-to-noise ratio and sparsity assumption the resolution may be improved. This can be, for example, employed to distinguish straight path echo and ground bounce echoes in a problem of elevation estimation [2].

A specific characteristics of the superresolution problem in PCL is that the dictionary consists of shifted copies of the same signal, which results in very high coherence when the shift granularity is fine. On the other side, in the ground bounce problem the sparsity value is very low – in [2] it is shown that typically $K = 2$ may be assumed.

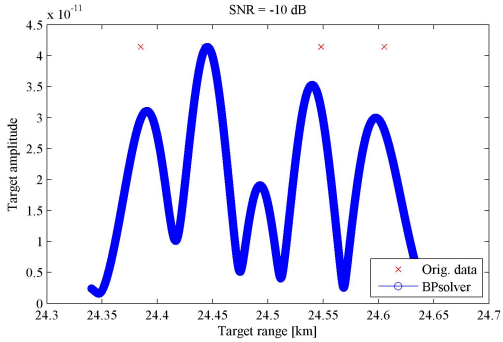

Fig. 2. Failures with BP algorithm [2]

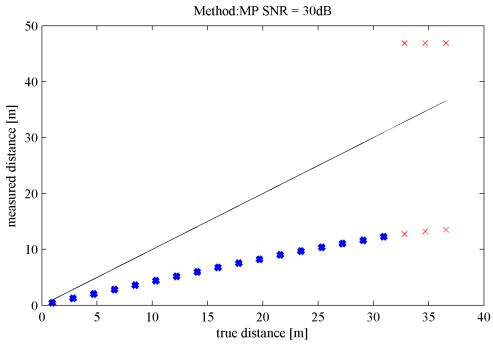

Fig. 3. Scatterplot of failures with MP algorithm [2]

III. Problems with Compressed Sensing Algorithms

Compressed Sensing algorithms, when applied to a problem which poses some challenge, tend to exhibit unwanted failure modes. These modes include:

- converging to a non-sparse solution, similar to classical LS one (this usually happens with BP with a challenging problem or simply with a wrong choice of Lagrangian parameter λ),
- wrong identification of sparse support (preferring too many atoms with large amplitude or sticking to a solution represented by a single strong atom – this frequently happens with MP in a superresolution setup).

An example of BP algorithm failure with superresolution problem is shown in Figure 2.

An example of MP algorithm failure when two echoes are too close is shown in Figure 3. The result of the algorithm is then one echo, approximately located in the middle between actual positions.

IV. Algorithm Evaluation

In this section, two popular algorithms will be evaluated, namely Basis Pursuit solved with linear programming methods [8] and Matching Pursuit [5]. In an effort to investigate the importance of coherence value, different dictionary granularities in spatial domain were used; the finer the granularity, the stronger is the correlation between atoms.

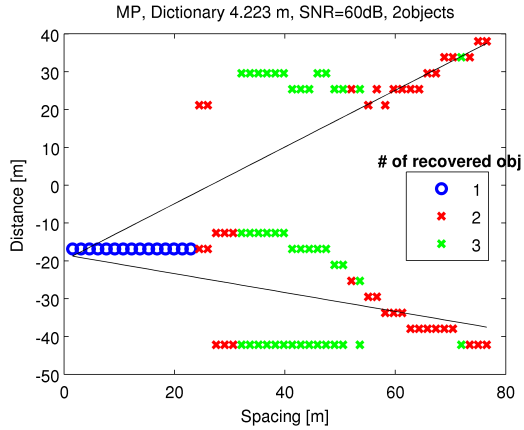

Fig. 4. Scatterplot of MP algorithm results for coarse dictionary

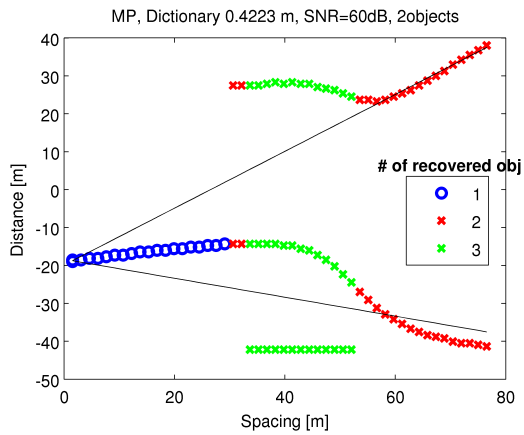

Fig. 5. Scatterplot of MP algorithm results for fine dictionary

The figures shown present the numerical experiment results in a form of scatterplot. The signal from a set of reflecting points in a DVB-T based passive radar is simulated, and it is then processed with use of the chosen algorithm.

The horizontal axis is the actual distance between reflectors, and the vertical axis shows the recovered position of a reflector. In the ideal case, the recovered position should be exactly on the black lines.

The BP algorithm appears very sensitive to the dictionary granularity (and to the resulting coherence). The MP algorithm is insensitive to granularity, however it fails (with a typical failure mode) when the echoes are closer to each other than approximately two range cells.

V. RoCoSaMP

A new algorithm called CoSaMP appeared in [6], which tries to combine ideas of Orthogonal Matching Pursuit with thresholding. Instead of using a predefined threshold, this algorithm selects the elements to be left in the solution vector as K largest elements of the intermediate solution. It means that an assumption has to be made about the expected sparsity

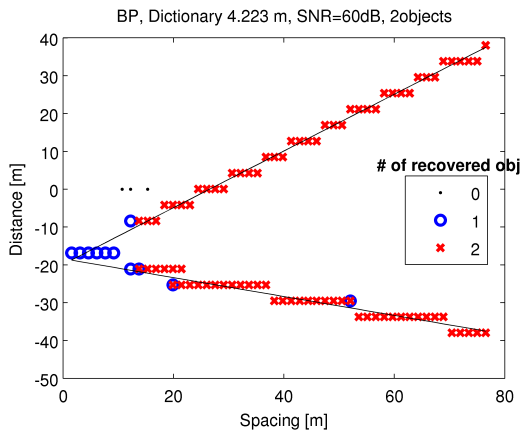

Fig. 6. Scatterplot of BP algorithm results for coarse dictionary

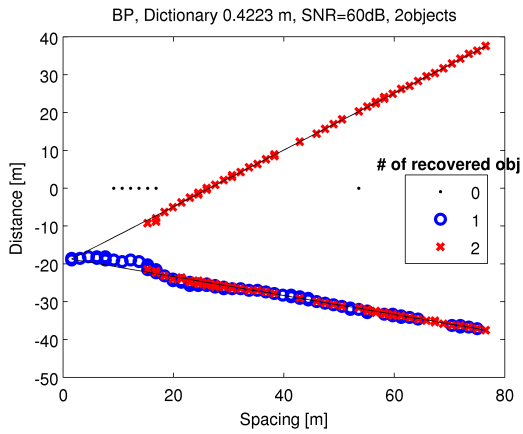

Fig. 7. Scatterplot of BP algorithm results for fine dictionary

of the solution, but in return a K-sparse solution is sought from the first iteration, which accelerates the process.

It should be noted that an assumption of a too large K does not ruin completely the result [7], as is a frequent case with parametric methods (e.g. MUSIC); thus, selecting K value just sets an upper limit on the solution sparsity.

In the CoSaMP algorithm, the main idea is to widen the selected support J^i for the LS operation to $(1+\alpha)K$ elements, where K of them come from the previous iteration and αK from analyzing the residue $\mathbf{r_i}$, and then narrow it again by pruning back to K elements for the residue update. Most frequently a value of $\alpha = 2$ is used, it is tunable; a larger value of α encourages exiting from local optima.

However, the CoSaMP algorithm applied to a superresolution problem often fails due to coherence (μ) of $\mathbf{P}$ matrix being close to unity; this is mainly due to usage of Least Squares minimization. If the support is chosen incorrectly (ex. rather than two points, one point in the middle is chosen), CoSaMP tends to stop in local minimum and is unable to recover the delays. In other cases it tends to grow result magnitudes to very large values.

The idea to create more robust version of CoSaMP, called here RoCoSaMP, came from following facts:

1) In radar case of separating close echoes, the high coherence of dictionary is due to high correlation of adjacent atoms,
2) the exhaustive search algorithm is the most robust one, however it induces unbearable computational complexity with searching of large expected support,
3) usually, very small number of strong scatterers is expected ($K \leq 3$).

In RoCoSaMP algorithm, two important changes are made. First, the LS minimization is substituted with more exhaustive (but also more accurate) basic l_0 search algorithm on truncated support. Second, the truncation of the support is similar to the original CoSaMP - the αK largest scalar product atoms are taken and their support (J^i) is added to the support from previous iteration. However, differently from original CoSaMP, the support is additionally extended by widening the previous estimated support (I^{k-1}) by 2β adjacent elements. It must be noted that this requires the "adjacency" to be defined, which is not a general assumption in Compressed Sensing . However, in the superresolution setup in one dimension this relation is obvious.

In the experiments presented in this paper, the values of $\alpha = 1$ and $\beta = 2$ were assumed.

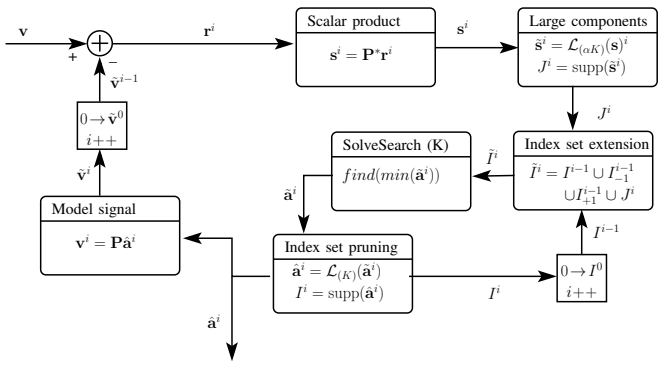

Fig. 8. RoCoSaMP algorithm

A schematic diagram of RoCoSaMP algorithm is shown in Figure 8. The $\mathcal{L}_{(k)}(x)$ symbol denotes an operator selecting k largest elements of vector x and zeroing the rest. Subscripts $+1$ and -1 indicate extension of the index set to the right and left respectively.

Results of evaluating RoCoSaMP in the same setup as in Section IV are shown in Figure 9 and Figure 10. Typically, few erroneous results may be seen, but the majority is correct, and the rate of failures does not depend significantly on the dictionary granularity.

The execution time of RoCoSaMP is plotted in Figure 11 in comparison to a full exhaustive search algorithm. The search algorithm has a deterministic run time, and RoCoSaMP was stopped at the moment when the residual power fails to diminish. With RoCoSaMP, three setups have been tested:

- with 3 actual echoes and 3 echoes assumed in the algorithm (RoCoSaMP (3))

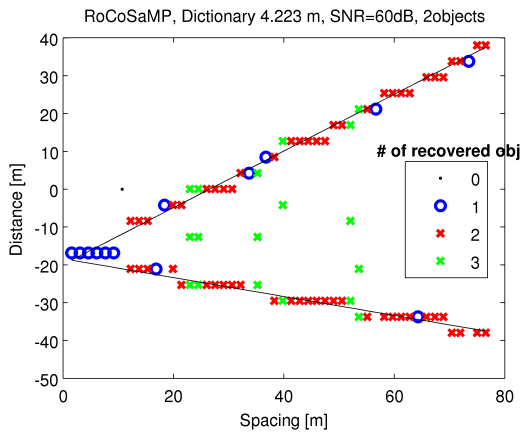

Fig. 9. Scatterplot of RoCoSaMP algorithm results for coarse dictionary

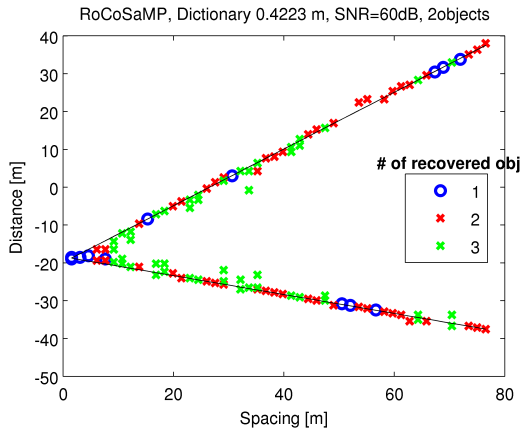

Fig. 10. Scatterplot of RoCoSaMP algorithm results for fine dictionary

- with 2 actual echoes and 2 echoes assumed in the algorithm (RoCoSaMP (2))
- with 2 actual echoes and 3 echoes assumed in the algorithm (RoCoSaMP (2+1))

As the sparsity value in the experiment is low, the search algorithm (run for $K = 3$) is faster with very coarse dictionary. However, when the dictionary gets finer, the search algorithm complexity grows, but RoCoSaMP run time is almost constant, as it performs the search only on a set with size not depending on the dictionary granularity.

VI. Conclusions

The superresolution setup in PCL radar poses a significant challenge for Compressed Sensing algorithms. However, as the algorithm failure is usually caused by wrong support guess, modifications may be made to encourage better search for correct support. One such modification has been proposed and the experiments show that it improves the correct resolution rate.

In the considered setup there are two problems to be solved before actual application. First, run times achieved are still too large – this may be solved with better algorithms or with new

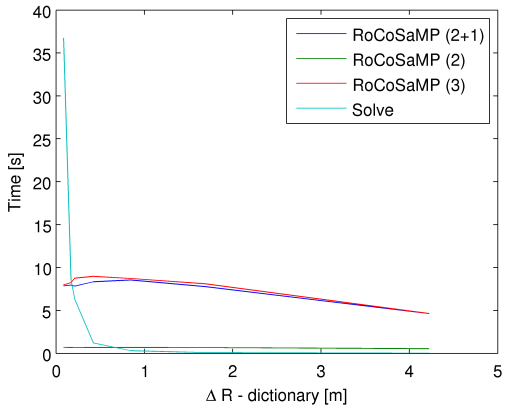

Fig. 11. Execution time of RoCoSaMP and full search (Solve) with respect to the dictionary granularity

processing hardware. Second, the solutions are sensitive to noise, which means that the superresolution may be achieved only with signals strong enough.

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