Retransmitted Jamming Method to LFM Radar Based on Compressed Sensing

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***Abstract*—The compressed sensing (CS) theory is a novel way to break through the existent difficulty in ultra-wideband jamming method development. In this paper, the application of the CS theory in linear frequency modulated signal processing is introduced, a new retransmitted jamming system based on CS is designed with its composition and workflow. Then, two generation modes of jamming signal are illustrated, i.e. the sensing matrix transformation and recovery atomic processing. At last, due to the comparative analysis through a simulation in these two modes, the feasibility and efficiency of this method are verified.**

Keywords-Compressed Sensing; LFM Radar; Retransmitted Jamming; Sensing Matrix Transformation; Recovery Atomic Processing

I. INTRODUCTION

As a typical non-stationary signal with both large time-width and bandwidth, linear frequency modulated (LFM) signal gets high pulse compression gain through matched filtering to achieve high resolution and strong anti-jamming capability, being widely used in SAR and other modern radar. While jamming technology to LFM radar is a hot topic, the active coherent interference is becoming the key and difficult point, among which, retransmitted deception jamming is one efficient method, raising jamming gain and reducing jamming-to-signal power ratio requirement.

Currently retransmitted jamming to LFM radar could be achieved by false target deception, intensive false target, frequency shift [1], interrupted sampling [2], smart noise [3], etc. Although the principles of achieving the jamming and signal forms are different, the sampling, storing and processing methods of the signal are similar. Due to the large time-width and bandwidth of the LFM signal, sampling and processing based on Nyquist theorem in the traditional jamming methods face enormous challenges. The interrupted sampling method is helpful to solve the problem, but it cannot address the issue completely.

To resolve the difficulty of sampling and processing large bandwidth signal, Donoho, Tao, et al. proposed the compressed sensing theory [4,5]. Compared to the Nyquist, CS theory is a novel way to sample the sparse signal at a lower rate and reconstruct it. When applying the CS theory, the requirements of hardware for data sampling, storing and processing are reduced. LFM signal being regarded as a kind of sparse signals, the CS theory has been applied in terms of LFM echo signal processing [6], parameters estimation [7], etc. This essay, based on the CS theory and the existing jamming technology, improves the means of signal sampling and processing to resolve the difficulty of dealing with large bandwidth radar signal, and proposes a new retransmitted jamming method to LFM radar.

II. PRINCIPLES OF THE CS THEORY AND THE RECONSTRUCTION METHOD OF LFM SIGNAL

A. *Introduction to the CS theory*

The CS Theory is applied to process K-sparse signal. If an N-dimensional signal $x(n)$ is K-sparse, it could be completely represented by a few sparse coefficients.

$$x_{N\times 1} = \Psi_{N\times N}\alpha_{N\times 1} \tag{1}$$

Where, Ψ is the signal basis in sparse domain, α is the sparse coefficient.

To process signal $x(n)$ with the CS theory, the first step is to sample $x(n)$ compressively by sensing matrix Φ .

$$y_{M\times 1} = \Phi_{M\times N}x_{N\times 1}, \qquad M \ll N \tag{2}$$

After calculating the product of sensing matrix Φ and sparse basis matrix Ψ , $\Theta = \Phi\Psi$, (2) is rewritten as follows.

$$y_{M\times 1} = \Phi_{M\times N}\Psi_{N\times N}\alpha_{N\times 1} = \Theta_{M\times N}\alpha_{N\times 1} \tag{3}$$

From the measured value $y(m)$, the sparse coefficients $\alpha(n)$ could be obtained by recovery algorithm, and signal $x(n)$ is reconstructed finally.

The compressive sampling achieves the reduction of the signal dimension and the sampling rate. Because the sensing matrix Φ and the sparse basis Ψ are both known already, the connection between the sparse coefficient $\alpha(n)$ and the measured value $y(m)$ is established in (3) without using the original signal $x(n)$. For the K-sparse signal $x(n)$, if matrixes Φ and Ψ satisfy restricted isometry property (RIP), the α could be confidence-recovered from $y(m)$ by nonlinear optimization algorithm [8], then the original signal could be reconstructed according to (1). The series of matching pursuit algorithm [9] is one of the most common optimization algorithms.

B. Method of Reconstructing LFM Signal

For wideband non-stationary signals, there are two forms of constructing atom dictionary, one is the over-complete time-frequency dictionary, and another is the signal waveform matching dictionary. The former one has wide application scope, but has drawbacks of enormous atomic number, high redundancy and complex calculation. The latter one, established according to priori knowledge of signal characteristics, has a better representation feature compared to the former. However, its scope and precision is limited.

1) Time-frequency Dictionary

There are several common forms of time-frequency dictionaries, such as Gabor, Chirplet, Laplace, Damped sin [10], among which, the Chirplet dictionary suits LFM signal as it can represent the sparse feature of LFM signal best. The expression of Chirplet atom is as follows.

$$g_r(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{j\left(\xi(t-u)+\frac{c}{2}(t-u)^2+\omega\right)}, g(t) = 2^{1/4} e^{-\pi t^2} \quad (4)$$

Where, the parameters s, u, ξ, c, ω, correspond to the stretching, the time shift, the frequency shift, the chirp rate and the phase of one atom, respectively.

2) Waveform Matching Dictionary

The priori knowledge of signal characteristic is required to build waveform matching dictionary. According to the form of LFM signal, the discrete basic atom is as follows.

$$g_r(f_u, K_v) = \exp\left\{ j2\pi \left[f_u \frac{n}{f_s} + \frac{K_v}{2} \left(\frac{n}{f_s} \right)^2 \right] \right\} \quad (5)$$

Where, $u = 1, 2, \cdots, U$, $v = 1, 2, \cdots, V$.

$$G_s = \begin{bmatrix} g_r(f_1, K_1) & g_r(f_1, K_2) & \cdots & g_r(f_1, K_V) \\ g_r(f_2, K_1) & g_r(f_2, K_2) & \cdots & g_r(f_2, K_V) \\ \vdots & \vdots & \ddots & \vdots \\ g_r(f_U, K_1) & g_r(f_U, K_2) & \cdots & g_r(f_U, K_V) \end{bmatrix} \quad (6)$$

The different atoms are obtained by traversing f_u and K_v at a certain step within the parameters range of signal, and the dictionary G_s is built in (6). The sparsity, accuracy and scope of G_s are related to the step between atoms and the range of parameters.

3) Sensing Matrix

If the product $\Theta = \Phi\Psi$ satisfies the RIP principle, the original signal would be obtained from M measurements. In general, when sensing matrix Φ and sparse basis matrix Ψ are irrelevant, the RIP is satisfied with a high probability. As the sparse basis, the dictionary of LFM signal is always fixed. So the sensing matrix should be designed to meet the RIP and the requirements of simulation or engineering realization. A random location sampling matrix is designed in this essay.

The sampling probability at every signal point is $p \sim (0,1)$, and the amount of 1 in the random (0,1) sampling sequence is controlled by changing the probability p. Rewrite the (0,1) sequence to matrix as follows.

$$\Phi_{M\times N} = \left[\phi_{m1}, \phi_{m2}, \phi_{m3}, \cdots, \phi_{mM}\right]^T, mi \in I_m, M \approx N \times p \quad (7)$$

I_m is the support set of matrix Φ, mi shows the position of the $i^{\text{-th}}$ 1. In the N-point (0,1) sequence, only position determined by mi is the 1, the others are 0. When the sampling probability of every point is 1, Φ would be an eye matrix, and $M = N$. Compared to other sensing matrixes, the random location sampling matrix has advantages of simpler structure, smaller computation and easier realization in engineering.

III. Generating Method of Jamming Signal

As other types of radar interference, the transmitting interference to LFM radar works to achieve two objectives. One is covering the real target echo by intensifying false targets or making smart noise, to block the real target that adversary try to get; while, the second is transferring the false information of the target to adversary with false target deception, to protect the real one.

A. Structure of Jamming System

The jamming signal form and the objective of the new retransmitted jamming method based on the CS theory are similar to traditional ones. The major differences between them are the means of sampling, processing and generating the jamming signal. The structure of retransmitted jamming system based on the CS theory is shown in Fig. 1.

The jamming control unit 1 modulates time delay or frequency shift of the receiving radar signal in the compressed sampling process, to make recovery signal own jamming characteristics. The jamming control unit 2 is in charge of managing the generating method of the jamming signal, and choosing jamming atoms and processing means to satisfy different jamming requirements.

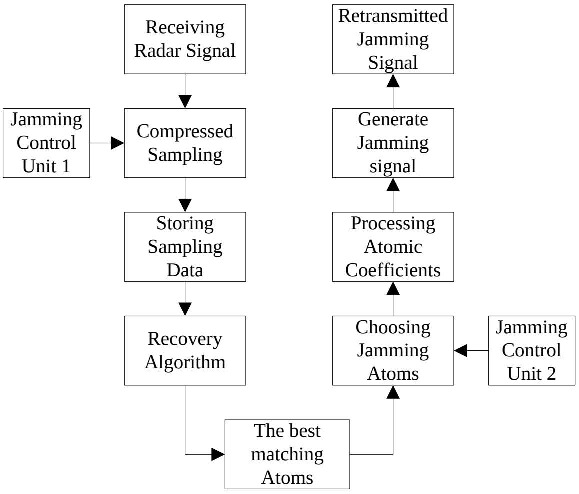

Figure 1. The structure of retransmitted jamming system based on CS theory

The workflow of this system is as follows:

a) Receiving wideband radar signal, compressing and sampling the date under the requirement of control unit 1, and then storing the data;

b) Processing the sampling data with recovery algorithm, and getting the best matching atoms and their coefficients;

c) Choosing the appropriate jamming atoms according to the requirement of control unit 2;

d) Assigning suitable coefficient to every atom, and generating jamming signal;

e) Transmitting the jamming signal to complete the workflow.

B. Generating Method of Jamming Signal

As shown in the structure of the jamming system, the two control units could modulate or process the sampling data to generate jamming signal. Here two generating methods are proposed, i.e. sensing matrix transformation and recovery atom processing.

1) Sensing Matrix Transformation

The sensing matrix transformation is controlled by the jamming control unit 1. Two matrixes are defined, the cyclic shift matrix P and the frequency shift matrix Q. Where, P is the cyclic shift of an eye matrix.

$$P\Big|_{p=1} = \begin{bmatrix} & & & 1 \\ 1 & & & \\ & \ddots & & \\ & & 1 & \end{bmatrix}_{N\times N} \tag{8}$$

Equation(8) shows the cyclic shift matrix P when the shift order $p=1$. Multiplying signal x by the matrix P could shift the signal. If the shift order is relatively less than the length of signal, the influence of signal shifting on pulse compression gain would be small, and the delay processing of signal would be done.

The frequency shift matrix Q is a series of signal points with a certain frequency.

$$Q\Big|_{f_q} = \begin{bmatrix} e^{j2\pi f_q \frac{1}{N}} & & & \\ & e^{j2\pi f_q \frac{2}{N}} & & \\ & & \ddots & \\ & & & e^{j2\pi f_q \frac{N}{N}} \end{bmatrix}_{N\times N} \tag{9}$$

Where, f_q is the shifting frequency. To multiply signal x by Q could shift the signal frequency. Because of the time frequency coupling character, the frequency shift of LFM signal leads to time shift of the pulse compression peak.

The sensing sample process is,

$$y = \Phi x \tag{10}$$

Put the cyclic shift matrix P and the frequency shift matrix Q in it,

$$y' = \Phi PQx \tag{11}$$

Alternate the sensing matrix by $\Phi' = \Phi PQ$, then $y' = \Phi' x$. When recovering signal by the original sensing matrix Φ, the recovery signal would contain time and frequency shifts to achieve interference to radar.

If adding several different P or Q respectively to transform sensing matrix, a series of jamming signal with different time delays or frequency shifts would be generated.

Compared with original compressed recovery process, that processing signal by sensing matrix transformation hardly increases the computation owing to it occurring in compressed sampling.

2) Recovery Atom Processing

When generating jamming signal by processing recovery atoms, every atom in the dictionary should be knowable and controllable. As the process of time frequency dictionaries, such as Gabor or Chirplet, is almost blind, under which, one atom could not be known and controlled specifically. To complete the mission, the waveform matching dictionary is needed, and three generating modes of jamming signal are presented..

a) False Target Deception

In false target deception jamming, the jamming signal must be similar to the original one to disturb the adversaries that they cannot recognize real or false target. The false signal can be made by using shift frequency jamming principle when choosing several atoms whose K equals to that of the original signal but f_0 owns certain shift. When the best matching atom is $g_r(f_u, K_v)$, then the jamming signal is,

$$s_j = \sum_i^m c_i g(f_i, K_v) \tag{12}$$

Where, m is the number of the jamming signal, K_v remains stable, f_i is chosen near f_u, and $\Delta f = f_i - f_u$ is the value of frequency shifting. As the coefficient of each atom, c_i is constant or random. m false targets are generated in this mode, and the purpose of interference is achieved.

b) Intensive False Target

When the number of the false targets is low, the real signal cannot be covered although the adversary is interfered. The intensive false target jamming could cover the real target in a certain range, and overload the processing capacity of adversary radar at the same time. It can also be achieved by frequency shift, two means adopted, i.e. the evenly space and the random location.

After getting the best matching atom and its coefficient, keep K of the chosen atom constant, shift the f_0 back or forth evenly spaced. Then, linearly superpose a set of atoms and transmit to complete the interference. The value of each frequency shifting is the step length of dictionary. When the best matching atom is $g_r(f_u, K_v)$, the jamming signal is,

$$s_j = \sum_i^m c_i g(f_i, K_v), \qquad i \in [u - \frac{m}{2}, u + \frac{m}{2}] \tag{13}$$

Because of the frequency shifting, the jamming signals around the real one have gained loss of pulse compression. Windowing the c_i to amplify the coefficients near the edge can enhance the gain relatively and compensate the loss.

The interference effect of evenly space mode is obvious, but the concentratedly and regularly distributed false targets are easily identified by adversary radar. The random location mode could improve randomness and authenticity of the false targets and reduce the data size. The jamming signal is,

$$s_j = \sum_i^m c_i p_i g(f_i, K_v), i \in [u - \frac{m}{2}, u + \frac{m}{2}], p \sim (0,1) \tag{14}$$

That the coefficient c_i whether 0 or not, is determined by p and the atom number is changed with probability of (0,1) distribution changing. When p following the equal probability (0,1) distribution, the jamming covering range is the same as (13), but the atom number reduces half. The needs of data size and jamming power are also reduced.

c) Smart Noise Jamming

Compressed to discrete peaks on the spectrum, interference only by frequency shift is easily identified. If shifting both K and f_0, randomly modulating the coefficients of the chosen atoms, the smart noise jamming would be achieved. When the best matching atom is $g_r(f_u, K_v)$, the jamming signal is,

$$s_j = \sum_{i,j}^m c_{i,j} p_{i,j} g(f_i, K_j), \quad p \sim N(0,1) \tag{15}$$

The i and j are chosen around the u and v, the coefficients $c_{i,j}$ are modulated by Gaussian distribution to add randomness.

IV. Simulation

A. Sensing Matrix Transformation

Initializing the original signal $T = 4\mu s$, $K = 13.31\text{MHz}/\mu s$, $f_0 = 51.47\text{MHz}$, bandwidth $B = KT = 53.24\text{MHz}$, sampling rate $f_s = 128\text{MHz}$, signal size $N = f_s T = 512$. The compressing rate is 4:1 by using the random location sampling matrix which $M = 128$. The Chirplet dictionary and OMP algorithm are adopted.

1) Cyclic Shift Matrix

The order of the cyclic shift matrix is $p = 20$, and the interference effect is shown as Fig.2.

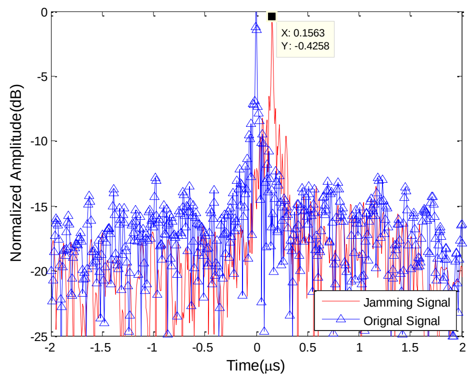

Figure 2. Jamming signal generated by cyclic shift matrix

In fig.2, The time delay of jamming signal is $\tau = Tp / N = 0.1563\mu s$, consistent with the simulation. Owing to the time center shifting, it losses pulse compression gain 0.4258dB. Then, Increasing the number of cyclic shift matrixes, superposing four matrixes including $p = -60$, -30, 20, 80, and compensating the coefficients, the interference effect is shown as Fig.3.

2) Frequency Shift Matrix

The shifting value is $f_q = 3\text{MHz}$, and the interference effect is displayed as Fig.4.

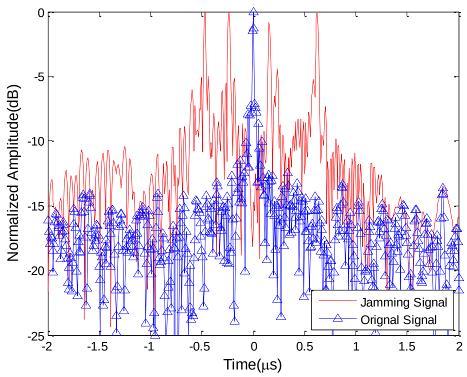

Figure 3. Jamming signal generated by four cyclic shift matrixes

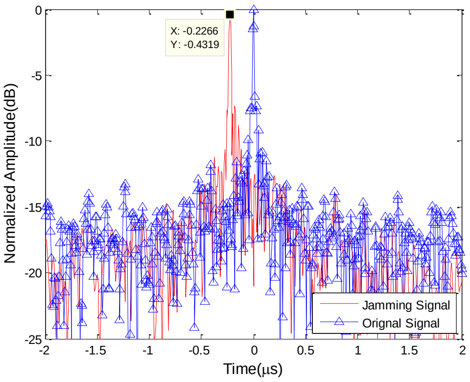

Figure 4. Jamming signal generated by frequency shift matrix

In fig.4, the time delay is $\tau = f_q / K = 0.2254\mu s$, consistent with the simulation. Owing to the frequency shifting, the gain losses 0.4319dB .Then, superposing Four matrixes that $f_q = -8MHz$, $-2MHz$, $3MHz$, $6MHz$, and compensating the coefficients, the interference effect is as follows.

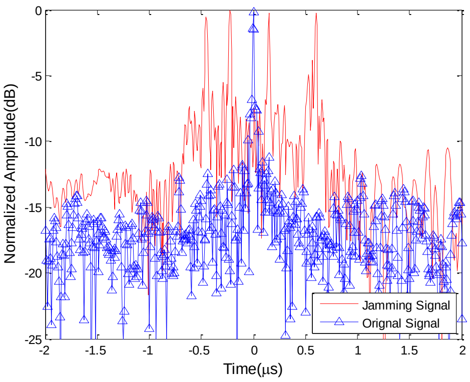

Figure 5. Jamming signal generated by four frequency shift matrixes

B. Recovery Atom Processing

To enhance the interference effect, extend the signal pulse width to $T = 16\mu s$, sampling rate $f_s = 512MHz$, and data size $N = 8192$. The variable step two-level dictionary is adopted. In the first-level dictionary, the range of f_0 is $0 \sim 100MHz$ with the step being $2MHz$, the range of K is $6 \sim 16MHz/\mu s$ with the step $0.2MHz/\mu s$, and the total number of atoms is 2500. The second-level dictionary expands after obtaining the best matching atom at first. While the range of f_0 is expanded to $5MHz$ with the step $0.1MHz$, the expanding range of K is $0.5MHz/\mu s$ and the step is $0.01MHz/\mu s$, the number of atoms is still 2500. Based on the above stimulation, the searching range of the two-level dictionary is $f_0 = 0 \sim 100MHz$ with precision being $0.1MHz$, $K = 6 \sim 16MHz/\mu s$ with precision $0.01MHz/\mu s$, equivalent to one million atoms. For the sensing matrix, random location sampling matrix with $M = 512$ is adopted while the compressed rate is 16: 1. Compared to the Chirplet dictionary, this dictionary is narrowed in terms of sampling rate and data size.

1) False Target Deception

The frequency shifting step is $f_0 = 10MHz$. Giving the same coefficients to 8 atoms, the interference effect is displayed on Fig.6.

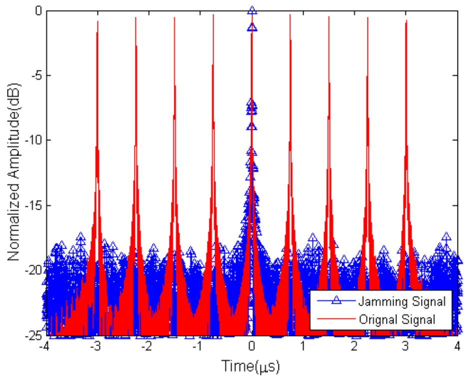

Figure 6. False target deception jamming

Every false target delay is set as $\tau = 0.0751\mu s$, and the sum delay of 8 atoms is about $\tau = \pm 3\mu s$. Since the characteristics of false targets are as the same as the real one, they are effective to deceive the adversary radar.

2) Intensive False Target

a) Evenly Spaced False Targets

Setting the step $\Delta f_0 = 1MHz$, shifting the f_0 back and forth evenly spaced, and modulating coefficients by the central part of an inversed Hamming window, the interference effect is as Fig.7 shown.There are 80 jamming atoms in the simulation, and the covering range of 80 false targets is $\pm 3\mu s$.

b) Random Located False Targets

Determining the location of former jamming atoms by equal probability (0,1) distribution, the interference effect is as Fig.8 shown.

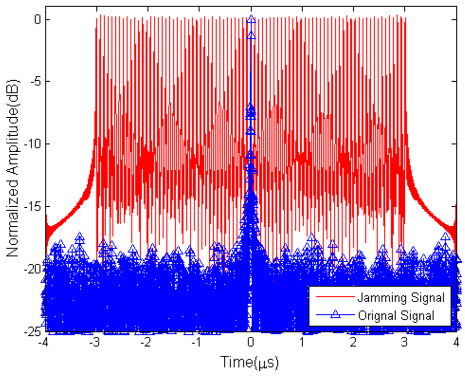

Figure 7. Intensive false targets evenly spaced

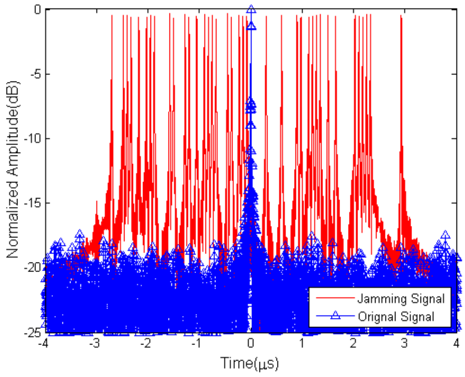

Figure 8. Intensive false targets random located

Only 40 atoms are needed in this simulation, achieving the same range by half of the atoms in Fig.7. The requirements of data size and jamming power are reduced, while the randomness of jamming distribution is increased.

3) Smart Noise Jamming

When K is shifted 5 times with step $\Delta K = 0.3\text{MHz}/\mu\text{s}$, f_0 is shifted 8 times with $\Delta f_0 = 10\text{MHz}$, and coefficients are modulated by Gaussian distribution $N(0,1)$, 40 effective jamming atoms are gotten. The interference effect is as Fig.9 shown.

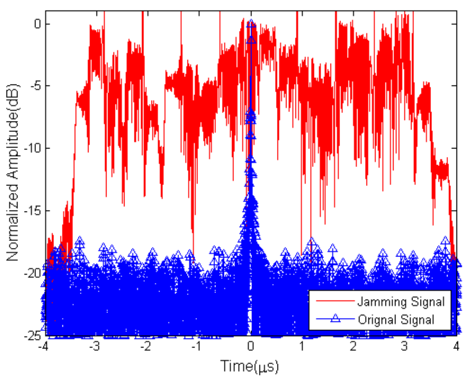

Figure 9. Smart noise jamming

The interference range is similar to Fig.7 with the same number of atoms, but the jamming signal is much more widely and randomly distributed. Because both K and f_0 are shifted, the parameters are not authentically and precisely estimated as in other modes. However, the drawbacks remain that modulating coefficients by Gaussian leads to the heavily fluctuating amplitude of jamming signal and reduce the availability of jamming power.

V. CONCLUSION

Comparing the sensing matrix transformation and the recovery atom processing, similarities can be shown on generating jamming signal, and displaying false target information on the adversary radar. In terms of the difference, the former has a wide scope of application concurring with a large amount of computation and difficult engineering realization because of the enormous over-completed time frequency dictionary, while the latter lowers the sampling rate and lessens the computation, however it cannot work when the signal is beyond the limited dictionary range since priori knowledge is needed to build the dictionary. In brief, each method has pros and cons, and could be chosen under the practical situation.

According to the analysis result, it can be concluded that processing LFM signal based on the CS theory would reduce the sample rate and the data size. Further, different interference effects could be achieved by adopting different generating methods of the jamming signal when applying retransmitted jamming to LFM radar.Compared to traditional methods, the retransmitted jamming method based on the CS theory has following advantages:

a) Reducing the sampling rate and narrowing the data size by compressed sampling;

b) Reducing the requirements of dynamic memory and arithmetic unit;

c) Simplifying the system structure, easing the situation of high engineering equipment standard and making a breakthrough of the dilemma when dealing with wideband signal in the traditional methods;

d) Decreasing the volume and power consumption of the jamming device;

e) Using flexible jamming signal modes by changing the command of jamming control units.

The CS theory has remarkable advantages of reducing sampling rate and computation, which is believed to resolve the difficult that traditional methods facing in retransmitted jamming application. The new retransmitted jamming method based on CS in this essay in significant in theory and in practice. The article is still limited in theoretical analysis and simulating experiment, realizing it in engineering field should be studied further.

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