Sub-Nyquist Sampling and Compressed Processing with Applications to Radar

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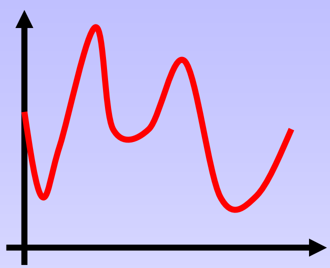
In collaboration with my students at the Technion

Sampling: "Analog Girl in a Digital World…" Judy Gorman 99

Analog world | Digital world



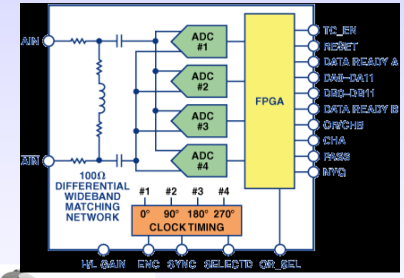
$x(t)$ → **Sampling ADC** → $c[n]$

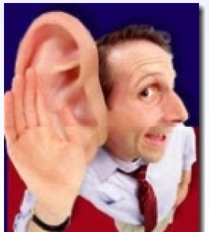
$\tilde{x}(t)$ ← **Reconstruction DAC** ←

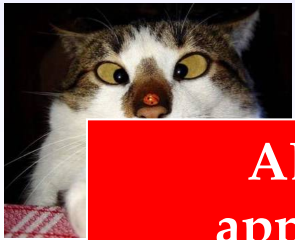
- Music
- Radar
- Image…

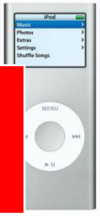
- Signal processing
- Image denoising
- Analysis…

- Very … tes: hard… olutions
- High DSP rates



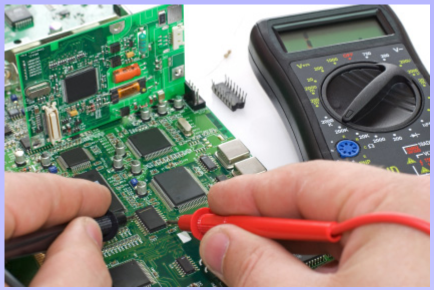






ADCs, the front end of every digital application, remain a major bottleneck

Today's Paradigm



The Separation Theorem:

- Circuit designer experts design samplers at Nyquist rate or higher
- DSP/machine learning experts process the data
  - Typical first step: Throw away (or combine in a "smart" way e.g. dimensionality reduction) much of the data …
  - Logic: Exploit structure prevalent in most applications to reduce DSP processing rates
  - DSP algorithms have a long history of leveraging structure: MUSIC, model order selection, parametric estimation …
  - However, the analog step is one of the costly steps

Can we use the structure to reduce sampling rate + first DSP rate (data transfer, bus …) as well?

Xampling:
Compression + Sampling

Exploit analog structure to improve processing performance:

- Reduce storage/reduce sampling rates
- Reduce processing rates
- Reduce power consumption
- Increase resolution
- Improve denoising/deblurring capabilities
- Improved classification/source separation

Goal:

- Survey the main principles involved in exploiting analog structure
- Provide a variety of different applications and benefits
- Applications to radar

Compressed Sensing and Hardware

- Explosion of work on compressed sensing in digital applications
- Many papers describing models for CS of analog signals
- Have these models made it into wideband hardware?
- CS is a digital theory – treats vectors not analog inputs, processing rates can be high, and are problematic in low SNR

	Standard CS	Analog CS
Input	vector x	analog signal $x(t)$
Sparsity	few nonzero values	?
Measurement	Random/det. matrix	RF hardware
Recovery	convex optimization greedy methods	need to recover analog input or specific data efficiently

We use CS only after sampling
Enables real hardware, low processing rates and low SNR

Talk Outline

- Part I: Motivation
- Part II: Xampling: Compressed sampling of analog signals
- Part III: Applications to radar
  - Pulse radar
  - Ultrasound imaging
  - LTV system identification

Part 1:

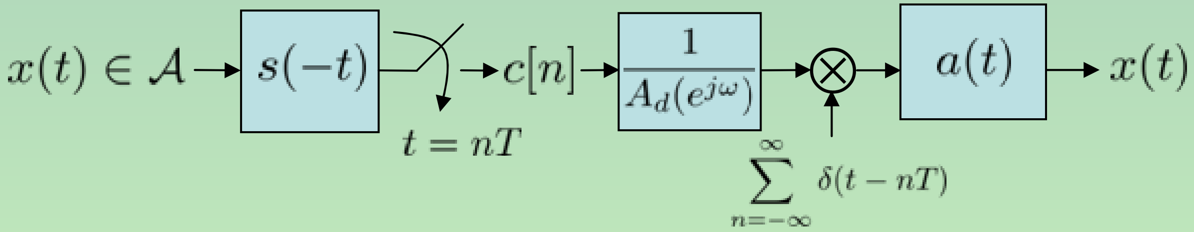
Motivation

Classical/Modern Sampling

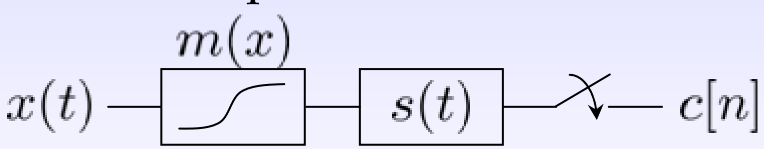
- Sampling theory has developed tremendously in the 60+ years since Shannon
- Recovery methods have been developed for signals in arbitrary subspaces
  (Unser,Aldroubi,Vaidyanathan,Blu,Jerri,Vetterli,Grochenig,Feichtinger,DeVore,Daubechies,Christensen,Eldar, …)

Sampling rate:
Degrees of freedom of subspace

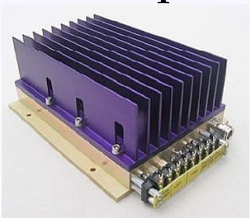
Perfect Reconstruction in a Subspace



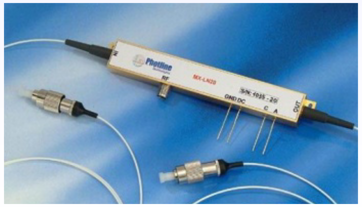
- Recovery from nonlinear samples as well (Dvorkind, Matusiak and Eldar 2008)



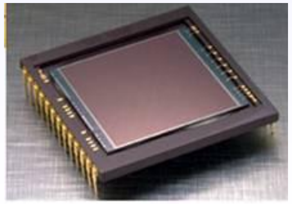
Power amplifiers



Optical modulators



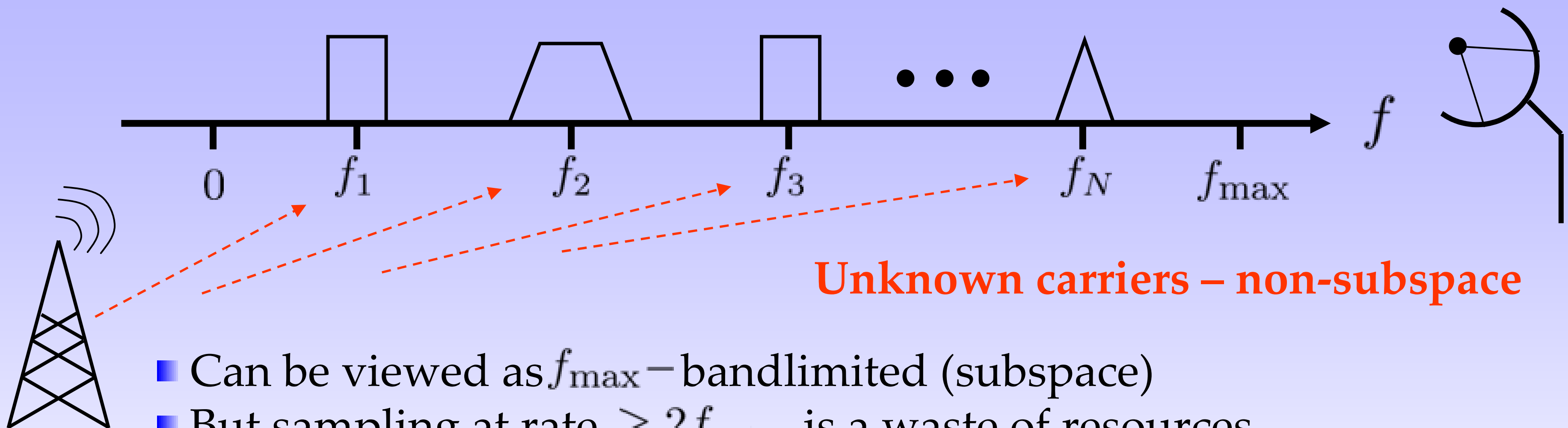
CCD arrays



Companding

Structured Analog Models

Multiband communication:



Unknown carriers – non-subspace

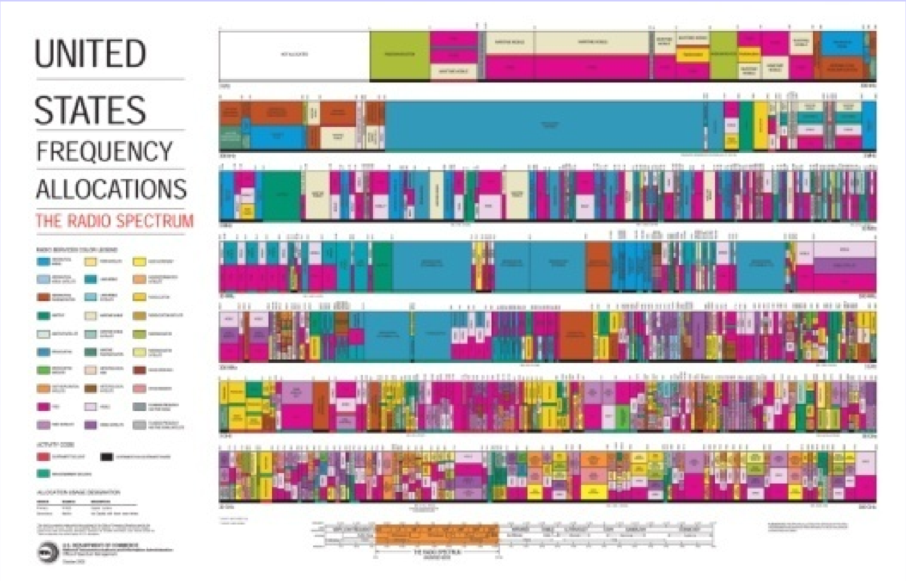
- Can be viewed as $f_{\max}-$bandlimited (subspace)
- But sampling at rate $\geq 2f_{\max}$ is a waste of resources
- For wideband applications Nyquist sampling may be infeasible

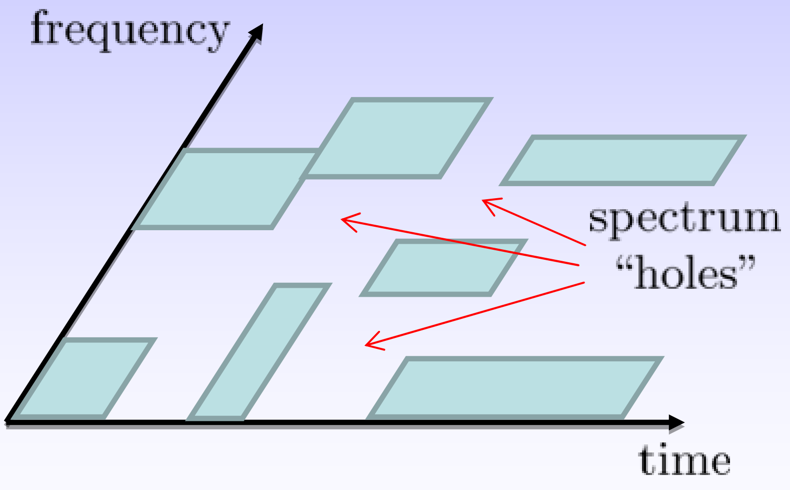
Question:
How do we treat structured (non-subspace) models efficiently?

Cognitive Radio

- Cognitive radio mobiles utilize unused spectrum ``holes''
- Spectral map is unknown a-priori, leading to a multiband model

Federal Communications Commission (FCC) frequency allocation

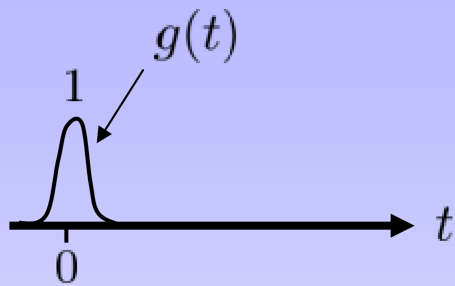


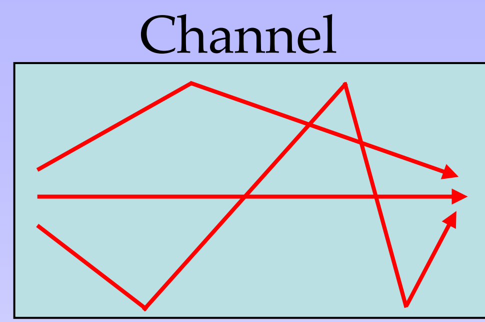


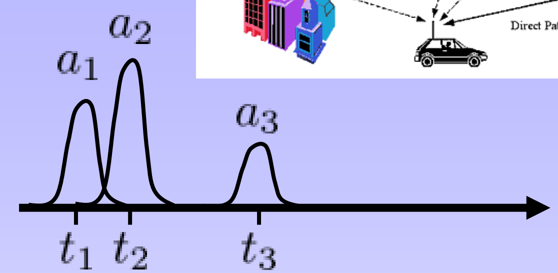
Licensed spectrum highly underused: E.g. TV white space, guard bands and more

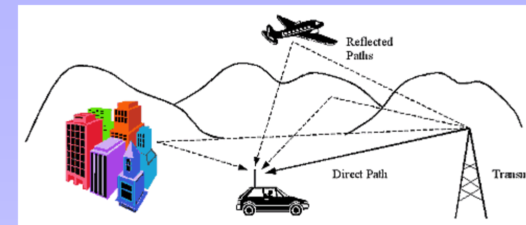
Structured Analog Models

Medium identification:









Similar problem arises in radar, UWB communications, timing recovery problems …

Unknown delays – non-subspace

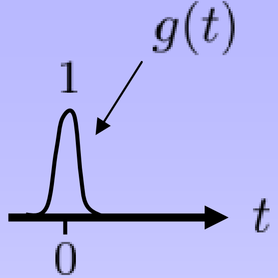
- Digital match filter or super-resolution ideas (MUSIC etc.) *(Quazi,Brukstein, Shan,Kailath,Pallas,Jouradin,Schmidt,Saarnisaari,Roy,Kumaresan,Tufts …)*
- But requires sampling at the Nyquist rate of $g(t)$
- The pulse shape is known – No need to waste sampling resources!

Question (same):
How do we treat structured (non-subspace) models efficiently?

Ultrasound

- High digital processing rates
- Large power consumption

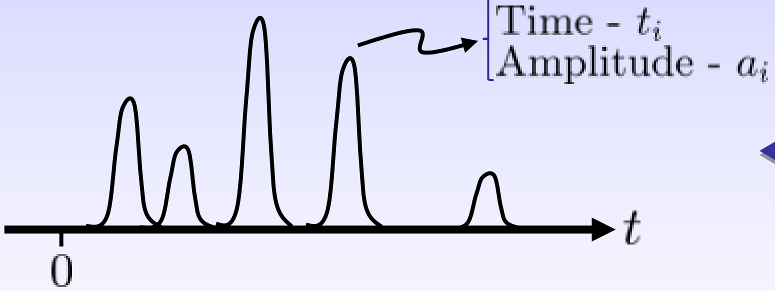
Tx pulse

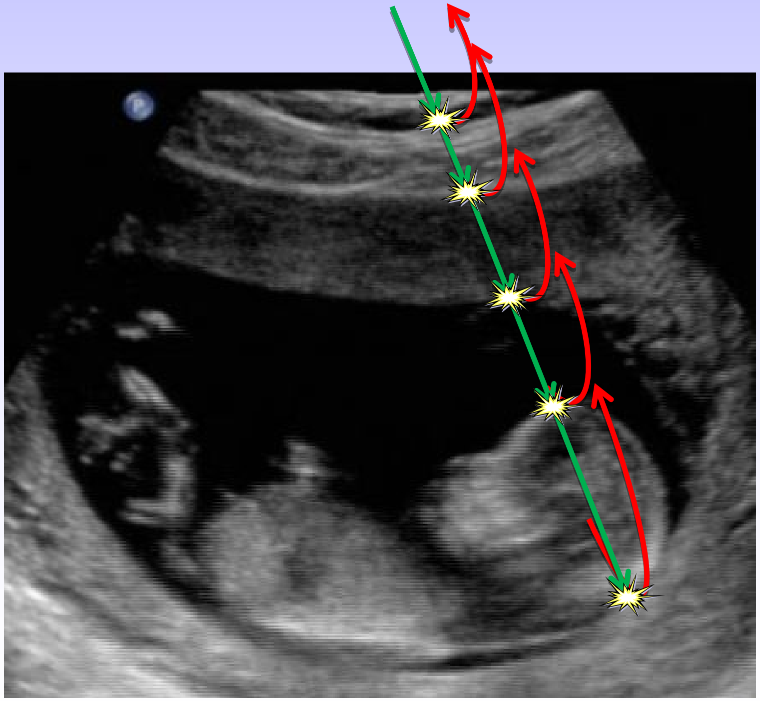


Ultrasonic probe

(Collaboration with General Electric Israel)

Rx signal

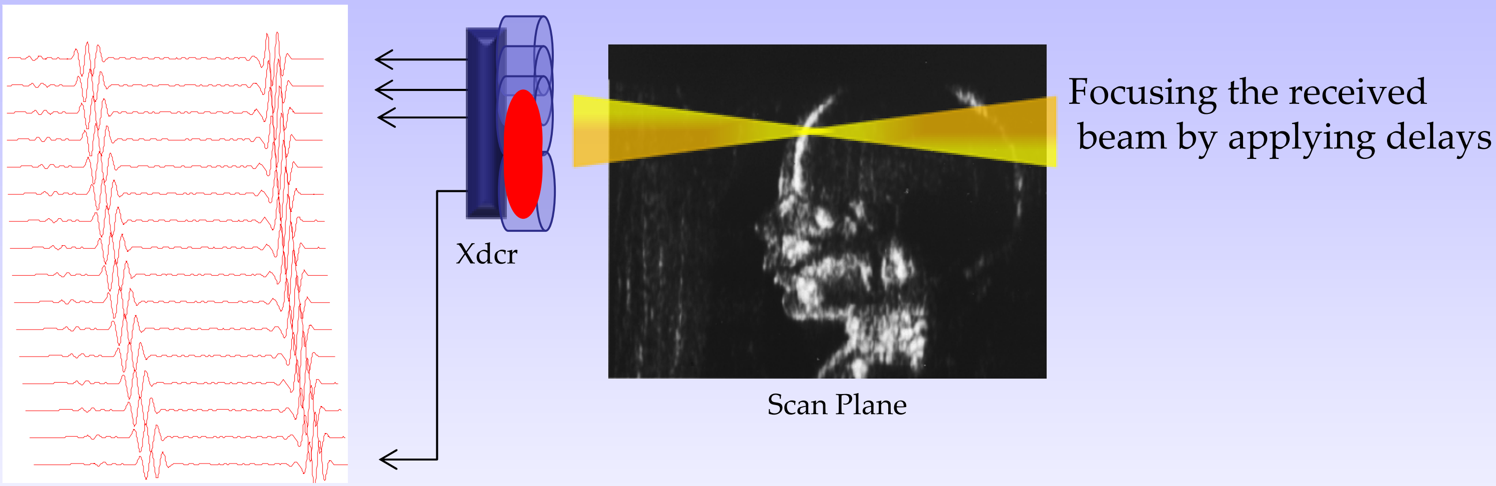




- Echoes result from scattering in the tissue
- The image is formed by identifying the scatterers

Processing Rates

- To increase SNR the reflections are viewed by an antenna array
- SNR is improved through beamforming by introducing appropriate time shifts to the received signals

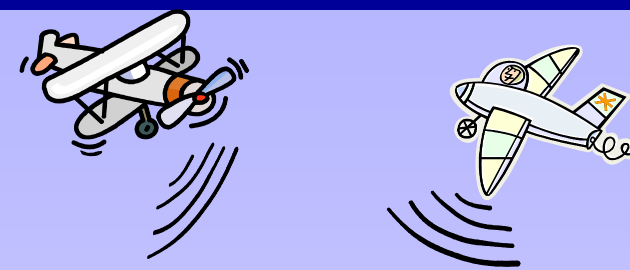


- Requires high sampling rates and large data processing rates
- One image trace requires 128 samplers @ 20M, beamforming to 150 points, a total of 6.3×10^6 sums/frame

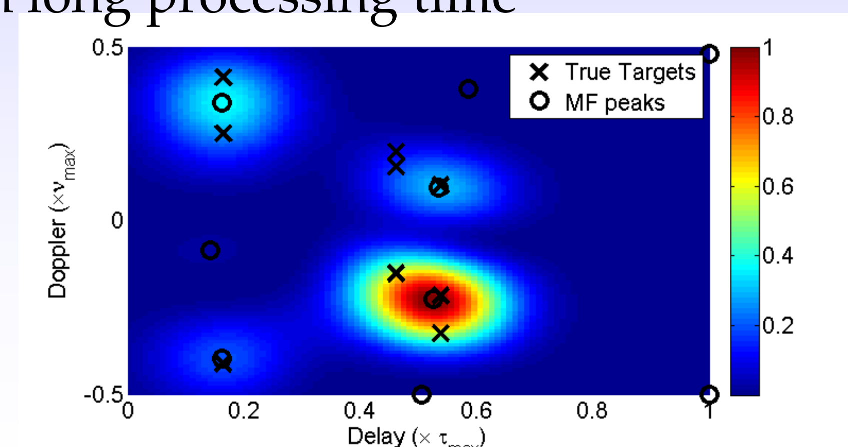
Compressed Beamforming

Resolution (1): Radar

- Principle:
  - A known pulse is transmitted
  - Reflections from targets are received
  - Target's ranges and velocities are identified



- Challenges:
  - Targets can lie on an arbitrary grid
  - Process of digitizing
    → loss of resolution in range-velocity domain
  - Wideband radar requires high rate sampling and processing which also results in long processing time



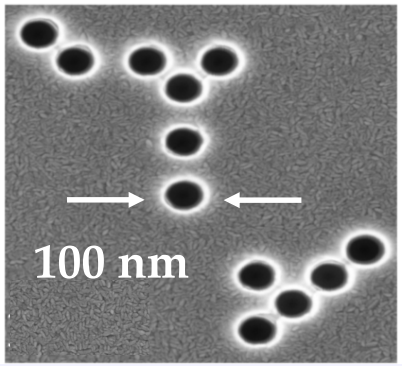
Resolution (2): Subwavelength Imaging

(Collaboration with the groups of Segev and Cohen at the Technion)

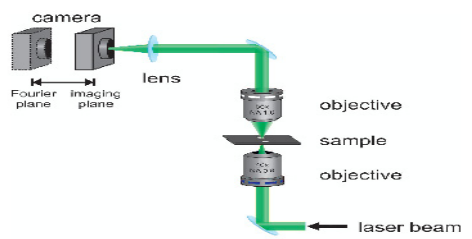
Diffraction limit: Even a perfect optical imaging system has a resolution limit determined by the wavelength λ

- The smallest observable detail is larger than ~ λ/2
- This results in image smearing

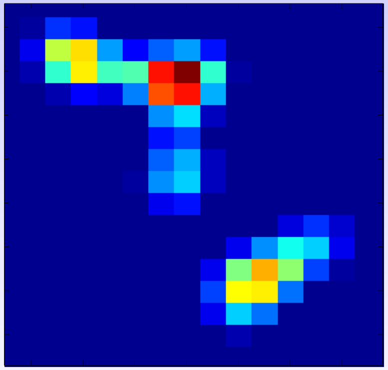
λ=514nm



Nano-holes as seen in electronic microscope



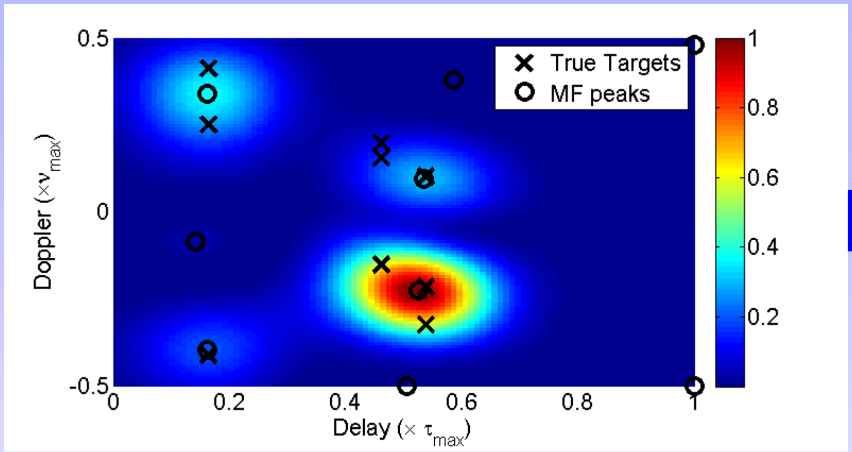
Sketch of an optical microscope: the physics of EM waves acts as an ideal low-pass filter

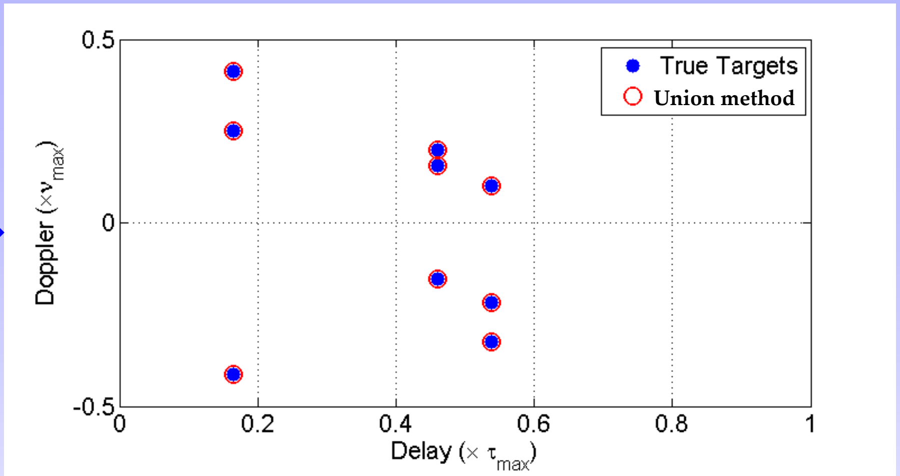


Blurred image seen in optical microscope

Imaging via "Sparse" Modeling

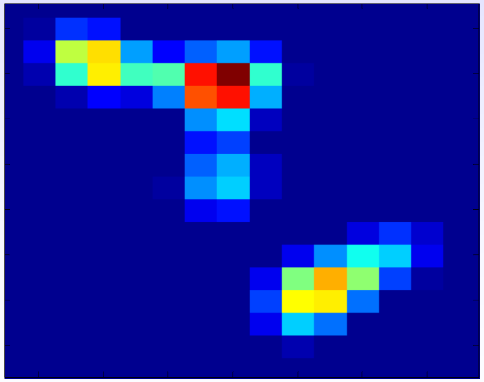
- Radar:

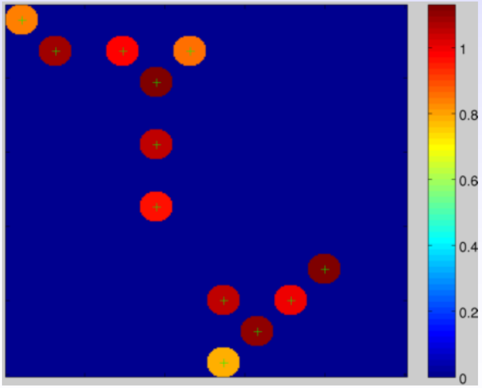




- Subwavelength Coherent Diffractive Imaging:

Bajwa *et al.*, '11





Recovery of sub-wavelength images from highly truncated Fourier power spectrum

Szameit *et al.*, Nature Photonics, '12

Part 2:

Xampling

Xampling

(Mishali and Eldar, 10)

Xampling: Compression+Sampling

- Prior to analog sampling reduce bandwidth by projecting data onto low dimensional analog space
- Creates aliasing of the data
- Sample the data at low rate in such a way that in the digital domain we get a CS problem
- Typically process in frequency: low rate processing, robustness
- Results in low rate, low bandwidth, simple hardware and low computational cost

x(t) →  → Acquisition → Compressed sensing and processing → recovery

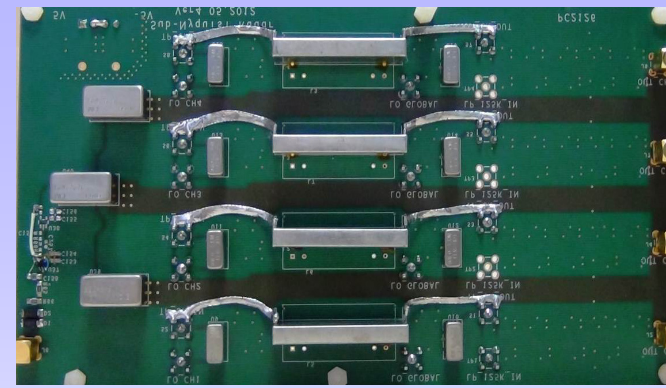
Analog preprocessing   Low rate (bandwidth)

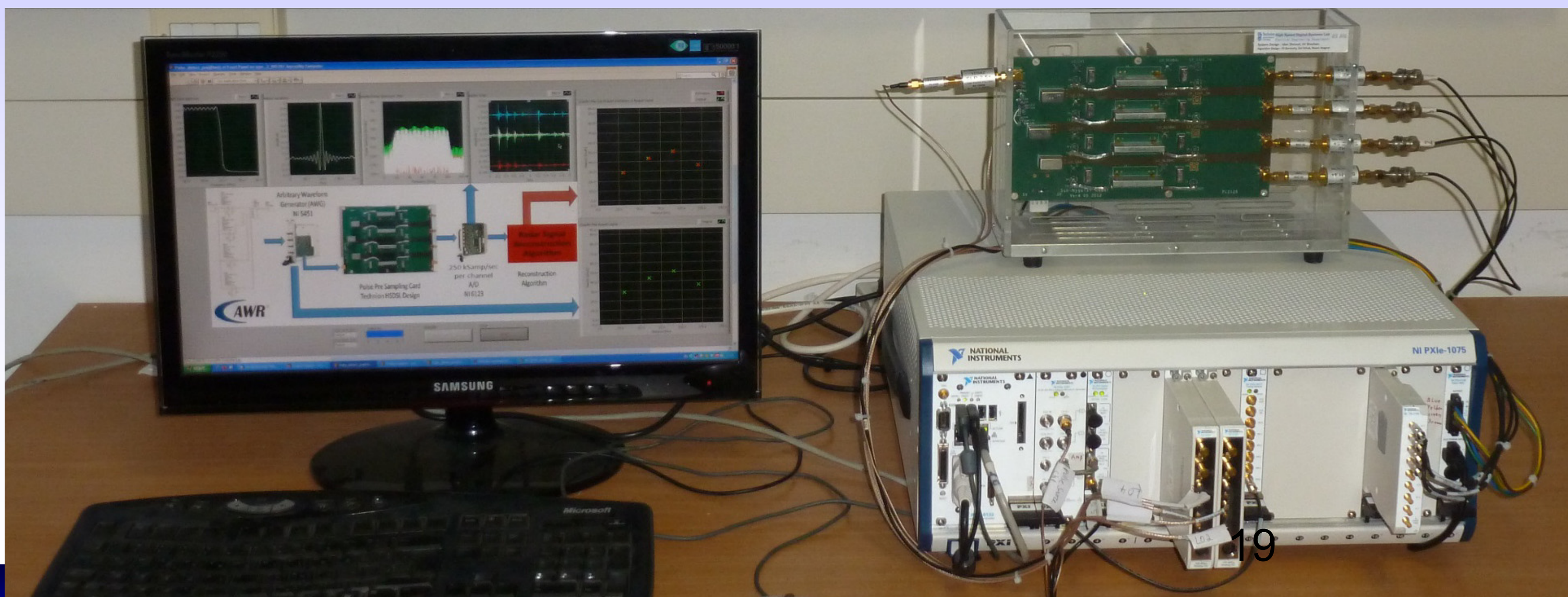
Xampling of Radar Pulses

Sampling at 1/30 of the Nyquist rate

(Baransky et. al 12, Bar-Ilan and Eldar 13)

Xampling-based hardware for sub-Nyquist sampling of radar signals (Recent demo at RadarCon in collaboration with NI)





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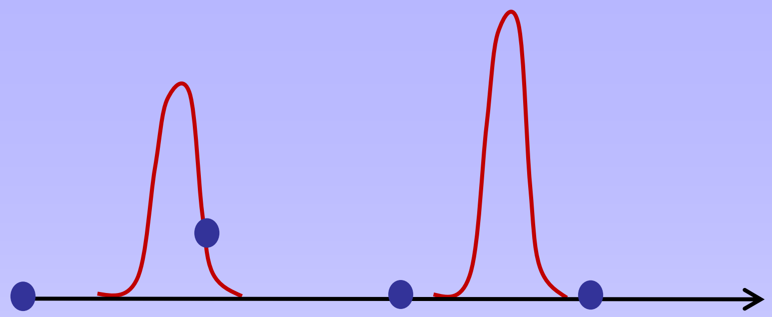
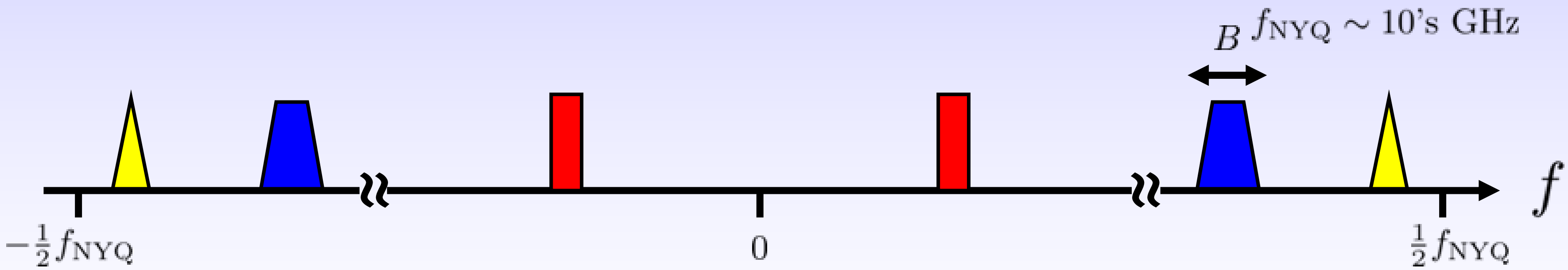
Difficulty

- Rate should be *2L* if we have *L* pulses
- Naïve attempt: direct sampling at low rate
- Most samples do not contain information!!

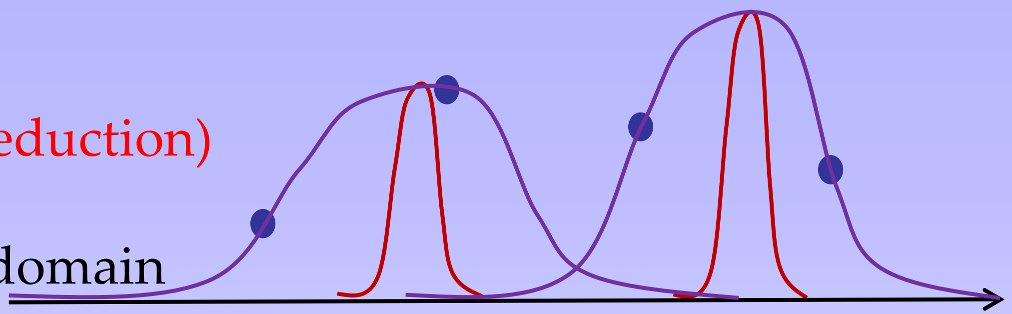


- Mulitband problem: Rate should be *2NB*
- Most bands do not have energy – which band should be sampled?



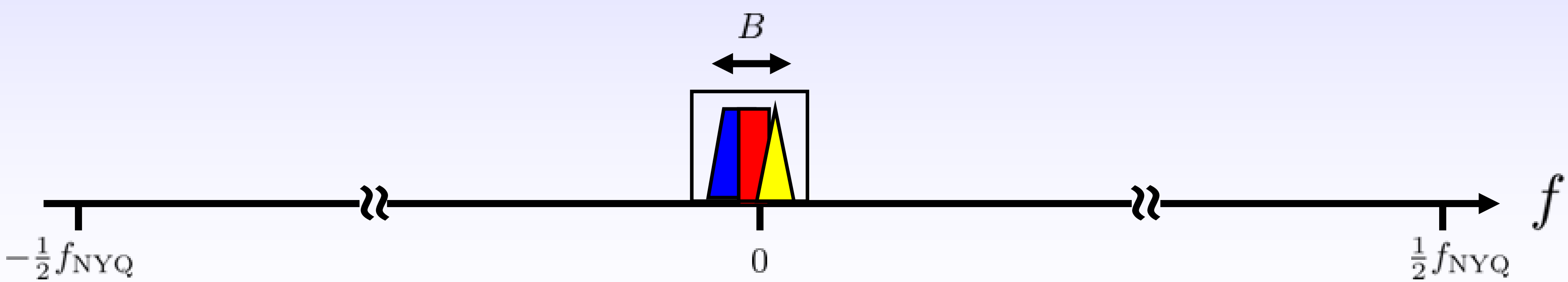
Intuitive Solution: Pre-Processing

- Smear pulse **before** sampling
  (analog projection – bandwidth reduction)
- Each sample contains energy
- Resolve ambiguity in the digital domain



- Use CS in digital, but set up problem in frequency

- Alias all energy to baseband **before** sampling (analog projection)
- Can sample at low rate
- Resolve ambiguity in the digital domain



Digital Recovery

- Subspace techniques developed in the context of array processing (such as MUSIC, ESPRIT etc.)
- Compressed sensing

Connections between CS and subspace methods:

Malioutov, Cetin, and Willsky , Davies and Eldar , Lee and Bresler,, Kim, Lee and Ye, Fannjiang, Austin, Moses, Ash and Ertin

For nonlinear sampling:

- Quadratic compressed sensing **(Shechtman et. al 11, Eldar and Mendelson 12, Ohlsson et. al 12, Janganathan 12)**
- More generally, nonlinear compressed sensing

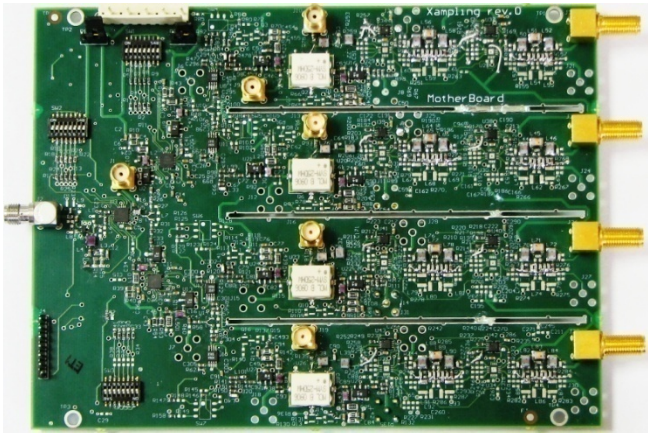
(Beck and Eldar 12, Bahman et. al 11)

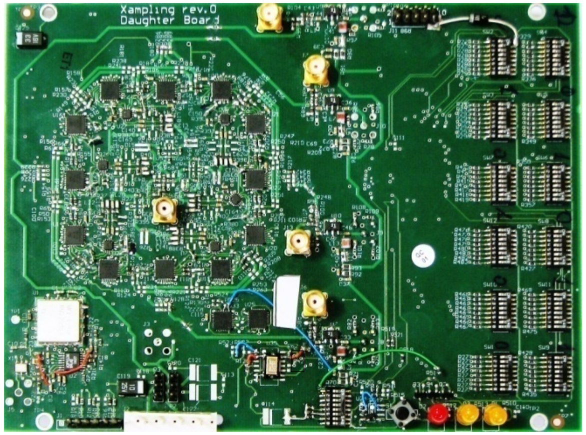
We use CS only after sampling
We set up problems in frequency and not in time
Enables efficient hardware and low processing rates

Nyquist: 2.4 GHz
Sampling Rate: 280MHz

~ 1/9 of the Nyquist rate

(Mishali, Eldar, Dounaevsky, and Shoshan, 2010)





Parameters:

- Nyquist rate: 2.4 GHz
- Band occupancy: 120 MHz (~1/20 of the Nyquist range)
- Sampling rate: 280 MHz (~1/9 of the Nyquist rate)

Rate proportional to the actual band occupancy!

Performance:

- Wideband receiver mode: 49 dB dynamic range, SNDR > 30 dB
- ADC mode: 1.2v peak-to-peak full-scale, 42 dB SNDR = 6.7 ENOB

Part 3:

Back to Radar

Joint work with Omer Bar-Ilan

1. O. Bar-Ilan and Y. C. Eldar, "Sub-Nyquist Radar via Doppler Focusing“
2. E.Baransky, G. Itzhak, I. Shmuel, N. Wagner, E. Shoshan and Y. C. Eldar, "A Sub-Nyquist Radar Prototype: Hardware and Algorithms"

Xampling of Radar Pulses

- Demand for high resolution radar requires high bandwidth signals on the order of 100s Mhz to several Ghz
- Classic matched filter processing requires sampling and processing the received signal at its Nyquist rate
- Hardware excessive solutions, large computational costs
- Previous CS works for this problem
- Either do not address sampling
- Require a prohibitive dictionary size: all delays and Dopplers
- Or perform poorly with noise, clutter and close Dopplers

We develop a sub-Nyquist radar prototype implemented in hardware which provides simple recovery and robustness to noise by performing beamforming on the low rate samples

Doppler Focusing

- Our sub-Nyquist method is based on the following concepts:

- Finite Rate of Innovation (FRI) (Vetterli et. al 02) enables modeling the analog signal with a small number of unknown parameters
- Xampling allows for low sampling rate
- Doppler Focusing is a method of beamforming the low rate samples in frequency which is numerically efficient and robust to noise
  - Optimal SNR scaling
  - CS size does not increase with number of pulses
  - No restrictions on the transmitter
  - Clutter rejection and the ability to handle large dynamic range

Radar Model

- Transmitted pulse train: $x_T(t) = \sum_{p=0}^{P-1} h(t - p\tau)$

RCS

Radial velocity

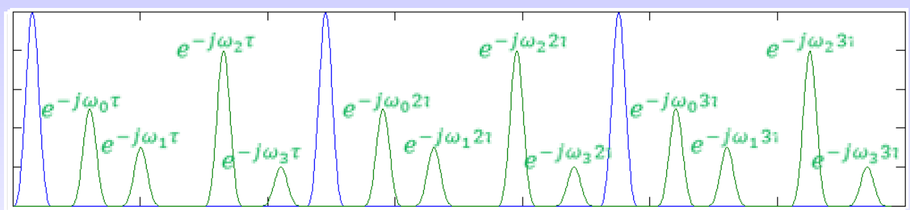
- Reflections from L targets, each defined by $\{\alpha_l, \tau_l, \omega_l\}$

Range

- Received signal: $x(t) = \sum_{p=0}^{P-1} \sum_{l=0}^{L-1} \alpha_l h(t - \tau_l - p\tau) e^{-j\omega_l p\tau}$

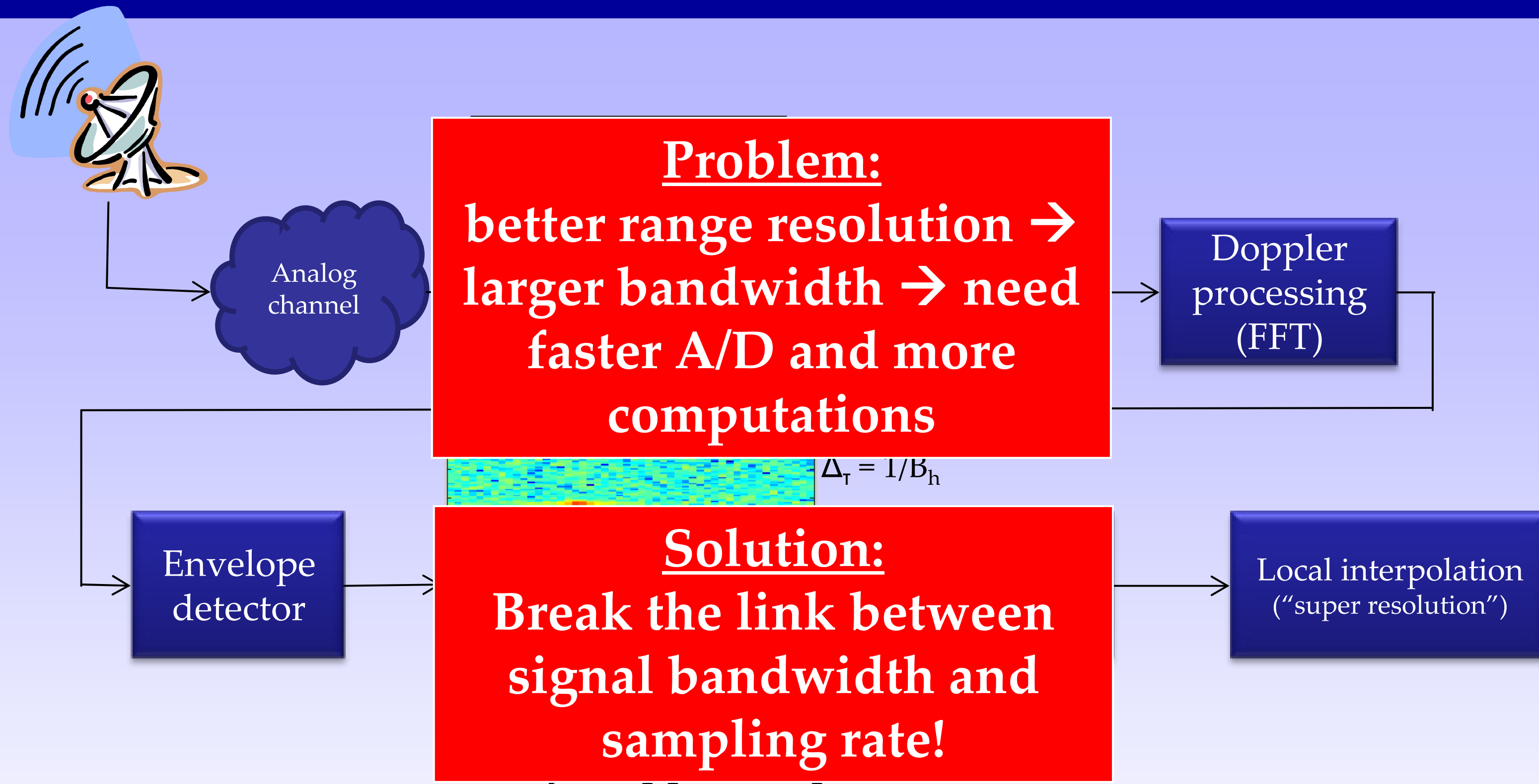
P=3
L=4

Tx
Rx



- Assumptions on targets:
  - "Far targets" – Swerling 0 targets' distance is large compared to the distance change during observation interval, allows for constant α_l
  - "No acceleration" – targets' constant velocity allows for constant ω_l
  - "Slow targets" – small target velocities allow for constant τ_l and constant Doppler phase during pulse time

Classic Pulse-Doppler Receiver



Problem:
better range resolution → larger bandwidth → need faster A/D and more computations

Solution:
Break the link between signal bandwidth and sampling rate!

- Sample rate: pulse's Nyquist rate B_h
- Computational cost
  - Matched filter: P convolutions of length τB_h
  - Doppler processing: τB_h FFTs of length P

How Do We Break the Link?

- Use the FRI framework to model the analog input using a small number of degrees of freedom

  $x(t) = \sum_{p=0}^{P-1} \sum_{l=0}^{L-1} \alpha_l h(t - \tau_l - p\tau) e^{-j\omega_l p\tau} \Rightarrow \rho = 3L/P\tau$

- In practice, P is at least on the order of L in order to allow for L distinct targets on Doppler grid
- Therefore we need $\approx L^2$ samples over the observation period

Theorem (Bar-Ilan and Eldar 13)

The number of samples required for perfect recovery of L targets when there is no noise is at least $4L^2$. In addition:

- The number of samples per period is at least $2L$;
- The number of periods $P \geq 2L$.

We will achieve this rate using Xampling and Doppler focusing

Fourier Processing

- Separate signal into frames: $x(t) = \sum_{p=0}^{P-1} x_p(t)$
- Express each frame as a Fourier series: $x_p(t) = \sum_{k\in\mathbb{Z}} c_p[k] e^{j2\pi kt/\tau}$
- Where $c_p[k] = \frac{1}{\tau} H(\frac{2\pi k}{\tau}) \sum_{l=0}^{L-1} \alpha_l e^{-j\omega_l p\tau} e^{-j2\pi k \tau_l/\tau}$

Known CTFT

Unknown model parameters

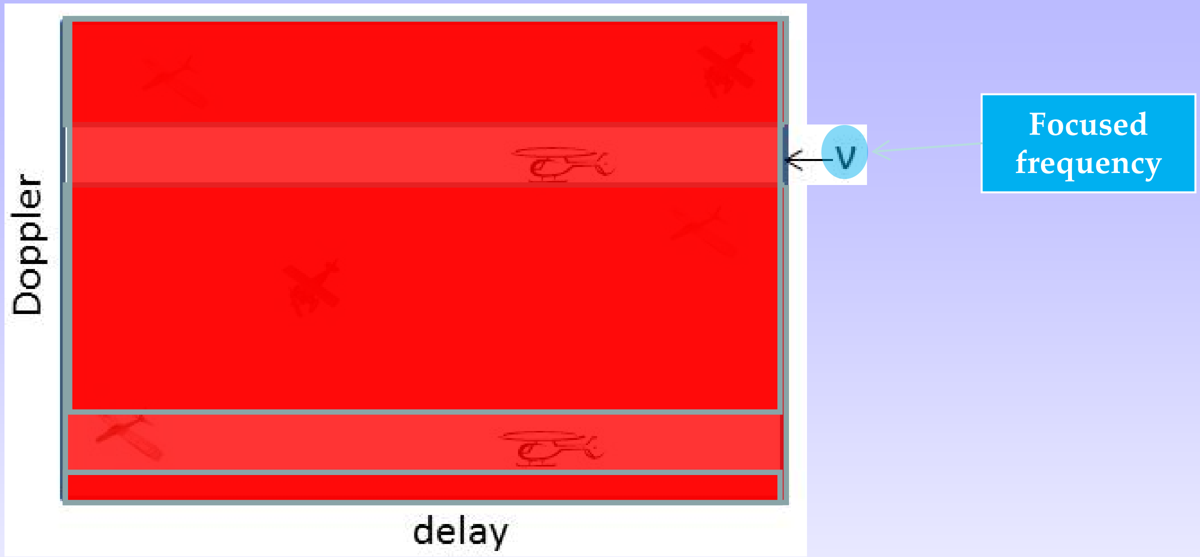
- All unknown parameters are embodied in the Fourier coefficients

We will show that the parameters can be recovered robustly from 2L Fourier coefficients per frame!

- Doppler focusing: low rate and robust processing from *2L* Fourier coeff.
- Only *2L* samples in time are needed – low rate

Doppler Focusing: Idea

Transform a delay-Doppler problem to a set of delay-only problems with specific Doppler frequency



Advantages:

- Reduce a hard 2D problem into several easier 1D problems
- Doppler focusing increases SNR by P which is the optimal scaling
- Improved resolution: Targets with different Doppler's do not interfere
- Fast to compute (FFT), operates on low rate samples
- Can use known delay estimation methods (CS, matrix pencil, MUSIC, etc'): No need to solve a 2D problem, typically few targets per frequency

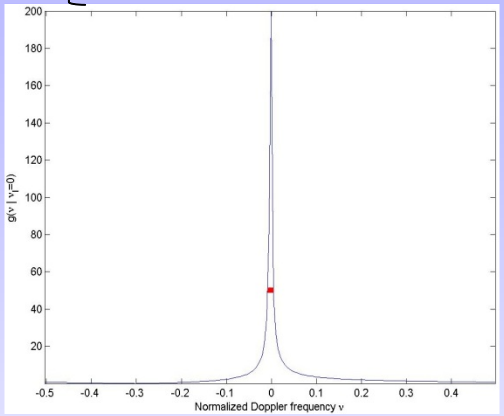
Doppler Focusing: Math

- Focusing for Doppler frequency ν: DFT over frame index

$$\psi_\nu[k] = \sum_{p=0}^{P-1} c_p[k] e^{j\nu p\tau}$$

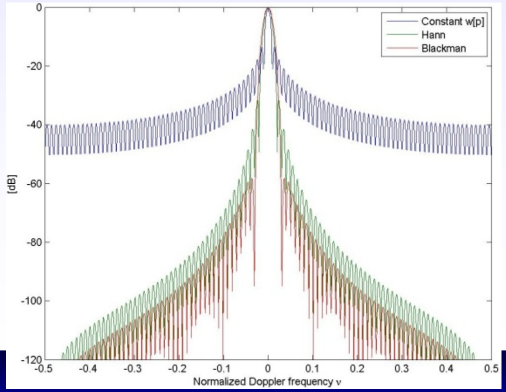
$$= \underbrace{\frac{1}{\tau} H\left(\frac{2\pi k}{\tau}\right) \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi\tau_l/\tau}}_{\text{Complex sinusoid problem}} \underbrace{\sum_{p=0}^{P-1} e^{j(\nu-\nu_l)p\tau}}_{\text{Exponent sum}}$$

$$\cong \begin{cases} P, & |\nu-\nu_l| > 2\pi/P\tau \\ 0, & |\nu-\nu_l| < 2\pi/P\tau \end{cases}$$



Windowing $\sum_{p=0}^{P-1} e^{j(\nu-\nu_l)p\pi}\omega[p]$

mitigates impact of "out-of-focus" targets:



- If the set of probed Doppler frequencies lies on a uniform grid:

  $\nu_n = 2\pi n/N_\nu\tau,\ n = 0, 1, \ldots, N_\nu - 1$

- Then $\Psi_\nu[k]$ can be created efficiently using an FFT:

  $\Psi_n[k] \triangleq \Psi_{\nu_n}[k] = DFT_{N_\nu}\{c_p[k]\}$

Recovery using Compressed Sensing

- For each Doppler frequency we have

$$\psi_\nu[k] \cong \frac{P}{\tau} H(2\pi k/\tau) \sum_{l:|\nu-\nu_l|<2\pi/P\tau} \alpha_l e^{-2j\pi k\tau_l/\tau}$$

SNR increase

Only “in focus” targets

- This is a spectral analysis problem, for which 2L frequency samples are enough to recover the unknown α’s and τ’s if there is no noise: $|k| \geq 2L$
- We solve by choosing a set of coefficients

$$\mathbf{\Psi}_\nu = [\psi_\nu[k_0] \ldots \psi_\nu[k_{|k|-1}]^T] \in \mathbb{C}^{|k|}$$

- Discretize the time delays: $\tau_l = q_l \Delta_\tau \quad N_\tau = \left\lfloor \frac{\tau}{\Delta_\tau} \right\rfloor$
- And using CS $\quad \mathbf{\Psi}_\nu = \frac{P}{\tau} \mathbf{H} \mathbf{V} x_\nu$

L(=num. of targets)-sparse vector of α’s

Diagonal matrix

|κ| Rows from a DFT matrix

Performance Guarantees

Optimal noise robustness:

Theorem (Bar-Ilan and Eldar 13)

Let $y(t) = x(t) + w(t)$ denote a noisy radar signal where $x(t) = \sum_{p=0}^{P-1} h(t - p\tau)$ and $w(t)$ is white noise. Then Doppler focusing increases the SNR by a factor of P which is the optimal SNR scaling obtained by the MF processing at the Nyquist rate

Minimal number of samples:

Theorem (Bar-Ilan and Eldar 13)

The minimal number of samples required for perfect recovery of L targets using Doppler focusing when there is no noise is $2LP$

Mid Summary

- Take 2L Fourier coefficients in each frame
- Use Doppler focusing to focus on specific Doppler values
- For each detected Doppler solve CS problem with CS matrix given by chosen frequencies

$$\mathbf{y} = P\mathbf{V}\mathbf{x}$$

where P is the focusing gain, $\mathbf{V}$ is a partial Fourier matrix with the chosen frequencies, and $\mathbf{x}$ is the sparse delay vector

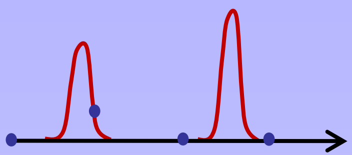
- Once delays are found, subtract them, and move on to next Doppler frequency

Questions:

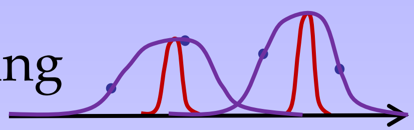
- We sample in time not in frequency: How to obtain the Fourier coeff. from low rate samples?
- Which frequencies should we choose?

Xampling Scheme

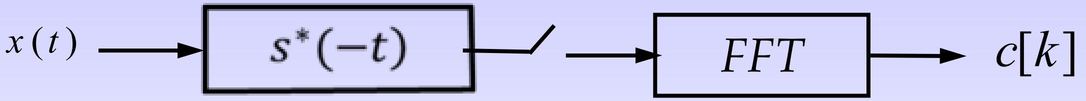
- Direct sampling at low rate is non-informative!



- Solution: Analog kernel prior to sampling to create aliasing



- Single channel: only *2L* samples needed



Theorem (Tur, Eldar and Friedman 11)

If the filter $s^*(-t)$ satisfies :

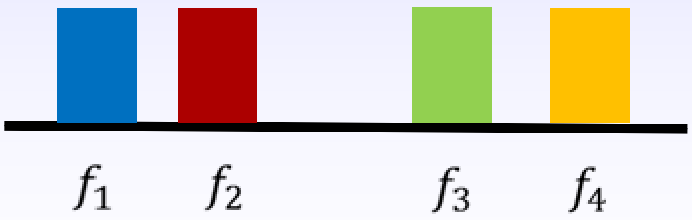
$$S^*(\omega) = \begin{cases} 0 & \omega = 2\pi k/\tau, k \notin \mathcal{K} \\ \text{nonzero} & \omega = 2\pi k/\tau, k \in \mathcal{K} \\ \text{arbitrary} & \text{otherwise,} \end{cases}$$

then $c[k]$ are the desired Fourier coefficients

Here $\mathcal{K}$ are the desired set of Fourier coefficients

Selecting The Active Frequencies

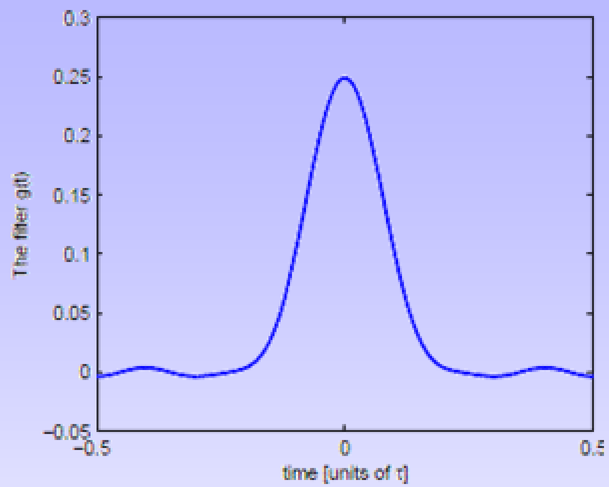
- For good resolution and good CS properties we need wide frequency aperture
- To avoid ambiguities we need at least two frequencies that are close to each other
- Can randomly place frequencies over wide aperture
- Our choice: Use a small set of bandpass filters spread randomly over a wide frequency range

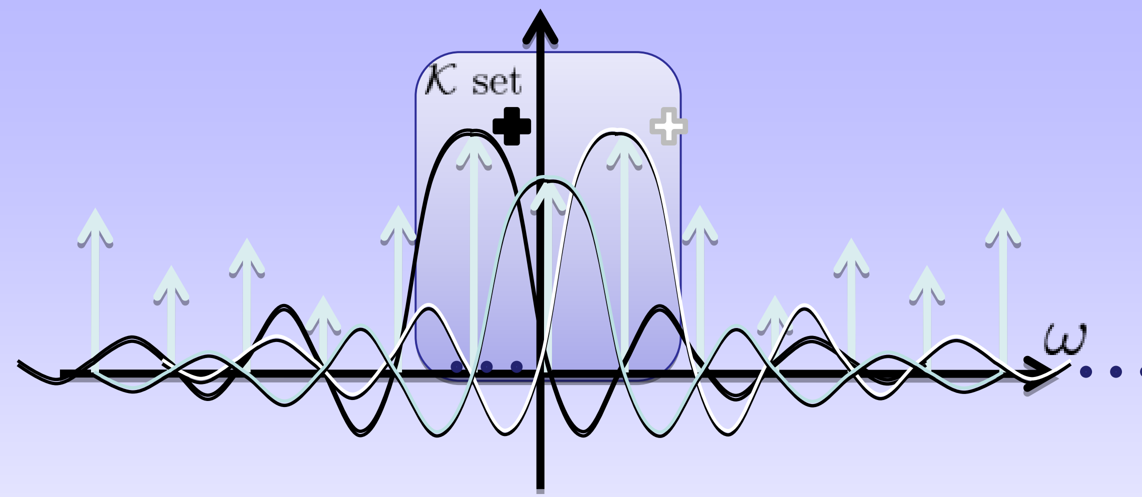


Examples of Filters

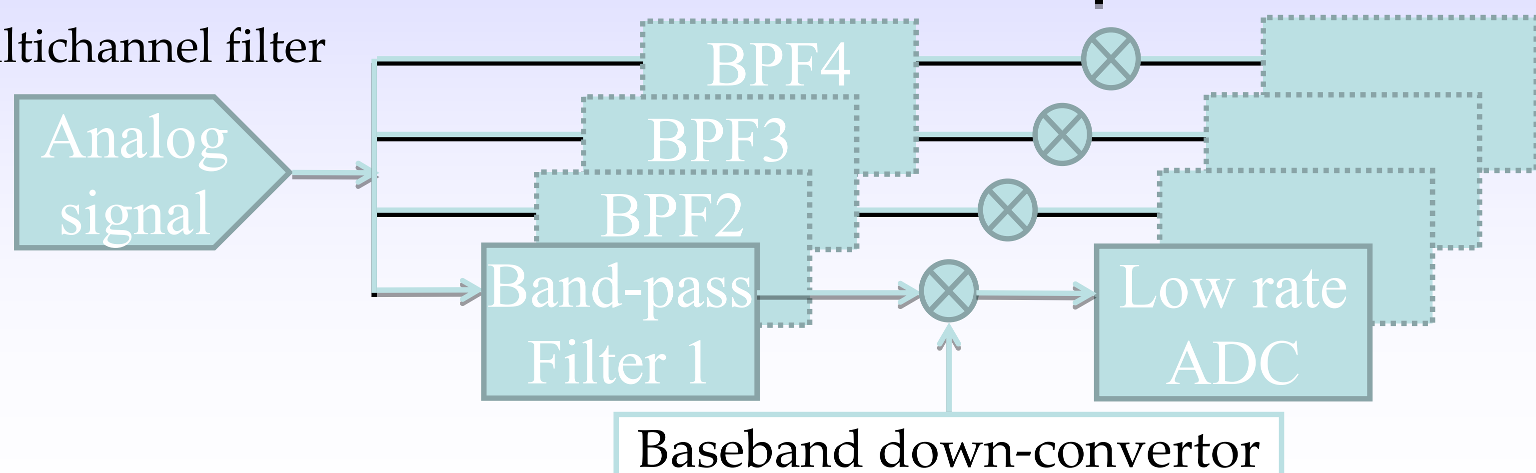
- Sum of Sincs filter – compact support

$$S(\omega) = \frac{\tau}{\sqrt{2\pi}} \sum_{k \in \mathcal{K}} b_k \mathrm{sinc}\left(\frac{\omega}{2\pi/\tau} - k\right)$$



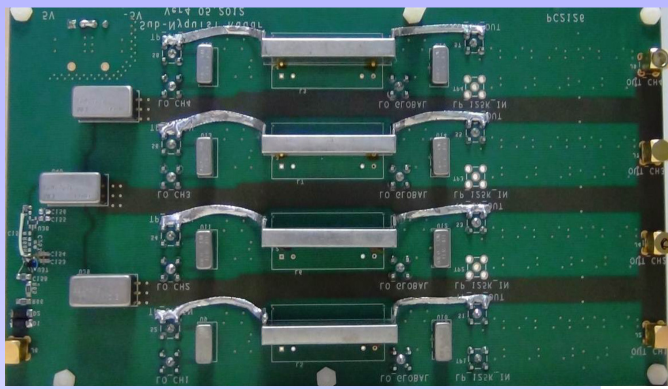


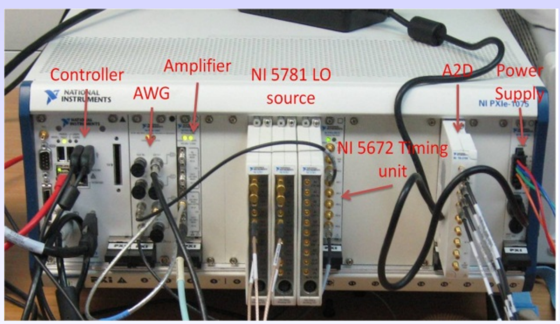
- Multichannel filter



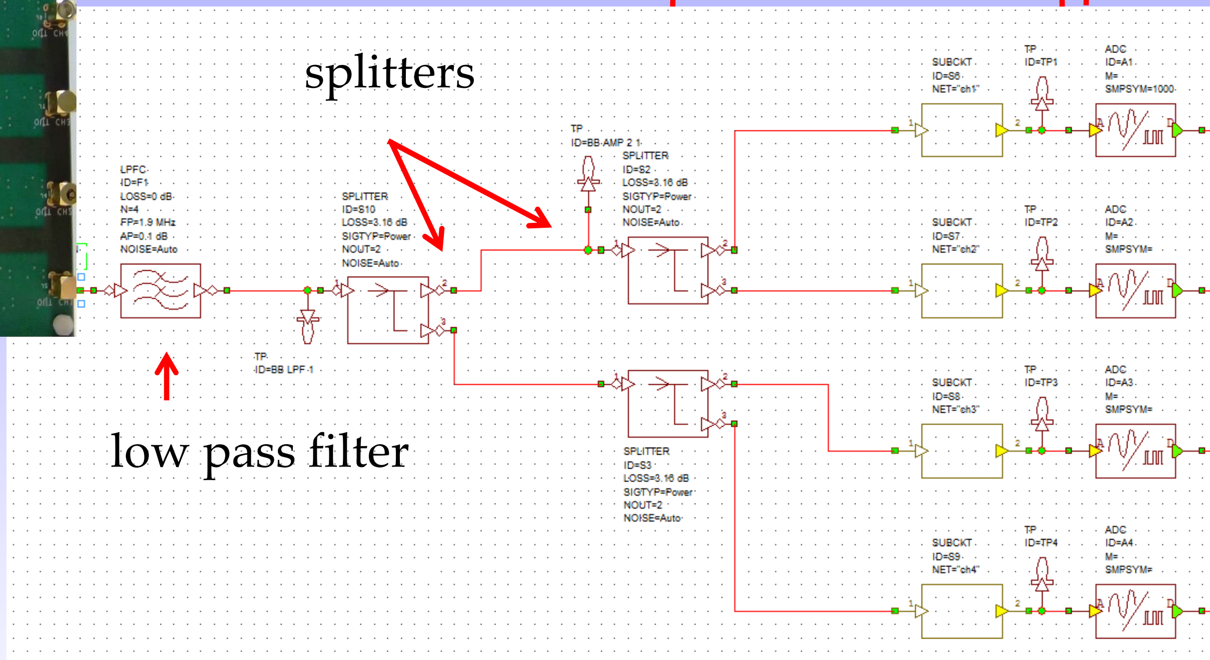
Xampling of Radar Pulses

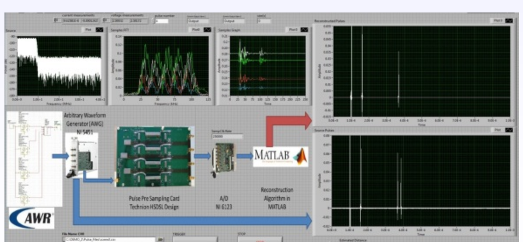
(Itzhak et. al. 2012 in collaboration with NI)





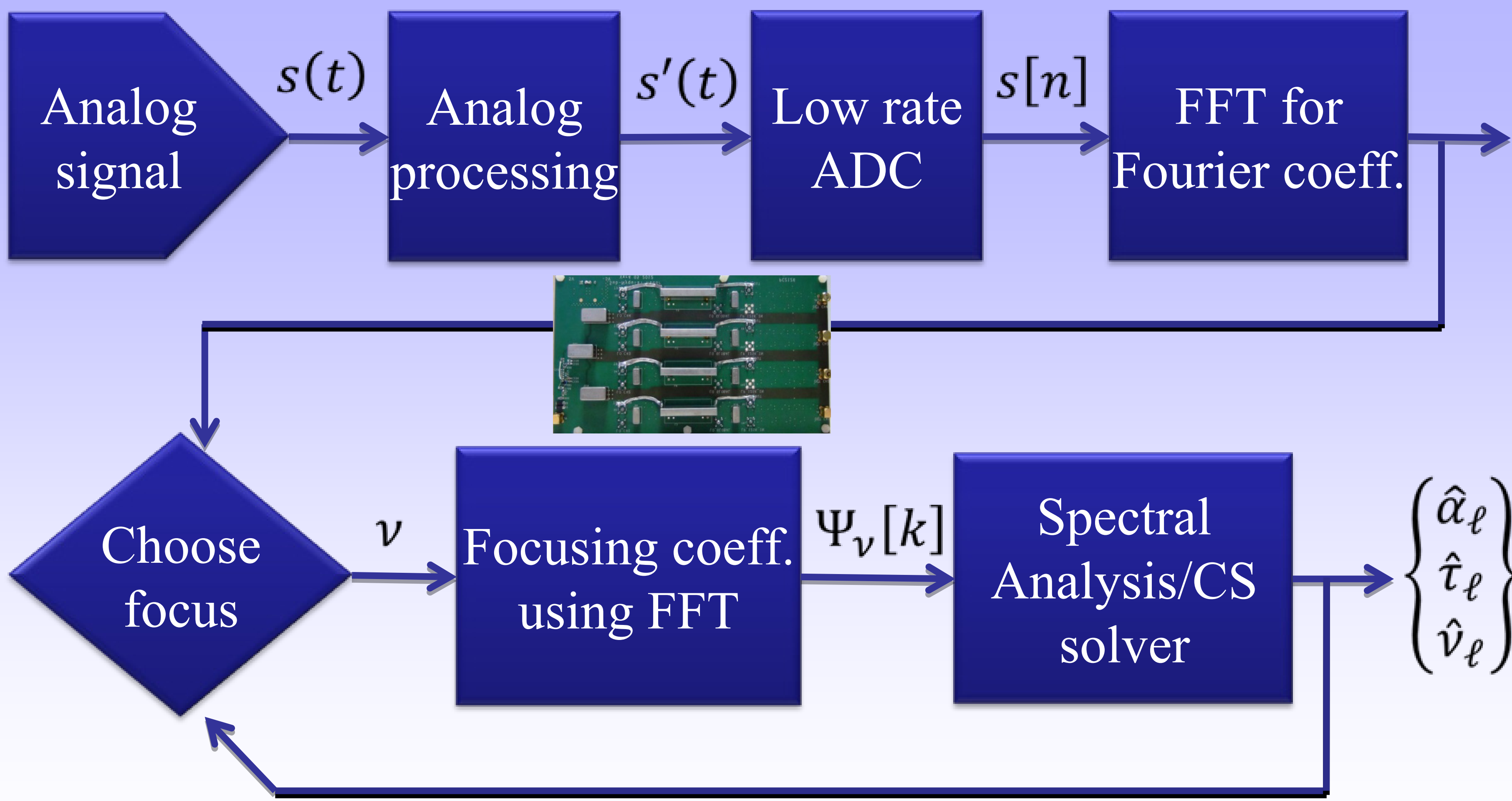
analog filter banks
ADCs



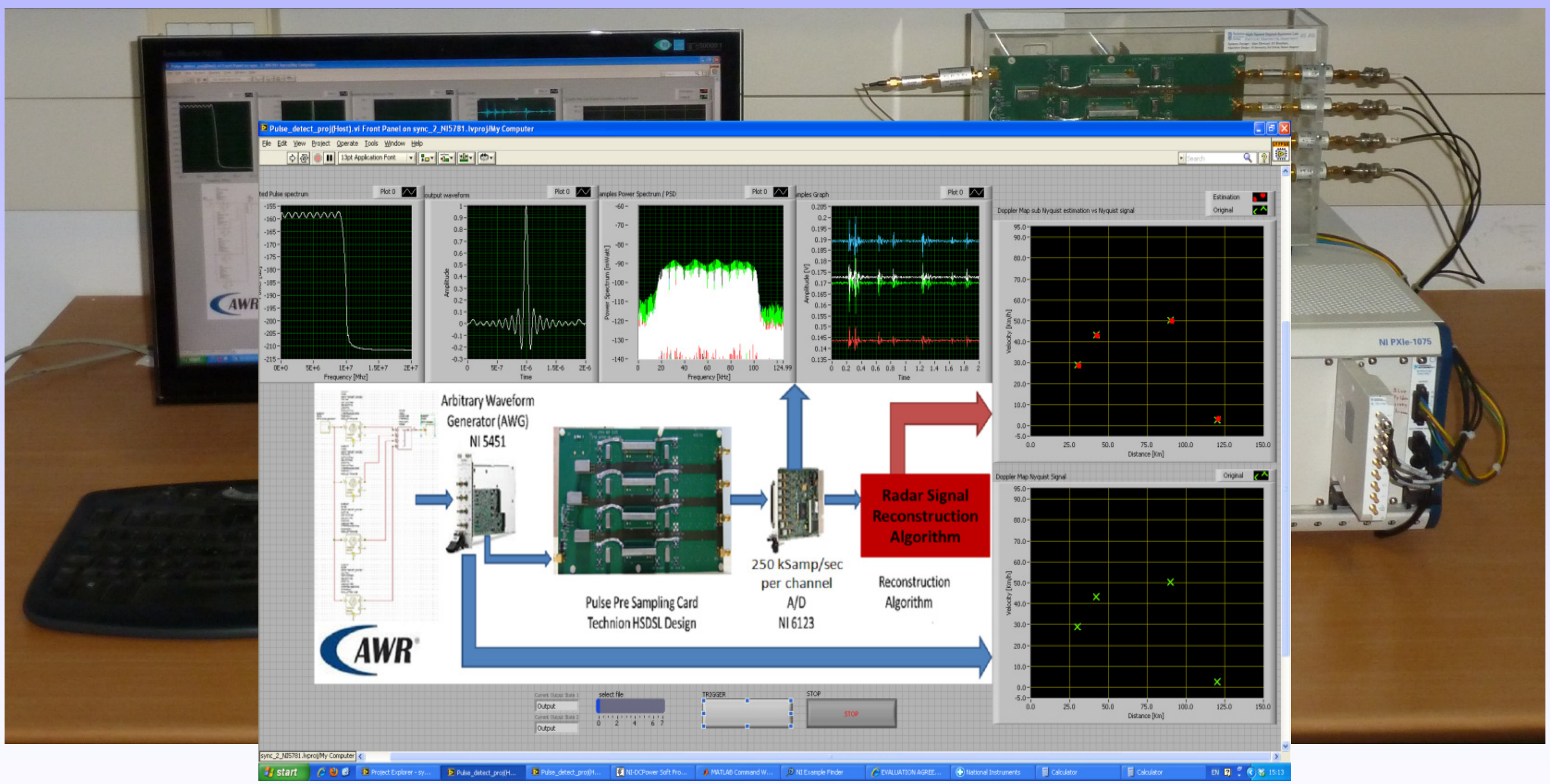


Demo of real-time radar at NI week

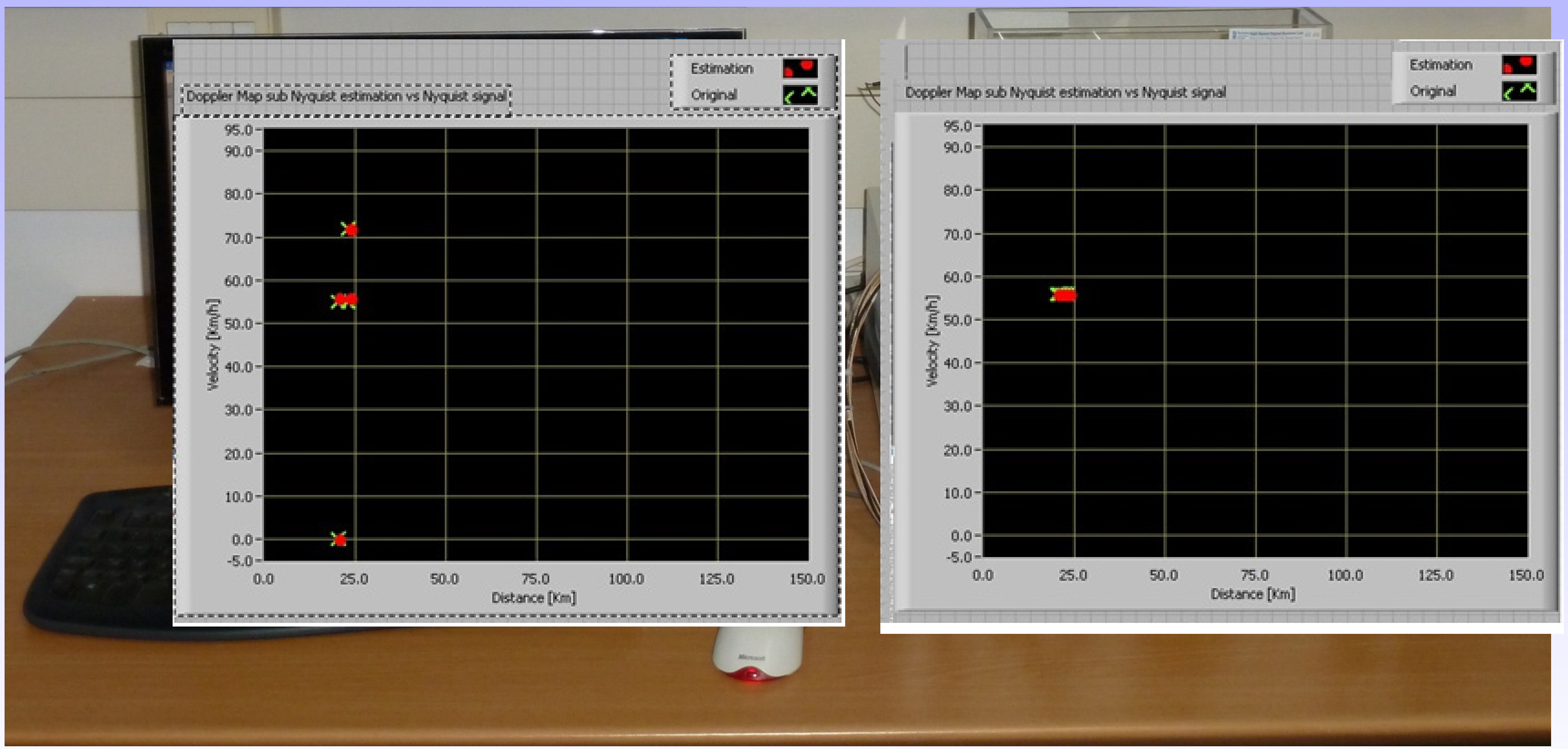
Final Scheme



Xampling of Radar Pulses



Xampling of Radar Pulses



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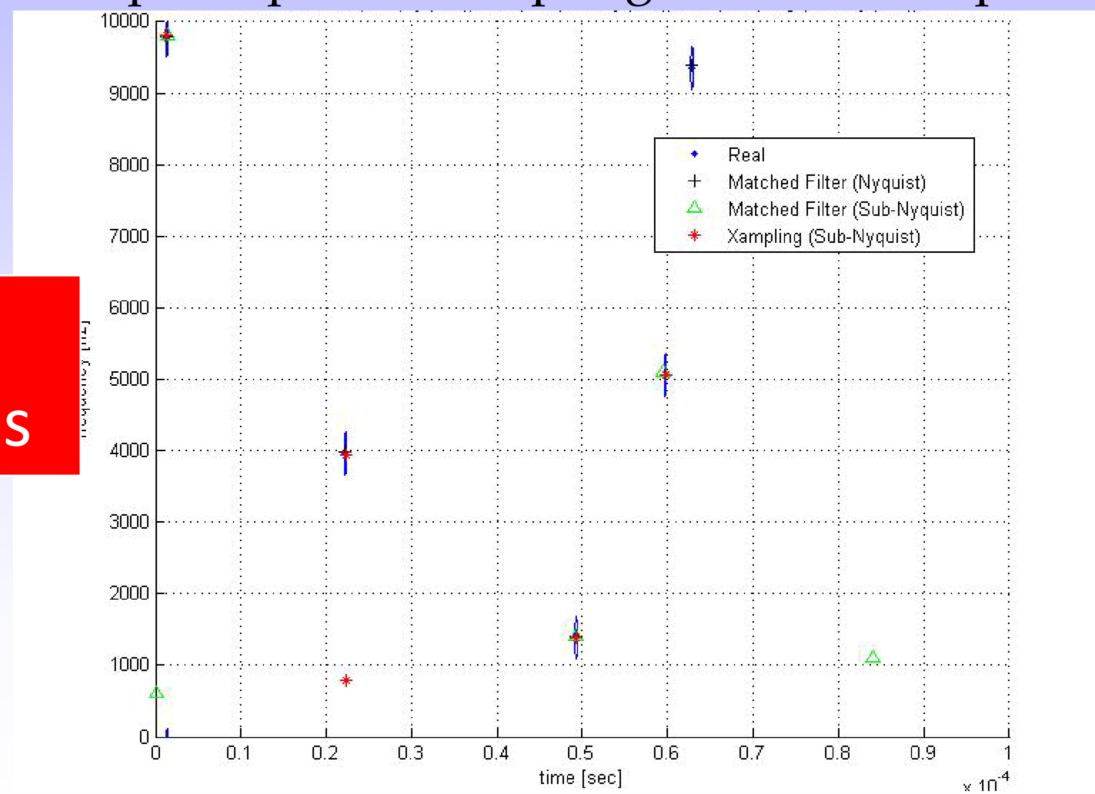
Low SNR: -25 dB

Sampling rate: 1/10 of the Nyquist rate

- Target delay and Doppler chosen randomly in the continuous unambiguous interval
- L =5, PRI = 0.1 mSec, P = 100 pulses, bandwidth B = 10MHz
- Nyquist rate generates 2000 samples / pulse, Xampling uses 200 samples



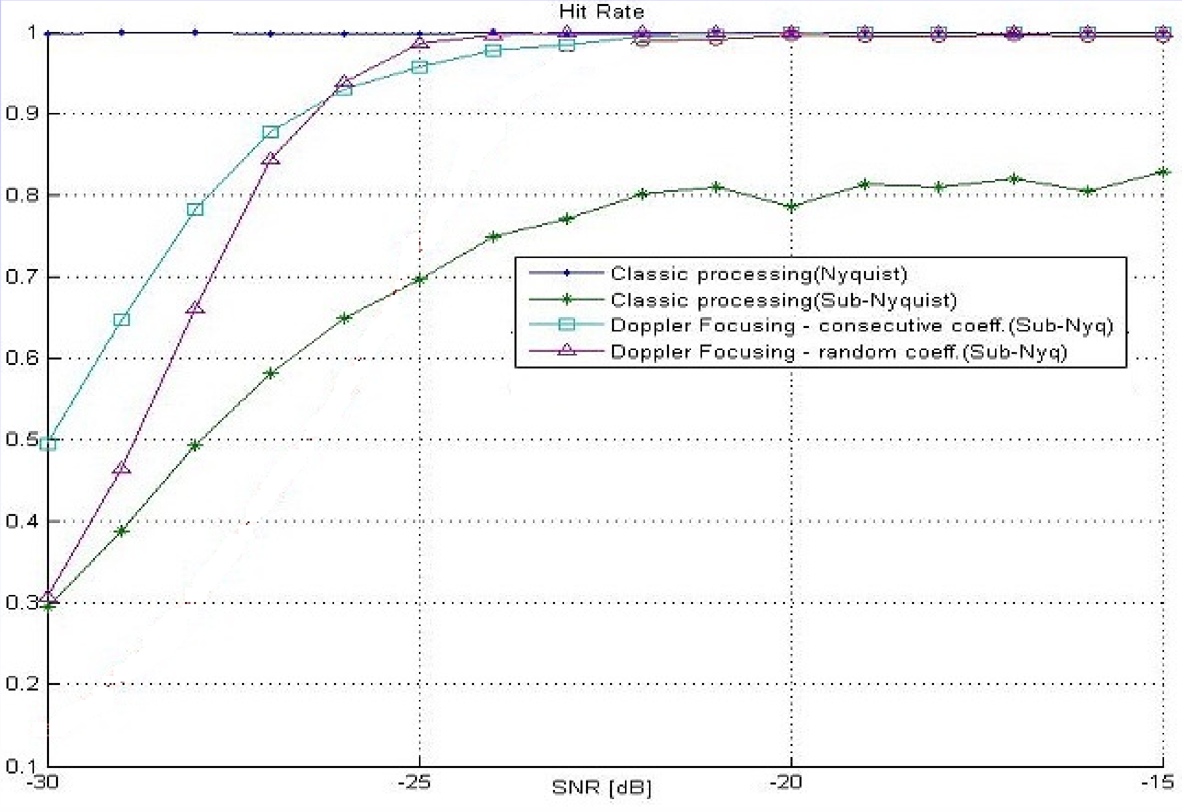
MF: 2/5 detections

Xampling: 4/5 detections

Low SNR

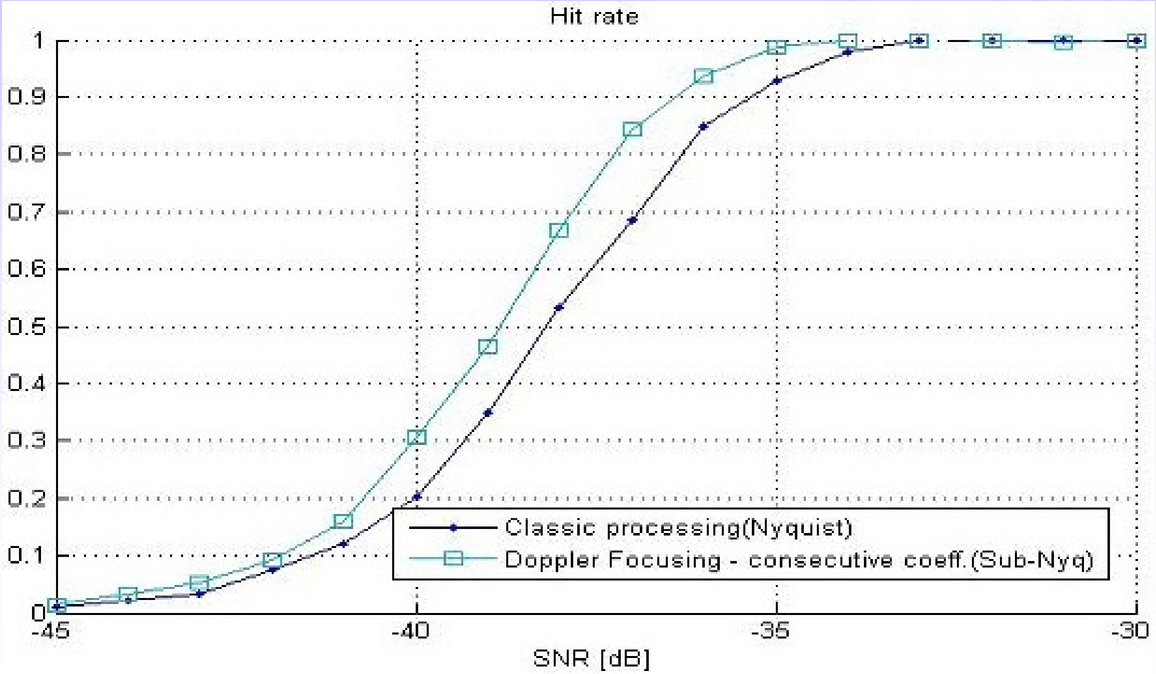
- Target delay and Doppler chosen randomly in the continuous unambiguous interval
- L = 5, PRI = 0.1 mSec, P = 100 pulses, bandwidth B = 10MHz
- Nyquist rate generates 2000 samples / pulse, Xampling uses 200 samples

Hit rate as a function of SNR



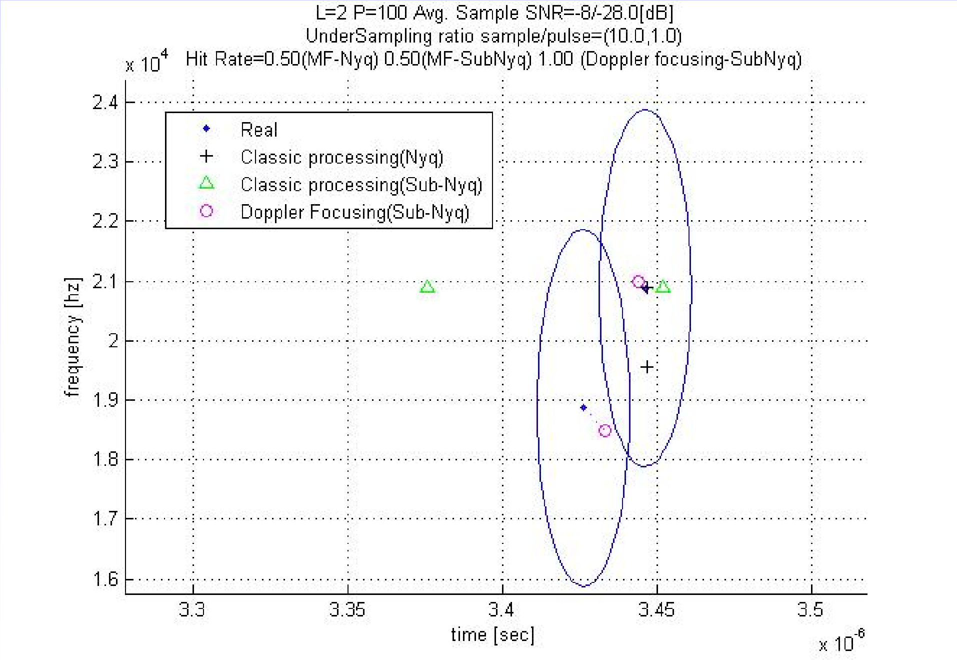
Controlling the Transmitter

- When we can control the transmitter, waveforms better suited for our recovery method can be used
- Since we perform sampling in frequency we use a waveform with its entire energy contents concentrated in these sampled frequencies
- In this setting, Doppler focusing achieves better performance than a Nqyuist rate matched filter



Target Dynamic Range

- The detection subtraction step in Doppler focusing helps detection of closely spaced targets with large dynamic range
- Here the left target is 20dB more powerful than the right target
- MF processing at both Nyquist and one tenth the Nyquist rate recovers only one target while Doppler focusing recovers both

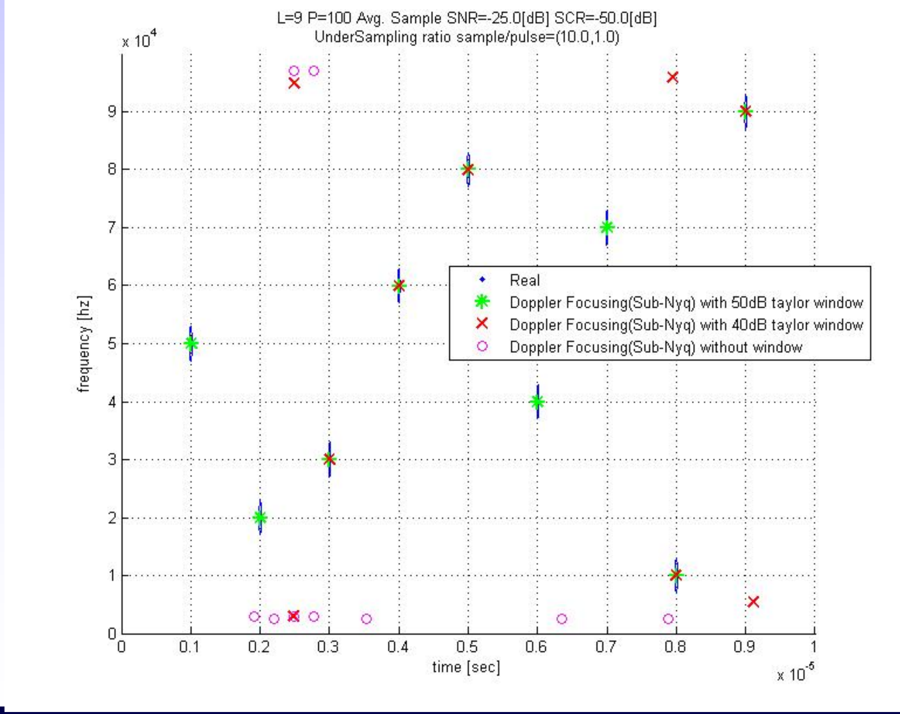


Clutter

- Clutter (land, sea, buildings…) size is usually much larger than target size – potentially masking target echoes and causing misdetections
- Clutter is not noise – cannot be mitigated with coherent integration
- Doppler focusing reduces the effects of clutter by creating isolation between signals with different Doppler frequencies

Example:

- Nine targets and almost static clutter
- Without windowing clutter sidelobes permeate the nonzero Doppler freq. area and cause misdetections
- With 40dB windowing five out of nine targets are recovered correctly
- With 50dB windowing entire scene is detected correctly



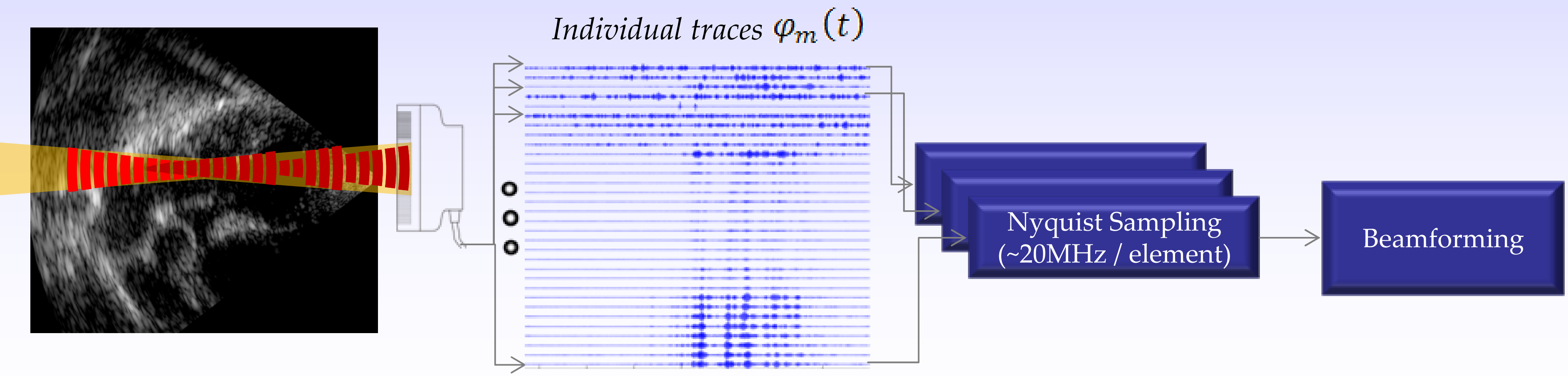
Previous Approaches

- Previous works do not address sample rate reduction feasible in hardware
- Various other works suffer from the following shortcomings:
- Impose constraints on the radar transmitter and do not treat noise (e.g. Baraniuk & Steeghs)
- Construct a CS dictionary with a column for each two dimensional grid point causes dictionary explosion for any practical problem size (e.g. Herman and Strohmer, Zhang et. al)
- Perform non-coherent integration over pulses, obtaining a sub-linear SNR improvement with P (e.g. Bajwa, Gedalyahu & Eldar)

Application to Ultrasound

Wagner, Eldar, and Friedman, '11

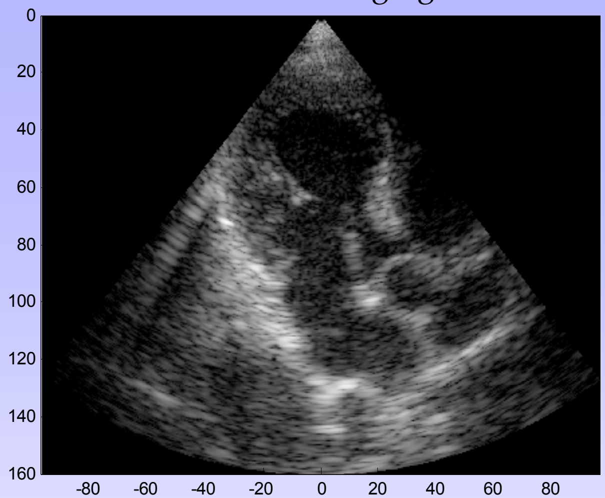
- Ultrasonic pulse is transmitted into the tissue
- Pulse is conducted along a relatively narrow beam
- Echoes are scattered by density and propagation-velocity perturbations
- Reflections detected by multiple array elements.
- Beamforming is applied – digital processing , signals must first be sampled at Nyquist rate (~20MHz)



Ultrasound Results

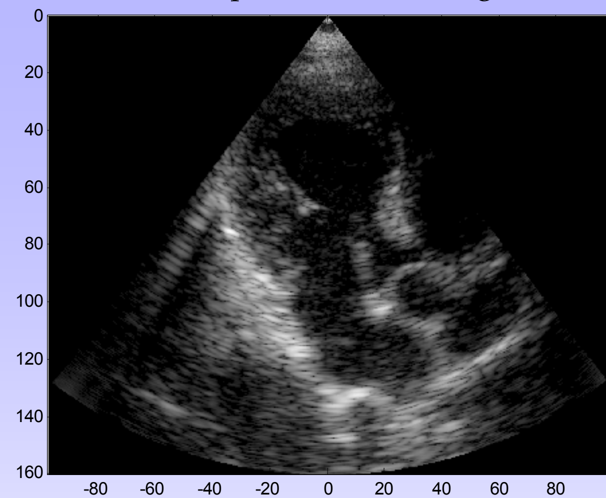
Chernyakova and Eldar 13

Standard Imaging



3328 real-valued samples, per sensor per image line

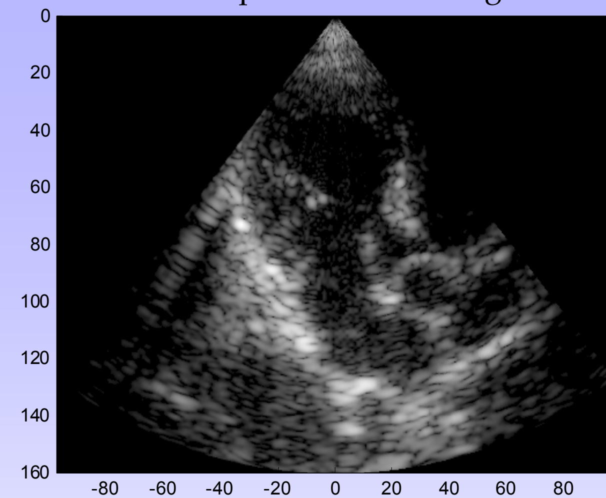
Xampled beamforming



360 complex-valued samples, per sensor per image line

~1/10 of the Nyquist rate

Xampled beamforming



100 complex-valued samples, per sensor per image line

~1/32 of the Nyquist rate

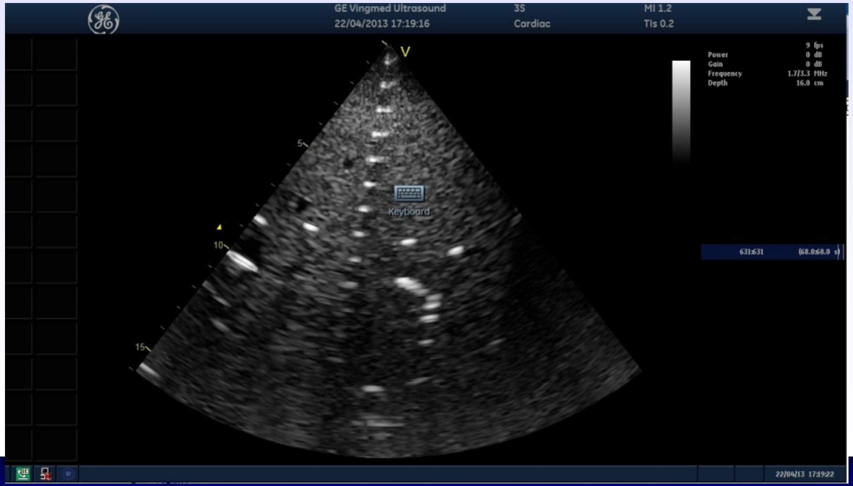
- We obtain a 32-fold reduction in sample rate and 1/16-fold reduction in processing rate
- All digital processing is low rate as well
- Almost same quality as full rate image

Sub-Nyquist Ultrasound Demo

GE Healthcare

- 32-fold reduction in sampling rate while retaining sufficient image quality
- Potential reduction in frame rate
- Improvement of radial resolution by transition of wideband pulse

Original image
time domain beamforming



Frequency domain beamforming
32 fold reduction in sampling rate

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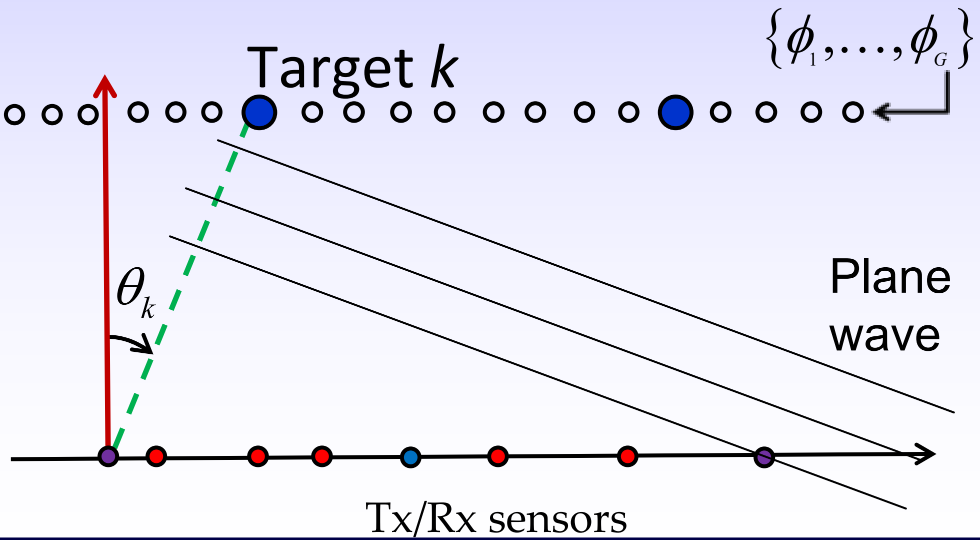
Spatial CS in MIMO Radar

Rossi, Haimovich and Eldar 13

- We can also use similar ideas in MIMO radar to reduce the number of antennas
- Using spatial Nyquist sampling the array aperture scales linearly with MN – the number of transmit and receive antennas
- Using CS we can get Nyquist resolution with MN scaling logarithmically with aperture



LTV System Identification

- Low rate sampling means the signal can be represented using fewer degrees of freedom
- Can be applied to linear time-varying (LTV) system identification

$x(t) \Rightarrow$ $\boxed{\text{LTV system}}$ $\Rightarrow y(t)=H(x(t))$

Identify LTV systems from a single output using minimal resources

Sub-Nyquist sampling of pulse streams can be used to identify LTV systems using low time-bandwidth product

LTV Systems

- Any LTV system can be written as *(Kailath 62, Bello 63)*

$$y(t) = \int_\nu \int_\tau a(\nu,\tau) x(t-\tau) e^{-j2\pi\nu t} d\tau d\nu$$

delay-Doppler spreading function

- Assumption: $a(\nu,\tau) = 0, \quad |\nu| \le \nu_0, |\tau| \le \tau_0$
- Underspread systems $\Delta = 4\nu_0\tau_0 < 1$
- Theorem (Kailath 62, Bello 63, Kozek and Pfander 05): LTV systems can be identified only if they are underspread

Difficulties:

- Proposed algorithms require inputs with infinite bandwidth *W* and infinite time support *T*
- *W* – System resources, *T* – Time to identify targets

Can we identify a class of LTV systems with finite *WT*?

Main Identification Result

- Probing pulse: $x(t) = \sum_{n=0}^{N-1} x_n g(t - nT_0), \quad 1 \leq t \leq T$
- $g(t)$ is a pulse of bandwidth W that is (essentially) supported on $[0, T_0]$
- x_n is a length-N probing sequence, with $N = T/T_0 \propto WT$

Theorem (Bajwa, Gedalyahu and Eldar 10):

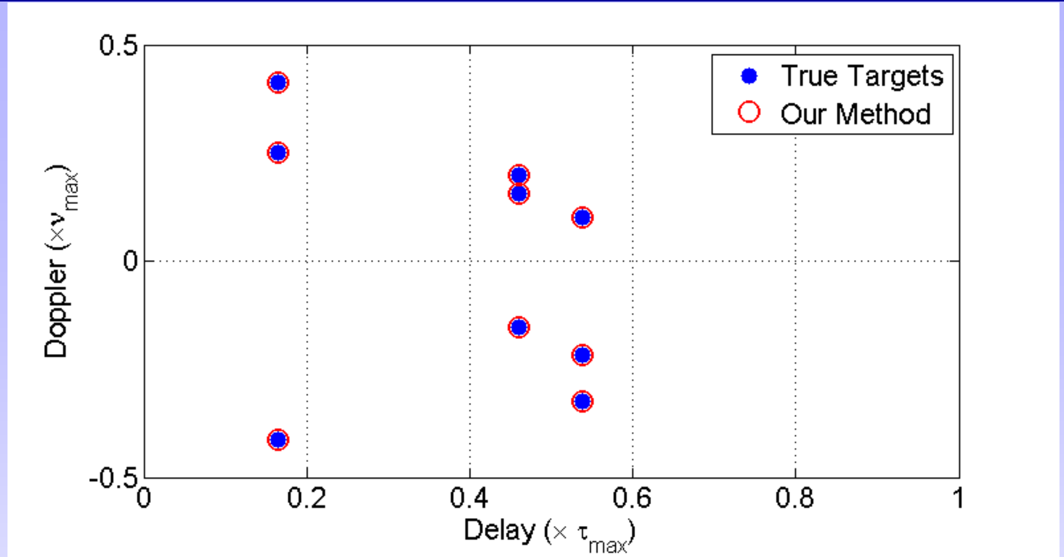
An underspread paramteric LTV system can be identified from a single observation, with infinite resolution, and in polynomial time if $|x_n| > 0$ and

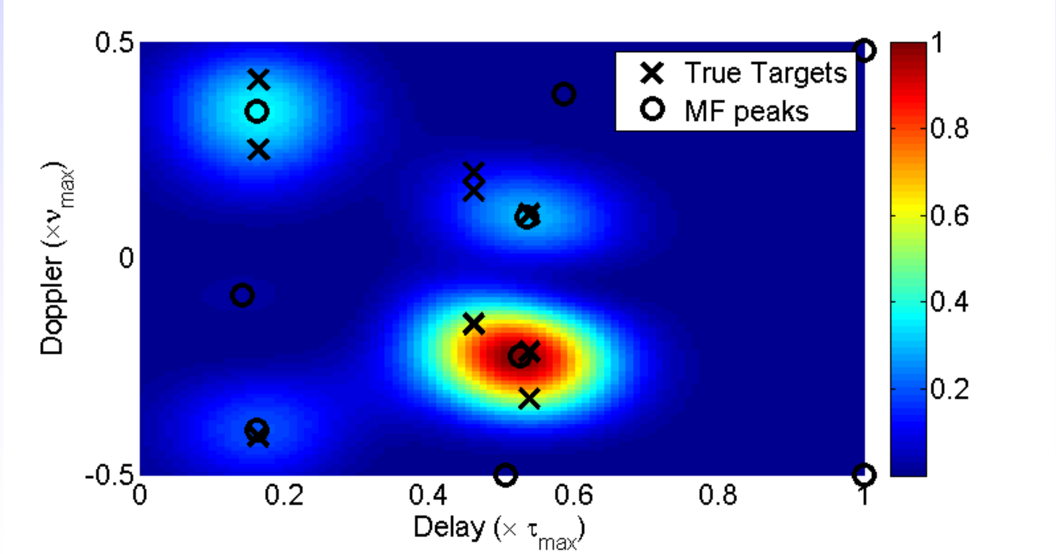
$$WT \geq 8\pi K_\tau K_\nu$$

***WT* is proportional *only* to the number of unknowns!**

Super-resolution Radar

Setup
- Nine targets
- Max. delay = 10 micro secs
- Max. Doppler = 10 kHz
- W = 1.2 MHz
- T = 0.48 milli secs
- N = 48 pulses in x(t)
- Sequence = random binary





Conclusions

- Compressed sampling and processing of many analog signals
- Wideband sub-Nyquist samplers in hardware
- Hardware prototype for sub-Nyquist radar processing
- Good SNR, clutter rejection and dynamic range capabilities
- Many applications and many research opportunities: extensions to other analog and digital problems, robustness, hardware …

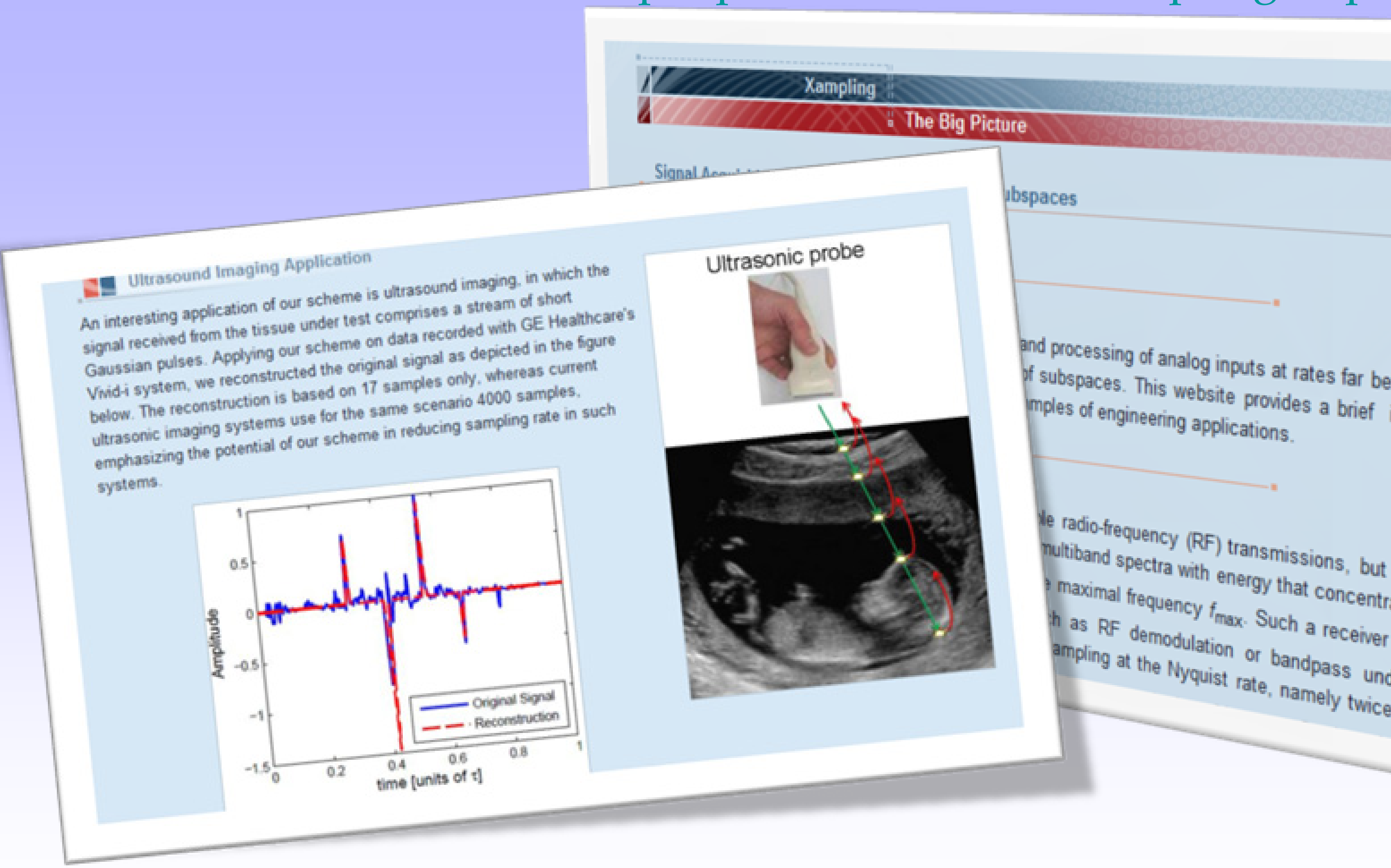
Exploiting structure can lead to a new sampling paradigm which combines analog + digital

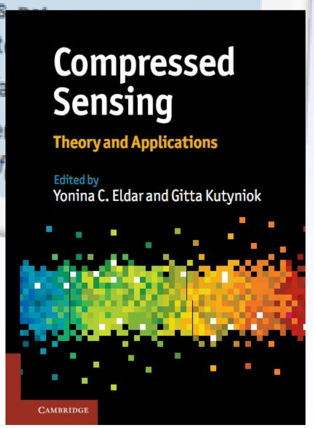
More details in:

M. Duarte and Y. C. Eldar, "Structured Compressed Sensing: From Theory to Applications," Review for TSP.
M. Mishali and Y. C. Eldar, "Xampling: Compressed Sensing for Analog Signals", book chapter available at http://webee.technion.ac.il/Sites/People/YoninaEldar/books.html

Xampling Website

webee.technion.ac.il/people/YoninaEldar/xampling_top.html





Y. C. Eldar and G. Kutyniok, "Compressed Sensing: Theory and Applications", Cambridge University Press, 2012

Thank you

If you found this interesting …

Looking for a post-doc!