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False Alarms in Radar Detection within Sparse-signal Processing

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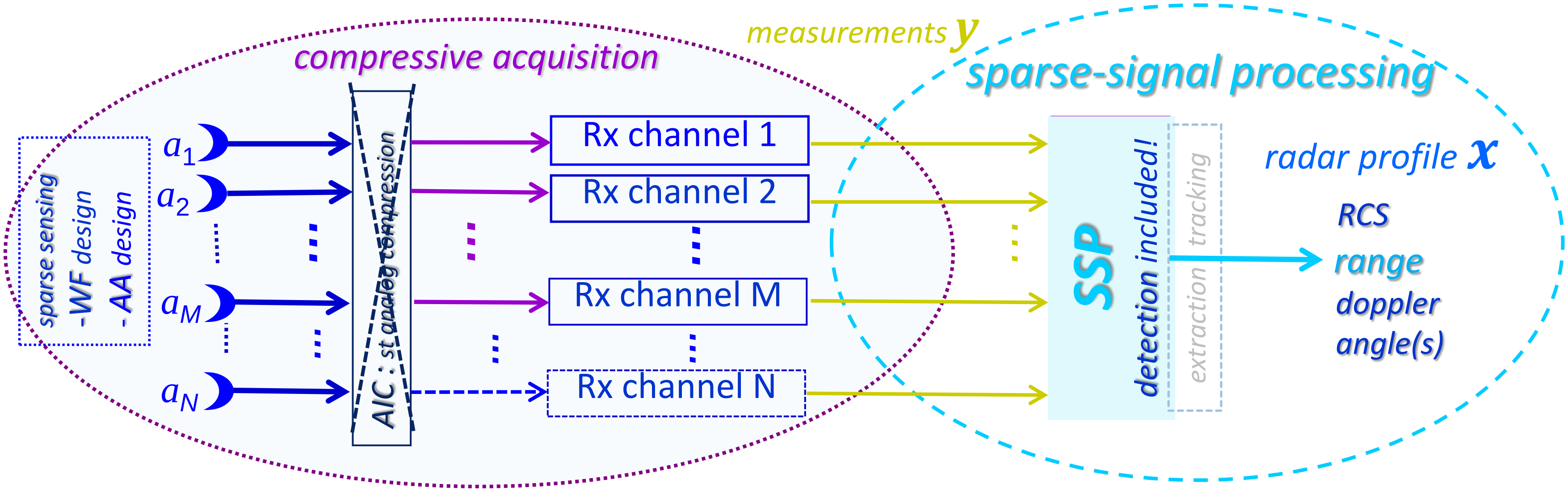
CS-radar receiver



measurements y = sensing-model A profile x + receiver-noise z , $z \sim CN(0, \gamma I_M)$

$$x_{SSP} = \arg\min_{x} \|y - Ax\|^2 + h\|x\|_1$$

Promising applications: **high resolution, multi-target detection, …**

Performance of x_{SSP} in radar in ***detection*** and *resolution*?

measurements $\boldsymbol{y}$ = sensing-model $\boldsymbol{A}$ profile $\boldsymbol{x}$ + receiver-noise $\boldsymbol{z}$, $z \sim CN\,(0, \gamma I_M)$

threshold ⇩

$$\boldsymbol{x}_{\mathrm{SSP}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|^2 + h\|\boldsymbol{x}\|_1$$

- ✓ sensing coherence $\mu(\boldsymbol{A}) = \max_{i,j,i\neq j} |\boldsymbol{a}_i^{\mathrm{H}} \boldsymbol{a}_j|$, $\|\boldsymbol{a}_n\| = 1$
- ✓ sparsity of $\boldsymbol{x}$, $K = \dim(\boldsymbol{T}), K < M \leq N$, $\boldsymbol{T}$ ... true support set

detection metrics: $P_{\mathrm{fa,SSP}}$ and $P_{\mathrm{d,SSP}}$ at h**?**

$$P_{\mathrm{fa,SSP}} = \mathrm{P}\{|x_{\mathrm{SSP},l}| \neq 0\},\ l \notin \boldsymbol{T}$$

$$P_{\mathrm{d,SSP}} = \mathrm{P}\{|x_{\mathrm{SSP},k}| \neq 0\},\ k \in \boldsymbol{T}$$

- *Existing detection : a single target at position k in* $\boldsymbol{y}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{z}$

$$H_1:\ \begin{matrix}\boldsymbol{a}_k x_k \neq 0\\ \boldsymbol{a}_l x_l = 0 \text{ for } \forall l: l\neq k\end{matrix} \qquad H_0:\ \boldsymbol{a}_n x_n = 0 \text{ for } \forall n$$

fixed $P_{\text{fa}} = \mathrm{P}\{|x_{\text{MF},n}| > \eta | H_0\}$ $\quad \eta = \sqrt{-\gamma \ln P_{\text{fa}}}$

coherence $\mu(\boldsymbol{A})$ ⇓

optimal $P_{\text{d}} = \mathrm{P}\{|x_{\text{MF},k}| > \eta | H_1\}$ $\quad x_{\text{MF},n} = \boldsymbol{a}_n^{\text{H}}\boldsymbol{y} = \boldsymbol{a}_n^{\text{H}}\boldsymbol{a}_k x_k + \boldsymbol{a}_n^{\text{H}}\boldsymbol{z}$

$P_{\text{fa,MF}} = \mathrm{P}\{|x_{\text{MF},l}| > \eta | H_1\} \geq P_{\text{fa}}$ *due to $\mu(\boldsymbol{A})$, i.e. realistic PSF*

- *Detection of multiple targets at positions k from* $\boldsymbol{T}, \dim(\boldsymbol{T}) = K$

$$H_1:\ \begin{matrix}\boldsymbol{a}_k x_k \neq 0, k\in \boldsymbol{T}\\ \boldsymbol{a}_l x_l = 0, l\notin \boldsymbol{T}\end{matrix} \qquad H_0:\ \boldsymbol{a}_n x_n = 0 \text{ for } \forall n$$

$$x_{\text{MF},n} = \boldsymbol{a}_n^{\text{H}}\boldsymbol{y} = \boldsymbol{a}_n^{\text{H}}\boldsymbol{A}_T\boldsymbol{x}_T + \boldsymbol{a}_n^{\text{H}}\boldsymbol{z}$$

False Alarms in Radar Detection within SSP : Background (2)

$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{z}$

$\boldsymbol{x}_{\mathrm{SSP}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|^2 + h\|\boldsymbol{x}\|_1$

- *Detection within SSP at positions* k *from* $\boldsymbol{T}, \dim(\boldsymbol{T}) = K$

H_1: $\boldsymbol{a}_k x_k \neq 0, \quad k \in \boldsymbol{T}$; $\boldsymbol{a}_l x_l = 0, \quad l \notin \boldsymbol{T}$ and H_0: $\boldsymbol{a}_n x_n = 0$ for $\forall n$

$$P_{\mathrm{fa,SSP}} = \mathrm{P}\{|x_{\mathrm{SSP},l}| \neq 0 | H_1\} = ?$$

$$P_{\mathrm{d,SSP}} = \mathrm{P}\{|x_{\mathrm{SSP},k}| \neq 0 | H_1\} = ?$$

SSP facilitates:

- *control parameter* h
- *sensing coherence* $\mu(\boldsymbol{A}) > 0$
- *multiple nonzeros,* $\dim(\boldsymbol{T}) = K$

⇒

Generic detection metrics:

$P_{\mathrm{fa,SSP}}$ and $P_{\mathrm{d,SSP}}$ *at* $\mu(\boldsymbol{A}) > 0$ and any K

- test statistics
- *detection interpretation*

SSP feasibility condition for $\boldsymbol{x}_{\mathrm{SSP}}$ in an estimated support set $\boldsymbol{S}$

subgradient $u_{\mathrm{SSP},n}$ identifies nonzeros: $|u_{\mathrm{SSP},i}| = 1, i \in \boldsymbol{S}$, i.e. $|x_{\mathrm{SSP},i}| \neq 0$

$$\boldsymbol{u}_{\mathrm{SSP}} = \boldsymbol{A}^{\mathrm{H}}(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}_{\mathrm{SSP}})/h, \qquad \|\boldsymbol{u}_{\mathrm{SSP}}\|_\infty \leq 1$$

$$u_{\mathrm{SSP},n} = \boldsymbol{a}_n^{\mathrm{H}}\left[\boldsymbol{A}_S\left(\boldsymbol{A}_S^{\mathrm{H}}\boldsymbol{A}_S\right)^{-1}\boldsymbol{u}_{\mathrm{SSP},S} + \left(\boldsymbol{I}_{\mathrm{M}} - \boldsymbol{A}_S\left(\boldsymbol{A}_S^{\mathrm{H}}\boldsymbol{A}_S\right)^{-1}\boldsymbol{A}_S^{\mathrm{H}}\right)\boldsymbol{z}/h\right]$$

no separation of FAs from targets

- proposed test statistic $v_{\mathrm{SSP},n}$ based on $u_{\mathrm{SSP},n}$ with reference to $\boldsymbol{T}$

$$v_{\mathrm{SSP},n} = \boldsymbol{a}_n^{\mathrm{H}}\left[\boldsymbol{A}_T\left(\boldsymbol{A}_T^{\mathrm{H}}\boldsymbol{A}_T\right)^{-1}\boldsymbol{u}_{\mathrm{SSP},T} + \left(\boldsymbol{I}_{\mathrm{M}} - \boldsymbol{A}_T\left(\boldsymbol{A}_T^{\mathrm{H}}\boldsymbol{A}_T\right)^{-1}\boldsymbol{A}_T^{\mathrm{H}}\right)\boldsymbol{z}/h\right]$$

$|v_{\mathrm{SSP},l}| > |u_{\mathrm{SSP},l}| = 1$, if $|x_{\mathrm{SSP},l}| \neq 0,\ l \notin \boldsymbol{T}$

$|v_{\mathrm{SSP},k}| = |u_{\mathrm{SSP},k}| \leq 1, k \in \boldsymbol{T}$

False Alarms in Radar Detection within SSP : Proposed Analysis (2)

✓ *proposed test statistic* $\upsilon_{\mathrm{SSP},n}$ *separates FAs from targets*

$$P_{\mathrm{fa,SSP}} = \mathsf{P}\{|x_{\mathrm{SSP},l}| \neq 0|H_1\} \equiv \mathsf{P}\{|\upsilon_{\mathrm{SSP},l}| > 1|H_1\},\ l \notin \boldsymbol{T}$$

$$P_{\mathrm{d,SSP}} = \mathsf{P}\{|x_{\mathrm{SSP},k}| \neq 0|H_1\} \equiv \mathsf{P}\{|\upsilon_{\mathrm{SSP},k}| = 1|H_1\},\ k \in \boldsymbol{T}$$

$$P_{\bar{\mathrm{d}},\mathrm{SSP}} = \mathsf{P}\{|x_{\mathrm{SSP},k}| = 0|H_1\} \equiv \mathsf{P}\{|\upsilon_{\mathrm{SSP},k}| < 1|H_1\},\ k \in \boldsymbol{T}$$

- *noise only*, i.e. under H_0 as $\boldsymbol{T} = \emptyset$, or under H_1 with $\mu(\boldsymbol{A})= 0$:

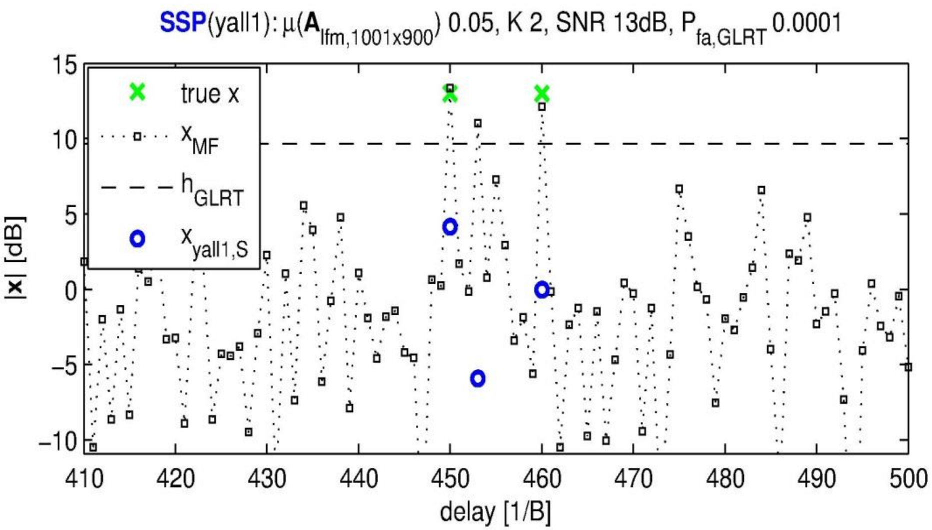
$$\upsilon_{\mathrm{SSP},l} = \boldsymbol{a}_l^{\mathrm{H}}\boldsymbol{z}/h \equiv \boldsymbol{a}_l^{\mathrm{H}}\boldsymbol{y}/h \equiv x_{\mathrm{MF},l}/h$$

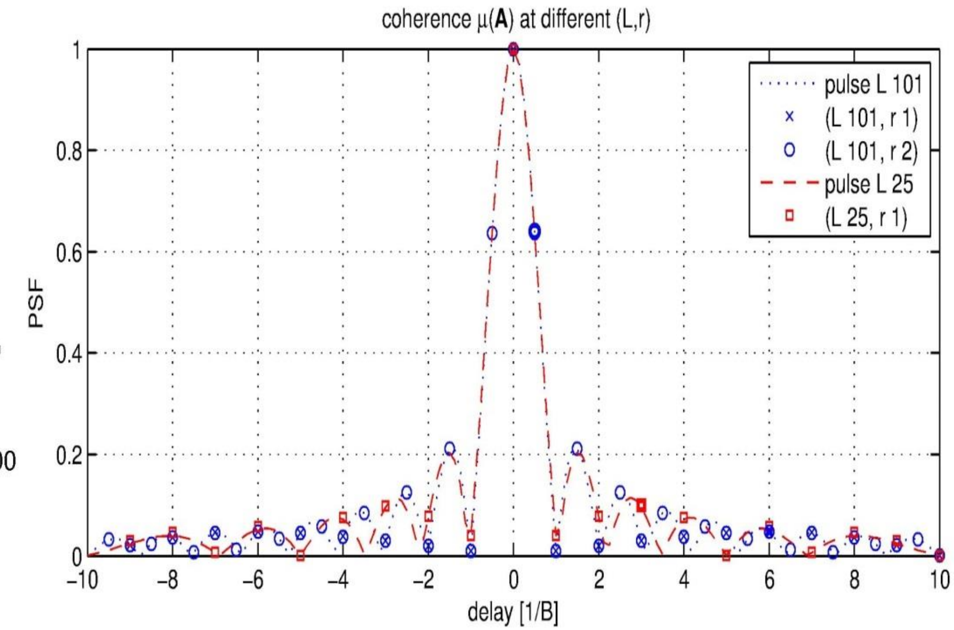
- *otherwise*, i.e. under H_1 with $\mu(\boldsymbol{A}) > 0$ and $\dim(\boldsymbol{T}) > 0$

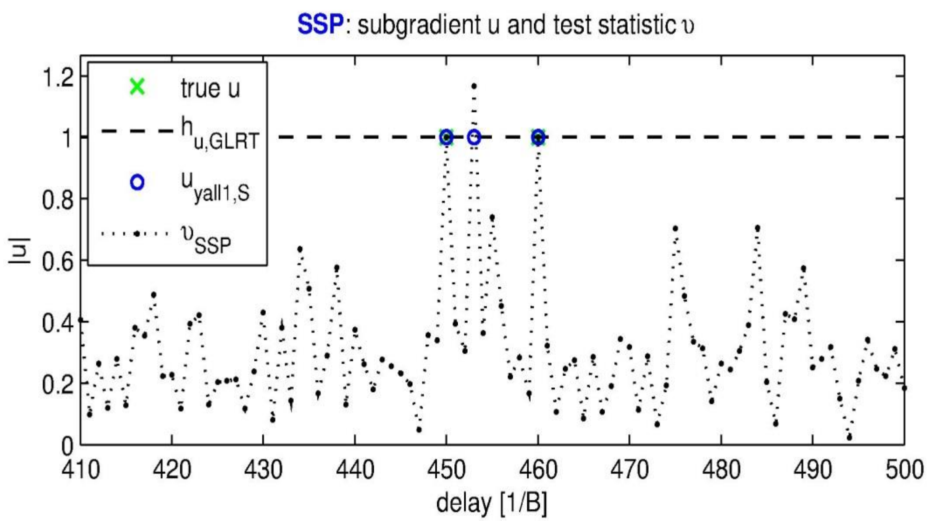
$$\upsilon_{\mathrm{SSP},l} = \boldsymbol{a}_l^{\mathrm{H}} \left[\boldsymbol{A}_T\left(\boldsymbol{A}_T^{\mathrm{H}}\boldsymbol{A}_T\right)^{-1}\boldsymbol{u}_{\mathrm{SSP},T} + \left(\boldsymbol{I}_{\mathrm{M}} - \boldsymbol{A}_T\left(\boldsymbol{A}_T^{\mathrm{H}}\boldsymbol{A}_T\right)^{-1}\boldsymbol{A}_T^{\mathrm{H}}\right)\boldsymbol{z}/h\right]$$

Range-only in pulse radar with LFM of unit bandwidth B:

- different $\mu(\boldsymbol{A})$, different K and different SNRs of SW0 targets







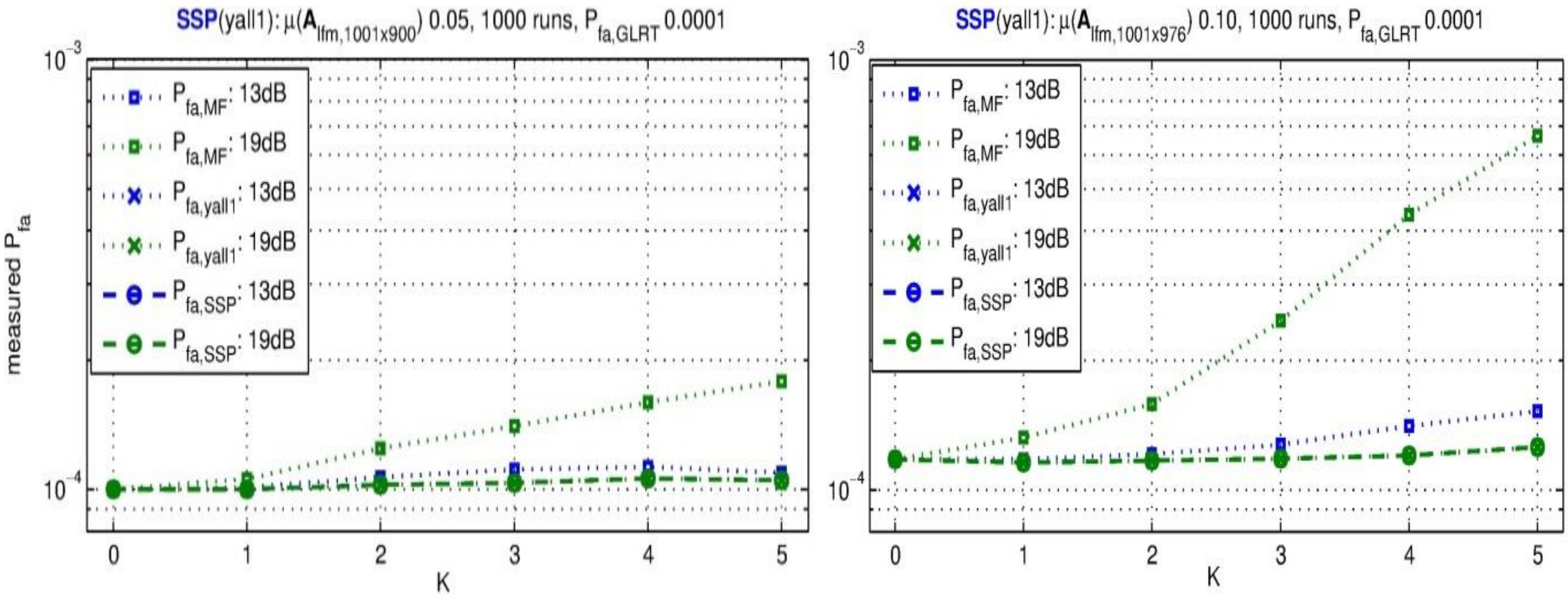
$x_{\mathrm{MF},n}$ → $P_{\mathrm{fa,MF}}$

$x_{\mathrm{yall1},n}$ → $P_{\mathrm{fa,yall1}}$

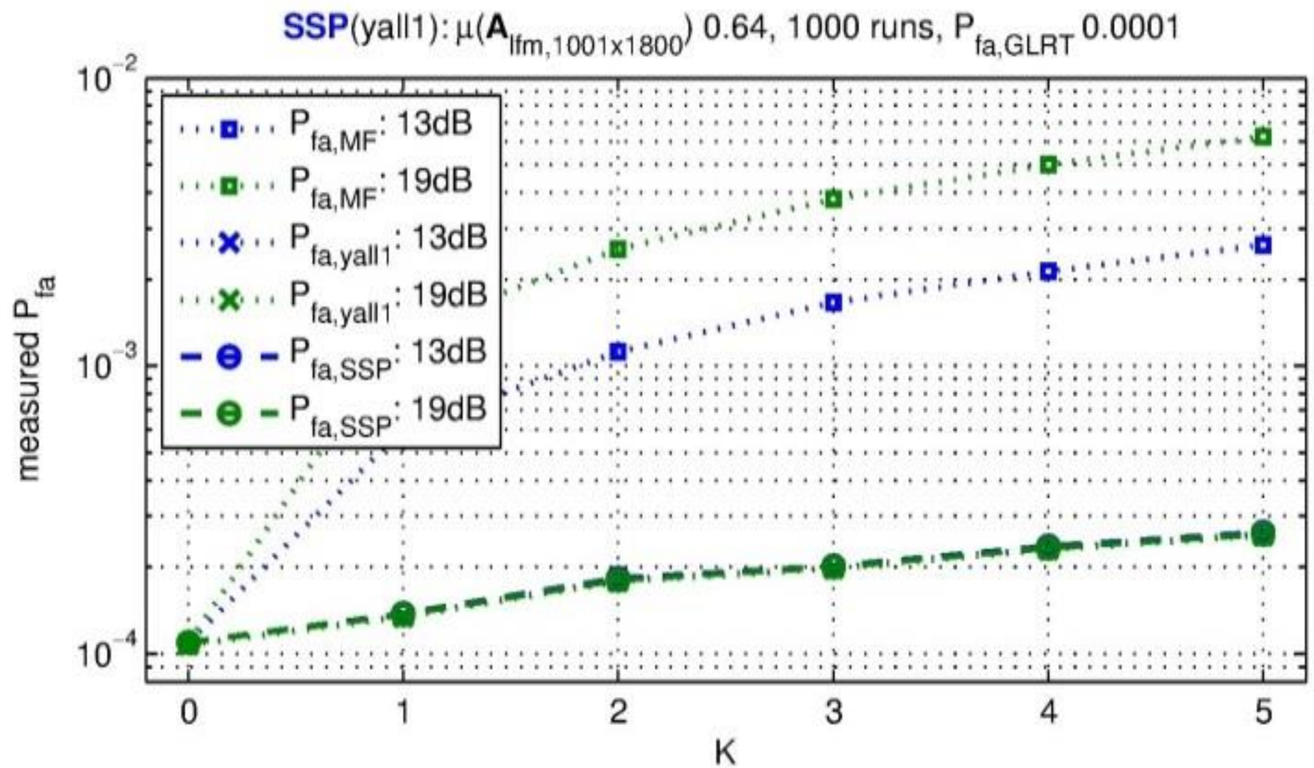
$\upsilon_{\mathrm{SSP},n}$ → $P_{\mathrm{fa,SSP}}$

False Alarms in Radar Detection within SSP : *Numerical Results* (2)

$\mu(\boldsymbol{A})$= 0.05 $\mu(\boldsymbol{A})$= 0.1



higher sensing coherence: $\mu(\boldsymbol{A})$= 0.64



higher $\mu(\boldsymbol{A})$, e.g. by up-sampling of the estimation grid, makes FAs increase, and hence, deteriorates the performance of detection within SSP

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Detection within SSP obtained with *metrics* $P_{\mathrm{d,SSP}}$ and $P_{\mathrm{fa,SSP}}$:

- ✓ **generic** : at given sensing coherence and any number of targets
- ✓ proposed test statistic separates targets from false alarms in SSP outcomes
- ✓ ***results with the test statistic coincide with actual SSP outcomes***
  - $P_{\mathrm{fa,SSP}}$ stable w.r.t. SNR
    - stable w.r.t. a number of targets at low sensing coherence
    - increase at higher sensing coherence but less than $P_{\mathrm{fa,MF}}$
- ✓ **outperforms existing radar detection**: $P_{\mathrm{fa,SSP}} \leq P_{\mathrm{fa,MF}}$ (while $P_{\mathrm{d,SSP}} = P_{\mathrm{d,MF}}$)
- ✓ ***better interpretation of existing radar-detection theory and practice***
  - $P_{\mathrm{fa,MF}}$ increases with SNR, sensing coherence and a number of targets
  - in such target cases: no fixed P_{fa} , i.e. no CFAR, is ensured

Thanks … ***Questions?***

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