ZESS

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Imaging
New Modalities

Adaptive Progressive Edge-Growth Construction of Low Density Sensing Matrices

Miguel Heredia Conde

heredia@zess.uni-siegen.de

Dr.-Ing. Klaus Hartmann

Prof. Dr.-Ing. Otmar Loffeld

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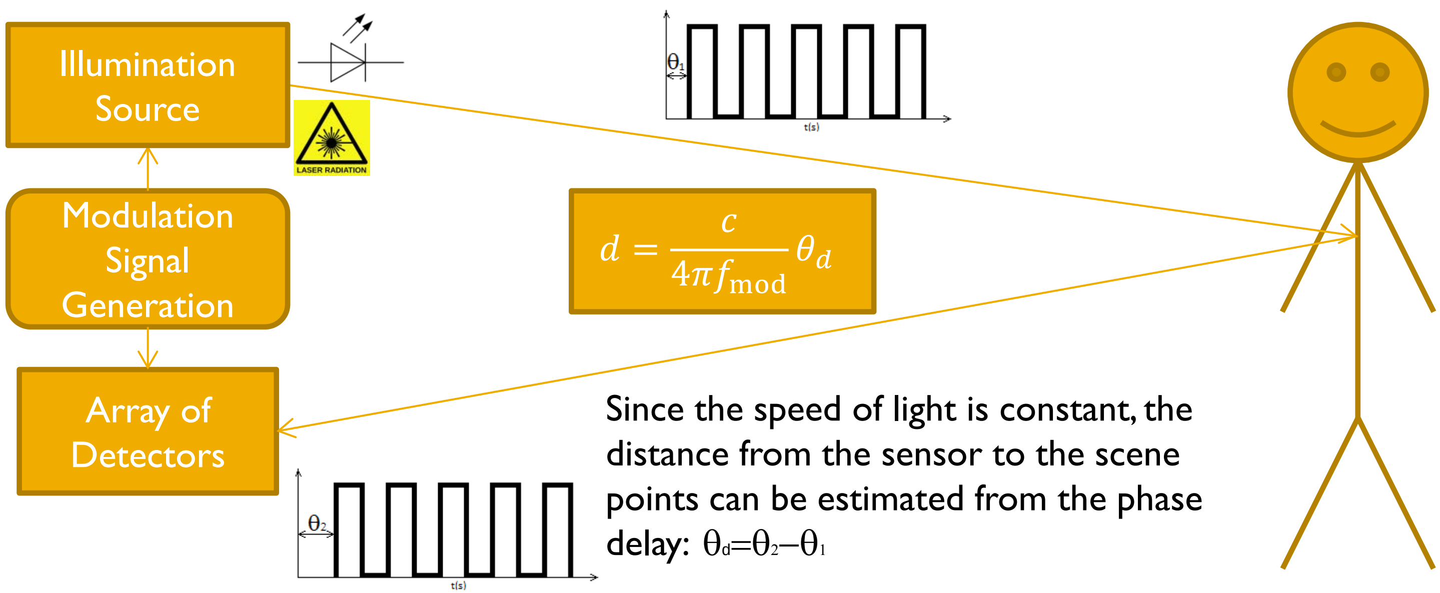
Contents

Phase-Shift-Based ToF

- Measure phase shift instead of time:
  - Modulated light, typically in the **NIR** band is emitted to the scene.
  - Modulation frequency in the radio frequency range (**20-120MHz**).
  - Reflected light reaches the camera with a delay that is proportional to the total distance from the illumination system to the scene point and then to the camera.
  - *Intelligent* pixels, featuring two or more channels (i.e., *multitap* pixels) are used to internally correlate the incoming signal with several reference signals.
  - Reference signals with different phase delays are used to sample the cross-correlation function.
  - The relative phase shift can be computed from few measurements (cross-correlation samples).

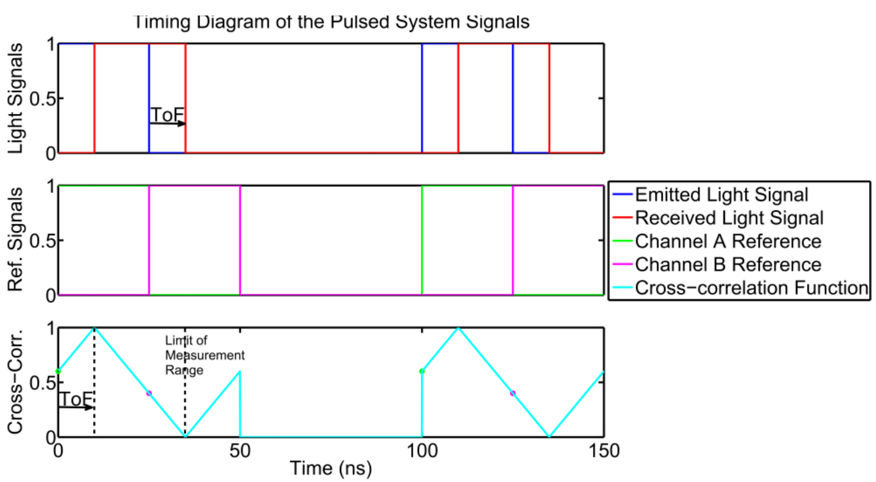


$$d = \frac{c}{4\pi f_{\mathrm{mod}}}\theta_d$$

Since the speed of light is constant, the distance from the sensor to the scene points can be estimated from the phase delay: $\theta_d = \theta_2 - \theta_1$

Motivation: Pulsed ToF

- Short pulses instead of CW:
  - Instead of continuously emitting a periodic signal, in pulsed ToF a single pulse is emitted and its echo is received.
  - After a period of inactivity the process is repeated until the desired SNR is attained.
  - **Reducing the pulse width yields better depth resolution**, at the time it allows attaining improved SNR without increasing the average illumination power. ✓
  - In a conventional pulsed system, the depth measurement range is fully determined by the pulse width. **Too short widths yield unacceptably short ranges**. ✗
  - One would desire attaining the depth resolution corresponding to the shortest pulse width allowed by the hardware, while keeping an arbitrarily large range. ☺



The triangular cross-correlation function eliminates the need for a costly arctangent transformation. The depth can be estimated linearly from the raw data.

$$d = \frac{c}{T}\frac{Q_B}{Q_A + Q_B}$$

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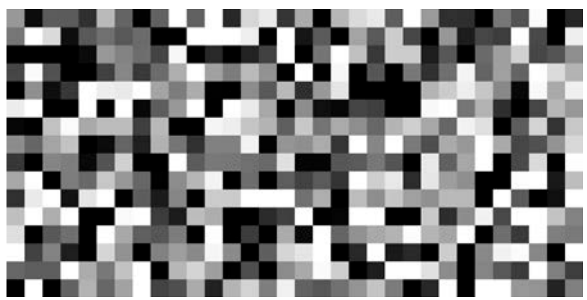
CS for Pulsed ToF

- Goals:
  - High depth resolution
    - Short pulses → Limits: drivers and light sources
    - Small discretization steps in time domain
  - Large depth range
    - Many discrete steps
- What is favorable?
  - Extreme sparsity of the pulse echo(es) in time domain
- What is unfavorable?
  - Intractable signal dimensionality
    - E.g., desired range: 10m at 1mm resolution yields $n = 10^4$ dimensions
  - Unknown signal support → Needle in the hay
    - Dense sensing matrices ensure that the we don't miss the support, but…
    - The SNR of the measurements tends to zero as $n \to \infty$

Random Sensing Matrices

- Gaussian:
  - Elements drawn from *i.i.d.* normal random variables, e.g., of zero mean and $1/m$ variance. Good CS matrices, from an RIP perspective.
- Bernoulli:
  - Bernoulli with elements $+1$ and -1, drawn from *i.i.d.* Bernoulli random variables with $p = q = 0.5$. Good CS performance, close to that of Gaussian matrices. Binary.
  - Bernoulli with elements $+1$ and 0, drawn from *i.i.d.* Bernoulli distributions with $p = q = 0.5$. Reportedly bad CS performance[1], but binary and *sparse*.



Gaussian sensing matrix

Bernoulli sensing matrix

Reduction of storage requirements by, at least, the bit depth used for quantization of the non-binary matrix elements.

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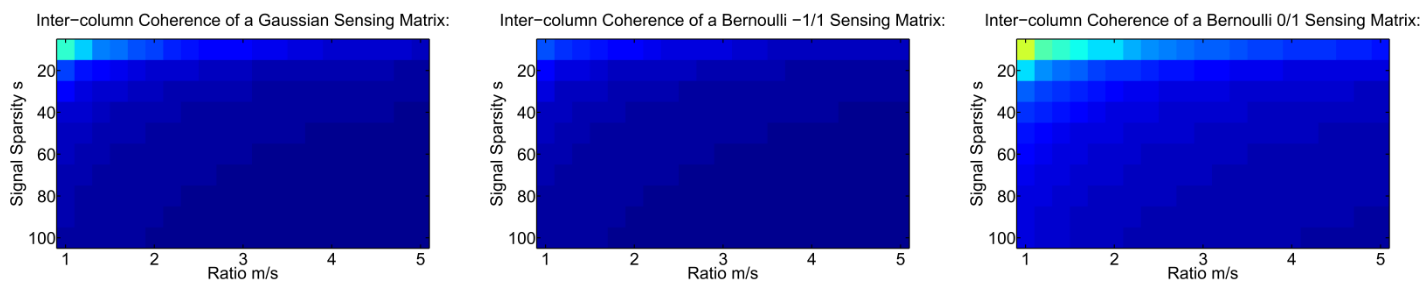
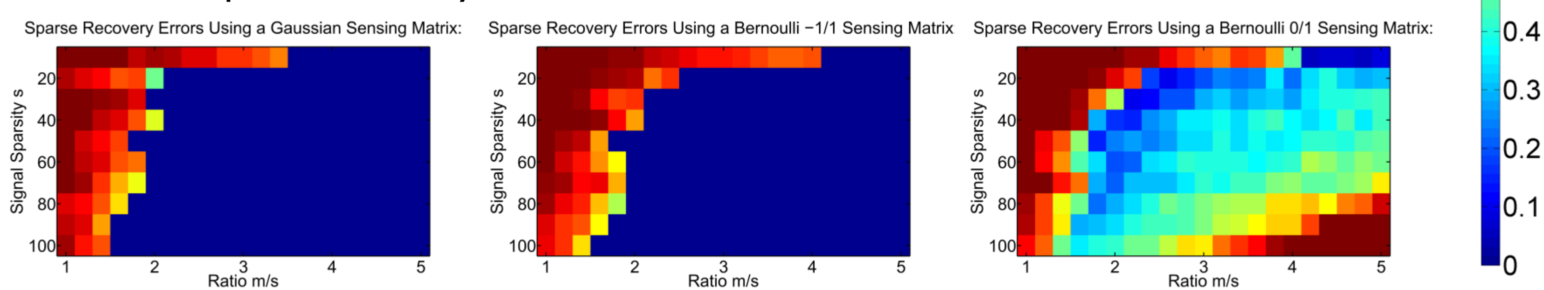
[1] V. Chandar, "A negative result concerning explicit matrices with the restricted isometry property," Tech. Rep., 2008.

Random Sensing Matrices

- Performance Evaluation:
  - Signal dimensionality $n = 1024$. Number of measurements m as a function of the sparsity s. $10 \leq s \leq 100, 1 \leq \frac{m}{s} \leq 5$.
  - Coherence Evaluation:



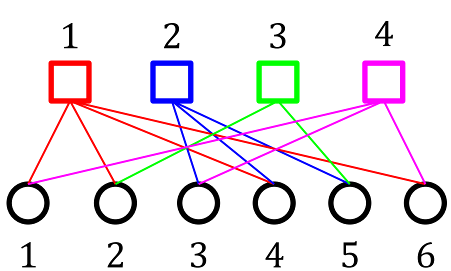
  - Sparse Recovery Evaluation:



Gaussian | Bernoulli +1/-1 | Bernoulli +1/0

Low-Density Binary Matrices

- Objectives:
  - Low density, i.e., as few nonzeros per row/column as possible
  - High regularity, i.e., the density can be globally defined
  - Low coherence
- Progressive deterministic construction:
  - Sequential addition of edges to the corresponding Tanner graph
  - Criterion: (local) *girth* maximization at each edge addition
  - Problem: a sequence of optimal decisions does not ensure attaining the global optimum, in terms of *girth* maximization
  - Example:



Tanner Graph

$g_l = 6$ $\quad n = 6$ columns

$m = 4$ rows

1	1	0	1	0	1
0	0	1	1	1	0
0	1	0	0	1	0
1	0	1	0	0	1

LDPC Matrix

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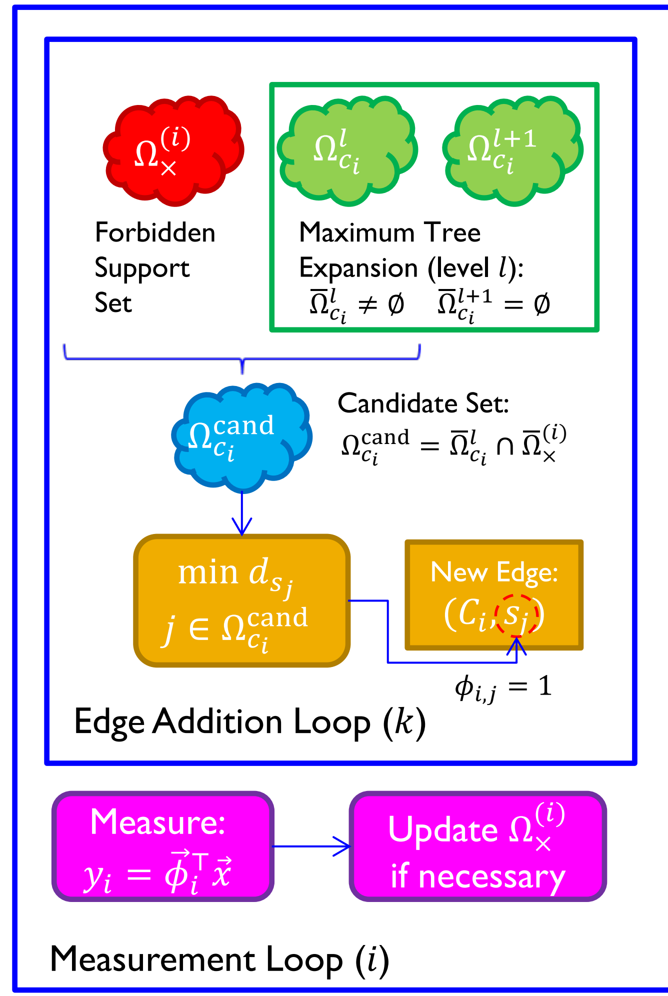
Adaptive Deterministic Construction (Algorithm)

Algorithm 1 Adaptive Progressive Edge-Growth (APEG)

Initialize: $\Omega_\times^{(1)} = \emptyset,\ \Pi_i^{(0)} = \emptyset\ \forall i,\ 1 \le i \le m$

for $i = 1; i := i + 1$ **to** $i = m$ **do**
  for $k = 1; k := k + 1$ **to** $k = d_c$ **do**
    if $k = 1$ **then**
      Candidate set: $\Omega_{c_i}^{\text{cand}} = \bar{\Omega}_\times^{(i)}$
    else
      Expand tree up to depth $l \mid \bar{\Omega}_{c_i}^l \neq \emptyset$ **and** $\bar{\Omega}_{c_i}^{l+1} = \emptyset$
      Candidate set: $\Omega_{c_i}^{\text{cand}} = \bar{\Omega}_{c_i}^l \cap \bar{\Omega}_\times^{(i)}$
    end if
    Select symbol node index: $j = \arg\min_{j \in \Omega_{c_i}^{\text{cand}}} d_{s_j}$
    Add new edge: $\Pi_i^{(k)} = \Pi_i^{(k-1)} \cup (c_i, s_j)$
  end for
  Sensing kernel: $\vec{\phi}_i \in \mathbb{R}^n \mid \phi_{i,j} = 1 \Leftrightarrow (c_i, s_j) \in \Pi_i$
  Measure: $y_i = \vec{\phi}_i^\top \vec{x}$
  if $y_i < \varepsilon$ **then**
    Update forbidden support: $\Omega_\times^{(i+1)} = \Omega_\times^{(i)} \cup \text{supp}\left(\vec{\phi}_i\right)$
  else
    Preserve forbidden support: $\Omega_\times^{(i+1)} = \Omega_\times^{(i)}$
  end if
end for



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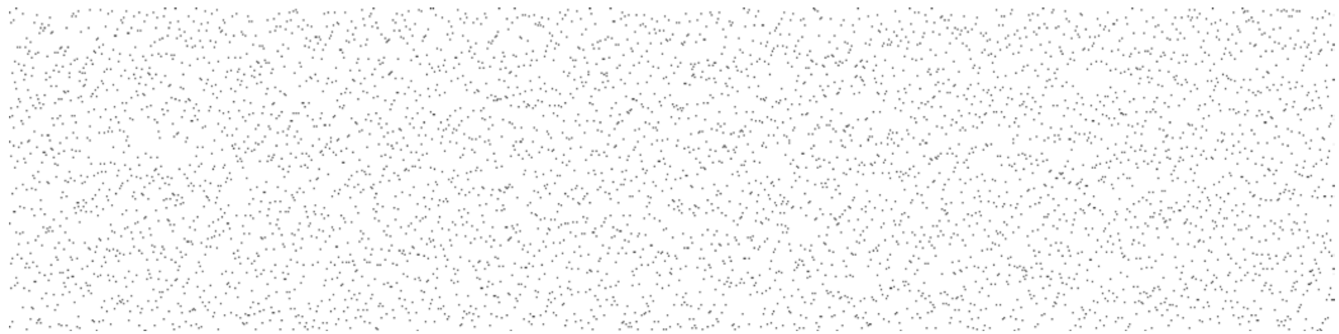
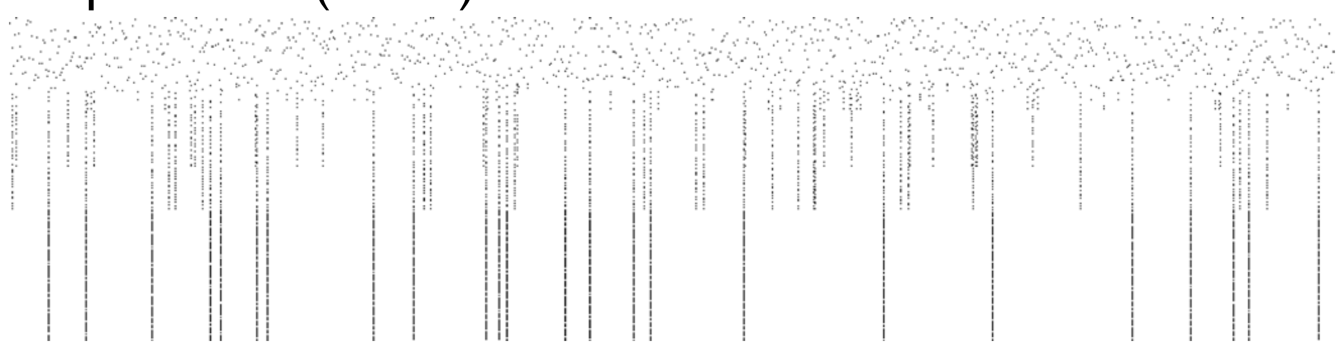
Adaptive Deterministic Construction

- How does the sensing matrix look like?
  - Progressive Edge Growth (PEG) Baseline (no adaptiveness):



  - Adaptive PEG (APEG) Construction:



$n = 1024$
$m = 250$
$d_c = 20$
$s = 10$

Strong overlap between sensing kernels support and signal support!

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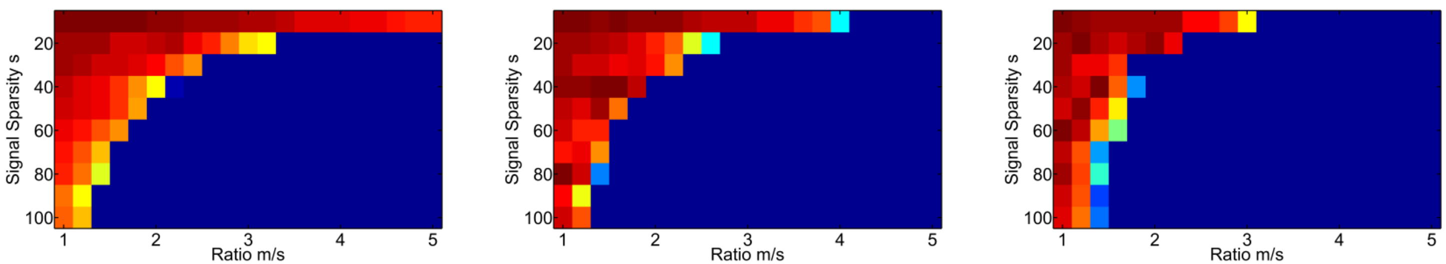
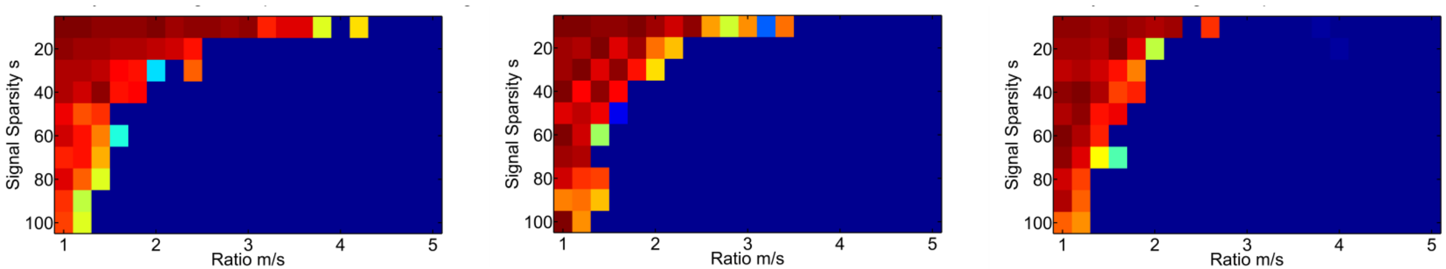
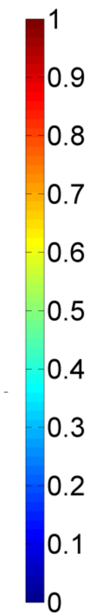
Adaptive Deterministic Construction

- Sparse Recovery Evaluation (normalized error):
  - Progressive Edge Growth (PEG) Baseline:



  - Adaptive PEG (APEG) Construction:





$d_c = 20$ $d_c = 40$ $d_c = 60$

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Conclusions

- Gaussian and ± 1 Bernoulli random matrices exhibit excellent CS recovery performance.
- A construction method of low-density binary sensing matrices has been proposed that is both *deterministic* and *adaptive*.
- Sensing matrices constructed using our APEG method exhibit better recovery performance than random and non-adaptive deterministic matrices.
- Densities between 1% and 5% have been observed to be sufficient when dealing with $n = 1024$ dimensions.

Graduiertenkolleg 1564
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Thank you for your attention.

Did you like the idea?
Then, download the sample code and test the performance of the APEG-LDPC matrices yourself!
https://uni-siegen.sciebo.de/index.php/s/mDKhRTClQoA77I9