Stochastic Resolution Analysis via a GLR Test in Radar

CoSeRa 2016

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THALES

Introduction

Resolution is primarily given by the minimum distance between two objects that still can be resolved (e.g.[1]). For the performance guarantees, a complete description needs also the probability of resolution at a given separation and signal-to-noise ratio (SNR).

For the stochastic completeness, exploring information geometry (IG, [2]-[4]) and compressive sensing (CS, [5]) suits radar ([6]).

This stochastic resolution analysis is aimed for CS radar because of critical SNR in its acquisition and high resolution from its processing.

CS (Compressive Sensing) Radar

performance: resolution, detection, …?

Compressive acquisition : *sparse sensing* [7]

- fewer measurements: processing gain and SNR critical!
- spatial–temporal measurements $\boldsymbol{y}$ for joint angle-Doppler

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{z}, \quad A_{mn} = e^{j(\mu_m \mathrm{u}_n + t_m \mathrm{v}_n)}, \ \boldsymbol{z} \sim CN(\boldsymbol{0}, \gamma \boldsymbol{I})$$

sampling: spatial μ and temporal t for angle u and Doppler v

Sparse-signal processing *(SSP,* e.g. [5] and [8]): *a sparse model-based refinement of existing radar signal processing* ([9]).

$$\boldsymbol{x}_{\mathrm{SSP}} = \arg\min_{\boldsymbol{x}} \|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|^2 + \eta \|\boldsymbol{x}\|_1$$

measured $P_{\mathrm{res,SSP}} = \mathrm{P}\{(x_{\mathrm{SSP},i} \neq 0) \wedge (x_{\mathrm{SSP},j} \neq 0) | H_1\}$

Stochastic Resolution Analysis

Information Geometry (IG) is stochastic signal processing where the stochastic inferences are structures in differential geometry ([2]-[3]).

The intrinsic geometrical structure of a data model is characterized locally by the Fisher information metric (e.g. [4], [6], [10], [11]):

$$G\left(\begin{bmatrix}\mathrm{u}\\ \mathrm{v}\end{bmatrix}\right) = -\mathrm{E}\left[\frac{\partial^2 \ln p(\boldsymbol{y}|\mathrm{u},\mathrm{v})}{\partial [\mathrm{u}\ \mathrm{v}]^T}\right] = 2\mathrm{SNR}\begin{bmatrix} M_t\|\boldsymbol{\mu}\|^2 & 0 \\ 0 & M_s\|\boldsymbol{t}\|^2\end{bmatrix} = \boldsymbol{G}_0$$

Information distances between $p(\boldsymbol{y}|\mathrm{u},\mathrm{v})$ and $p(\boldsymbol{y}|\mathrm{u}+\delta\mathrm{u}, \mathrm{v}+\delta\mathrm{v})$:

$$d_{\boldsymbol{\theta}} = d(\boldsymbol{\theta}, \boldsymbol{\theta}+\delta\boldsymbol{\theta}) \equiv \min_{\boldsymbol{\vartheta}(t)} \int_0^1 \sqrt{\dot{\boldsymbol{\vartheta}}(t)^T G(\boldsymbol{\vartheta}(t)) \dot{\boldsymbol{\vartheta}}(t)}\ dt$$

$$d\left(\begin{bmatrix}\mathrm{u}\\ \mathrm{v}\end{bmatrix}, \begin{bmatrix}\mathrm{u}+\delta\mathrm{u}\\ \mathrm{v}+\delta\mathrm{v}\end{bmatrix}\right) = \sqrt{2\,\mathrm{SNR}(M_t\|\boldsymbol{\mu}\delta\mathrm{u}\|^2 + M_s\|\boldsymbol{t}\delta\mathrm{v}\|^2)}.$$

Resolution: testing $H_0: \delta\boldsymbol{\theta} = \boldsymbol{0}$ and $H_1: \delta\boldsymbol{\theta} \neq \boldsymbol{0}$ [2]

$$\mathrm{GLR} = p(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{\theta}+\delta\boldsymbol{\theta}) / p(\boldsymbol{y}|\boldsymbol{\theta})\,|_{\delta\boldsymbol{\theta}=\delta\boldsymbol{\theta}_{\mathrm{ML}}}$$

GLR test: asymptotic $\ln \mathrm{GLR} \sim \chi^2_{\varepsilon,\dim(\boldsymbol{\theta})}$ [12]

$$\varepsilon \equiv d^2_{\boldsymbol{\theta}}$$

Probability of resolution at separation $\delta\boldsymbol{\theta}$ and SNR

$$P_{\mathrm{res,GLRig}} = \mathrm{P}\{\ln \mathrm{aGLR} > \rho \mid H_1\}$$

$\rho = \chi^{2,\mathrm{inv}}_{0,\dim(\boldsymbol{\theta})}(P_{\mathrm{fa}})$ and in SSP: $\eta = \sqrt{-\gamma \ln P_{\mathrm{fa}}}$

Numerical Results

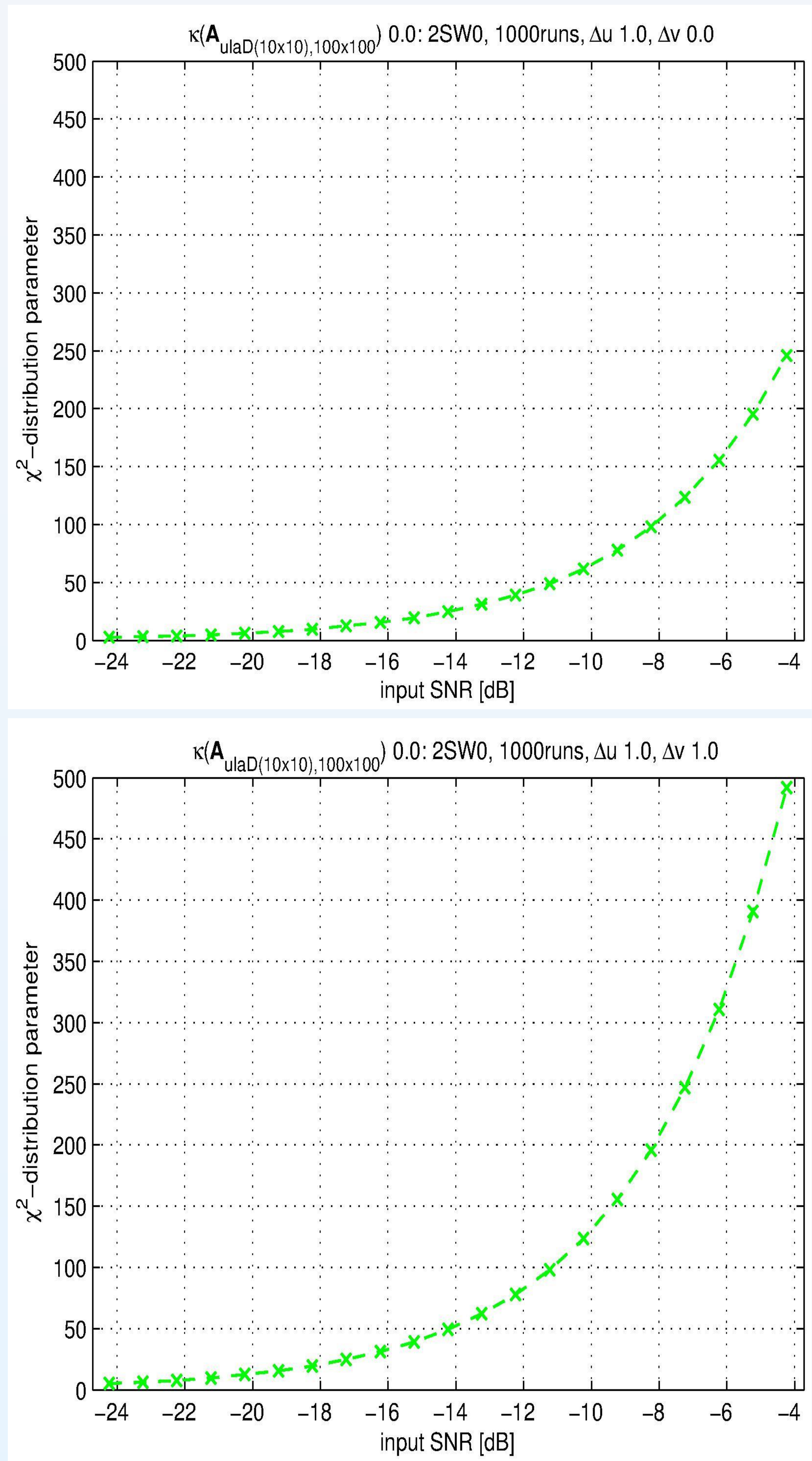

Figure 1. Parameter ε of the χ^2-distribution obtained from a test case with a uniform LA of size M_s over M_t time samples, both equal to 10, $M_s = M_t = 10$. Two point-targets are separated in u by du of $2\pi/M_s$ and in v by dv of 0 (top) and $2\pi/M_t$ (bottom).

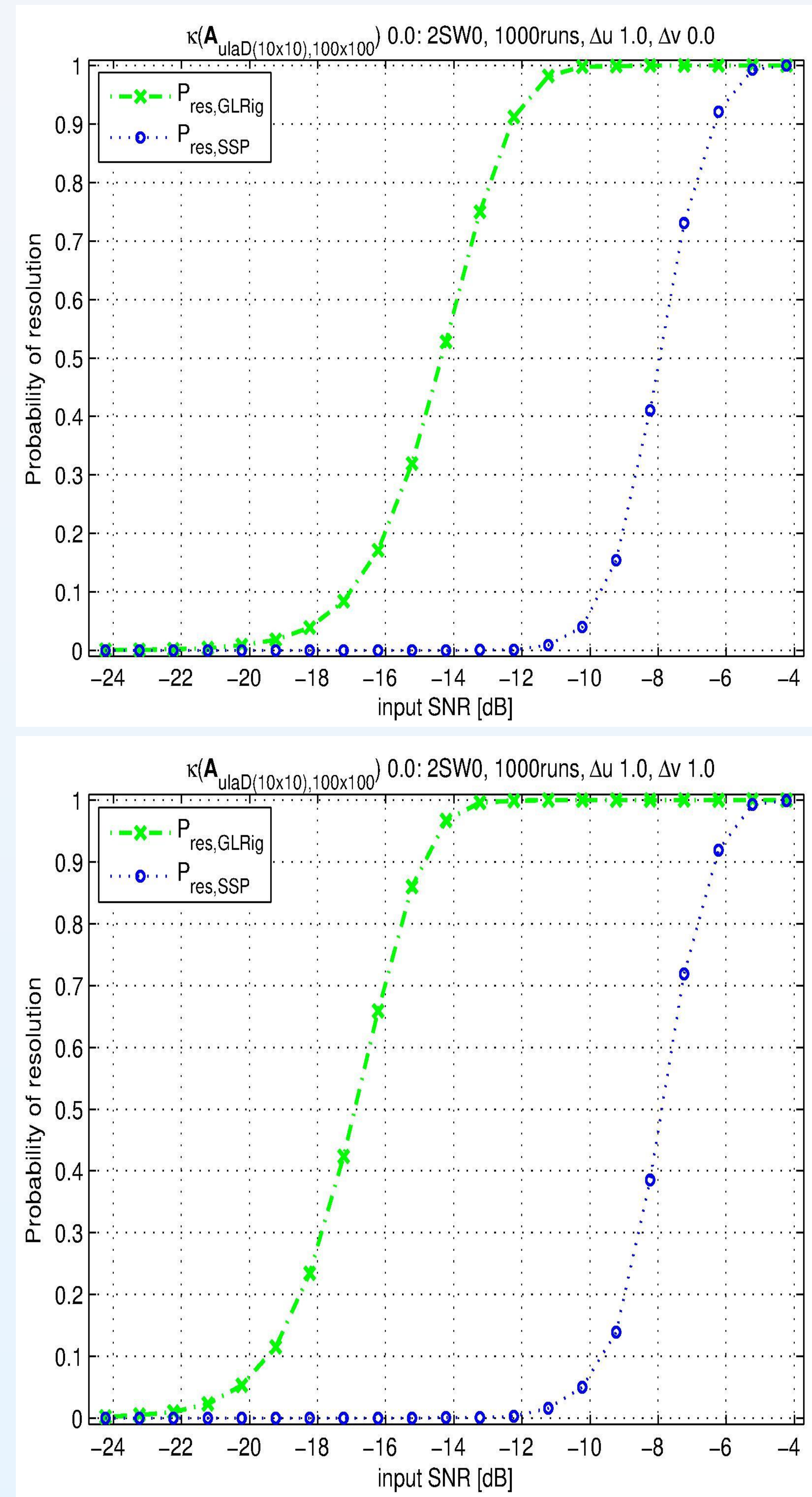

Figure 2. Stochastic resolution bounds obtained from the same test cases as in Figure 1. The probability $P_{\mathrm{res,GLRig}}$ (green crosses) obtained via IG-based GLR test with $\ln \mathrm{aGLR}_{\mathrm{IG}} \sim \chi^2_{\varepsilon,2}$ is compared with the SSP resolution $P_{\mathrm{res,SSP}}$ (blue cricles).

Contributions and Conclusions

- ❖ multi-dimensional parameter space
- ❖ probability of resolution assessed via a GLR test based on information distances, and compared with SSP resolution

This stochastic resolution analysis suits radar (as well as other sensors).

The performance guarantees are given by all the crucial impacts: spatial-temporal array design as well as input SNR, separation and a probability of resolution.

Future work

- further interpretation of stochastic resolution bounds
- all radar parameters: range, Doppler and angles
- sparse sensing in the front end, and
- links to the information theory

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IEEE CoSeRa 2016, Aachen: Poster Session A7-III **Sparse Reconstruction Aspects,** Wednesday, 21 September, 2016