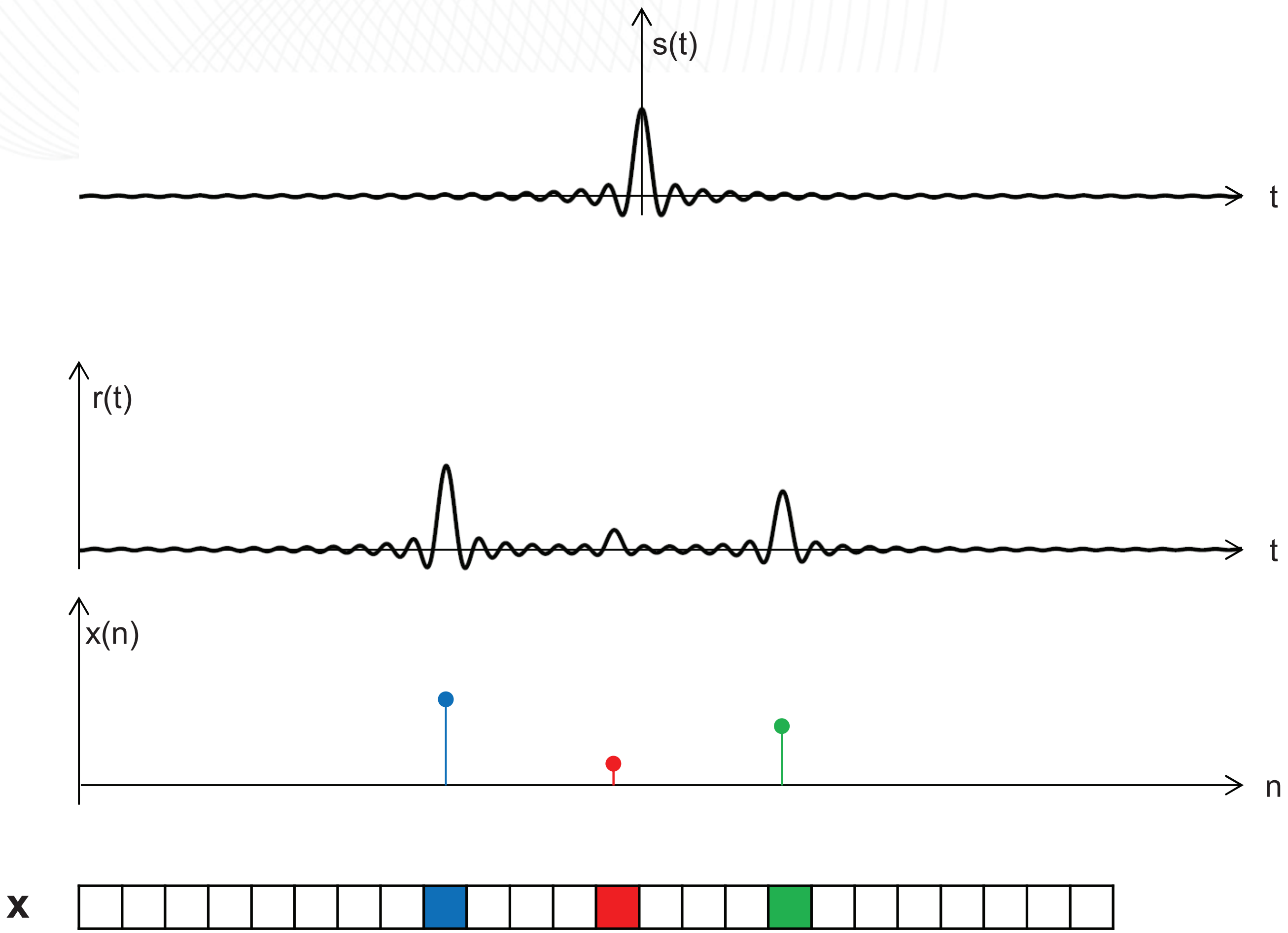
Sub-Nyquist Radar with Optimized Sensing Matrices – Performance Evaluation Based on Simulations and Measurements

Hannes Stahl	AIRBUS DS Electronics and Border Security GmbH, D-89077 Ulm, Germany
Jan Mietzner	AIRBUS DS Electronics and Border Security GmbH, D-89077 Ulm, Germany
Martin Kirscht	AIRBUS DS Electronics and Border Security GmbH, D-88039 Friedrichshafen, Germany
Robert F.H. Fischer	Institute of Communications Engineering, Ulm University, D-89081 Ulm, Germany

September 22, 2016

Motivation

- Pulse radar can be considered sparse in time: Typically few distinct target objects over range



Motivation

- Bandwidth/ range resolution of modern digital radars has gradually increased
- Nyquist sampling of received echo signals produces large amounts of data while goal is to extract small number of targets ⇒ Classic approach more and more inefficient
- Compressed Sensing (CS) can reduce sampling rates below Nyquist limit while still capturing the essential received information:
  - Generalized sampling (rather than classical ADCs)
  - Non-linear recovery algorithms (rather than conventional signal processing)

Motivation

- Bandwidth/ range resolution of modern digital radars has gradually increased
- Nyquist sampling of received echo signals produces large amounts of data while goal is to extract small number of targets ⇒ Classic approach more and more inefficient

- Compressed Sensing (CS) can reduce sampling rates below Nyquist limit while still capturing the essential received information:
  - Generalized sampling (rather than classical ADCs)
  - Non-linear recovery algorithms (rather than conventional signal processing)

- Preliminaries:
  - Received signals are sparse w.r.t. some basis ✓ (time domain)
  - Generalized sampling should be conducted in an incoherent domain e.g. frequency domain

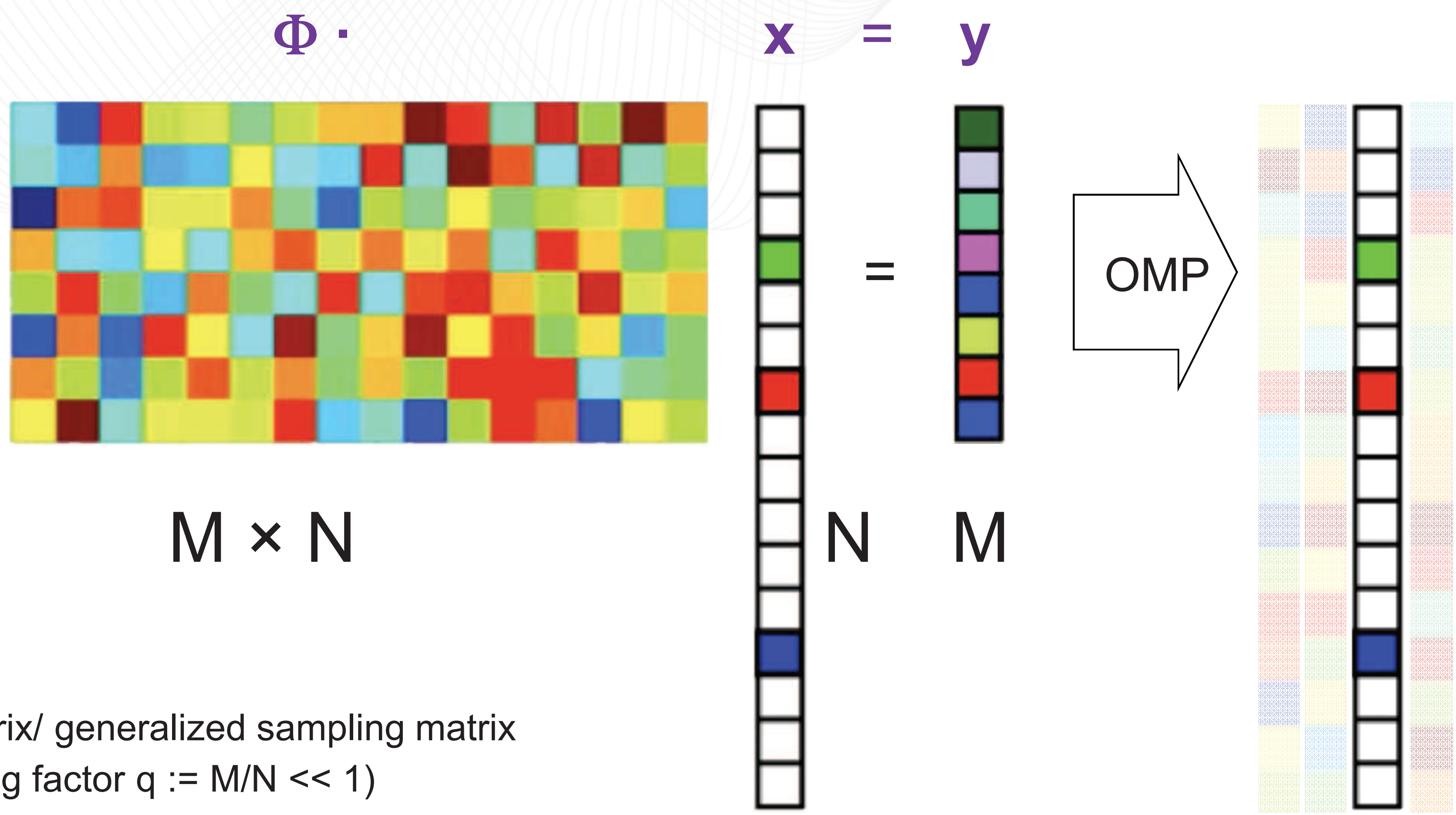
- Practical issues:
  - Received data contain noise and (possibly) clutter components
  - Target objects may appear extended rather than point-like (due to physical size and/or off-grid effects)

Outline

- Sub-Nyquist Radar Scheme with Generalized Sampling in Fourier domain
  - Optimized Design based on Sparse Rulers
  - Practical Realization using Bandpass Filters
- Performance Analysis based on Simulation Results
  - Perfect Support Reconstruction
  - Relaxed Support Reconstruction
- Application to Real-World Radar Data
  - Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar
  - Reconstruction Results
- Conclusions and Future Work

Sub-Nyquist Radar with Generalized Fourier Domain Sampling

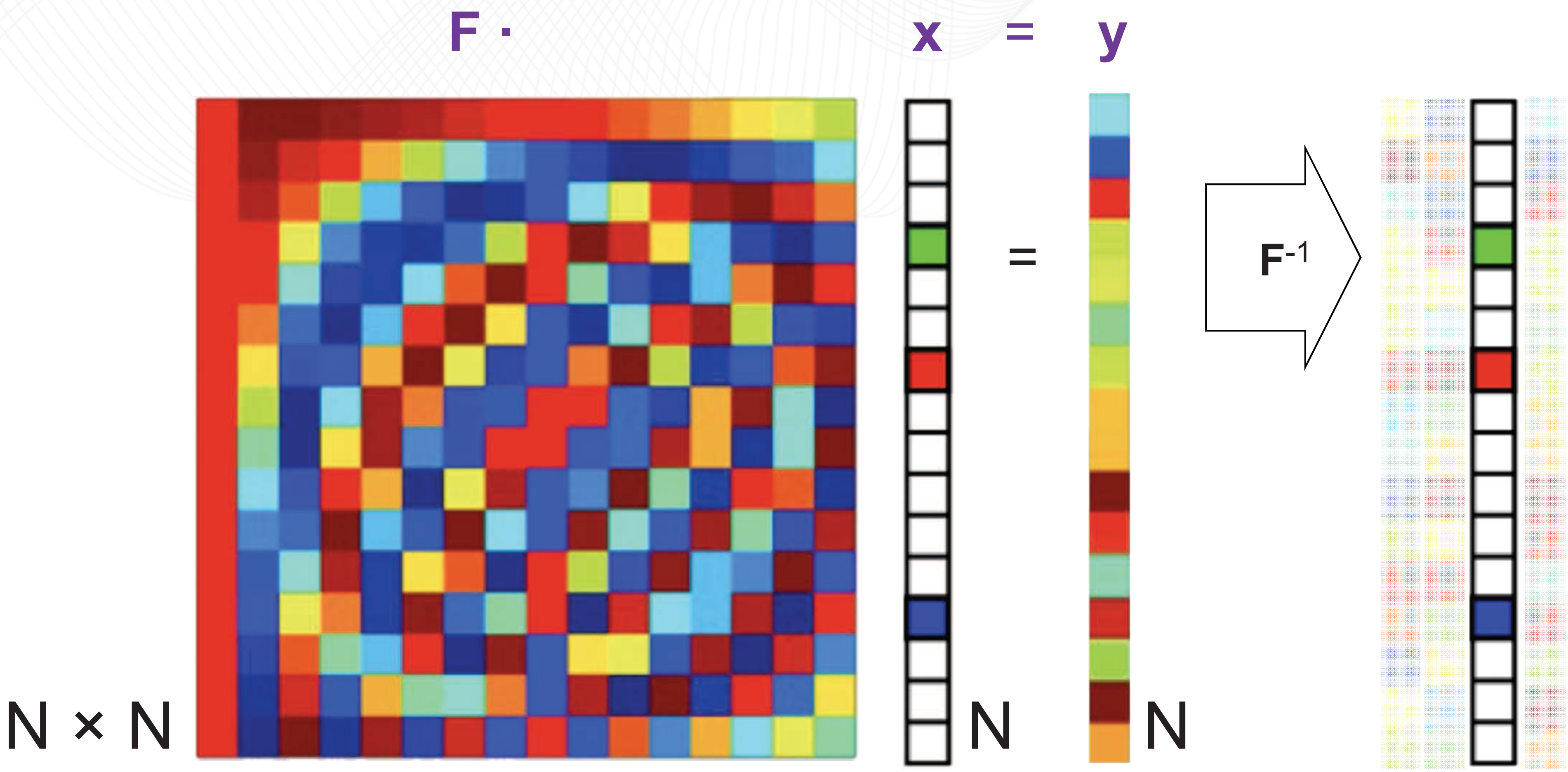
General Sub-Sampling Scheme



- **Φ:** Sensing matrix/ generalized sampling matrix
  (sub-sampling factor $q := M/N << 1$)
- OMP: Orthogonal Matching Pursuit for non-linear recovery

Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Generalized Nyquist Sampling in the Fourier Domain



- **F:** Discrete Fourier Transform (DFT) matrix ⇒ Fourier domain incoherent to time domain

Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Idea

- Sub-sampling in Fourier domain by selecting a sub-set of M << N rows of **F**
- CoSeRa 2015 paper considered selection of
  (a) individual rows
  (b) sub-blocks of B subsequent rows (c sub-blocks, M := c·B)

  H. Stahl, J. Mietzner, and R.F.H. Fischer, "A sub-Nyquist radar system based on optimized sensing matrices derived via sparse rulers," in *Proc. Int. Workshop on Compressed Sensing Theory and its Appl. to Radar, Sonar, and Remote Sensing (CoSeRa'15)*, Pisa, Italy, June 2015

Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Idea

- Sub-sampling in Fourier domain by selecting a sub-set of $M << N$ rows of $\mathbf{F}$
- CoSeRa 2015 paper considered selection of
  (a) individual rows
  (b) sub-blocks of B subsequent rows (c sub-blocks, $M := c \cdot B$)

  H. Stahl, J. Mietzner, and R.F.H. Fischer, "A sub-Nyquist radar system based on optimized sensing matrices derived via sparse rulers," in *Proc. Int. Workshop on Compressed Sensing Theory and its Appl. to Radar, Sonar, and Remote Sensing (CoSeRa'15)*, Pisa, Italy, June 2015

- In practical system sub-sampling would be conducted in analog domain
- Option (b) seems more attractive for hardware implementation, since it can be realized with few bandpass filters (BPFs), e.g. $c < 10$

Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Idea

- Sub-sampling in Fourier domain by selecting a sub-set of M << N rows of **F**
- CoSeRa 2015 paper considered selection of
  (a) individual rows
  (b) sub-blocks of B subsequent rows (c sub-blocks, M := c·B)

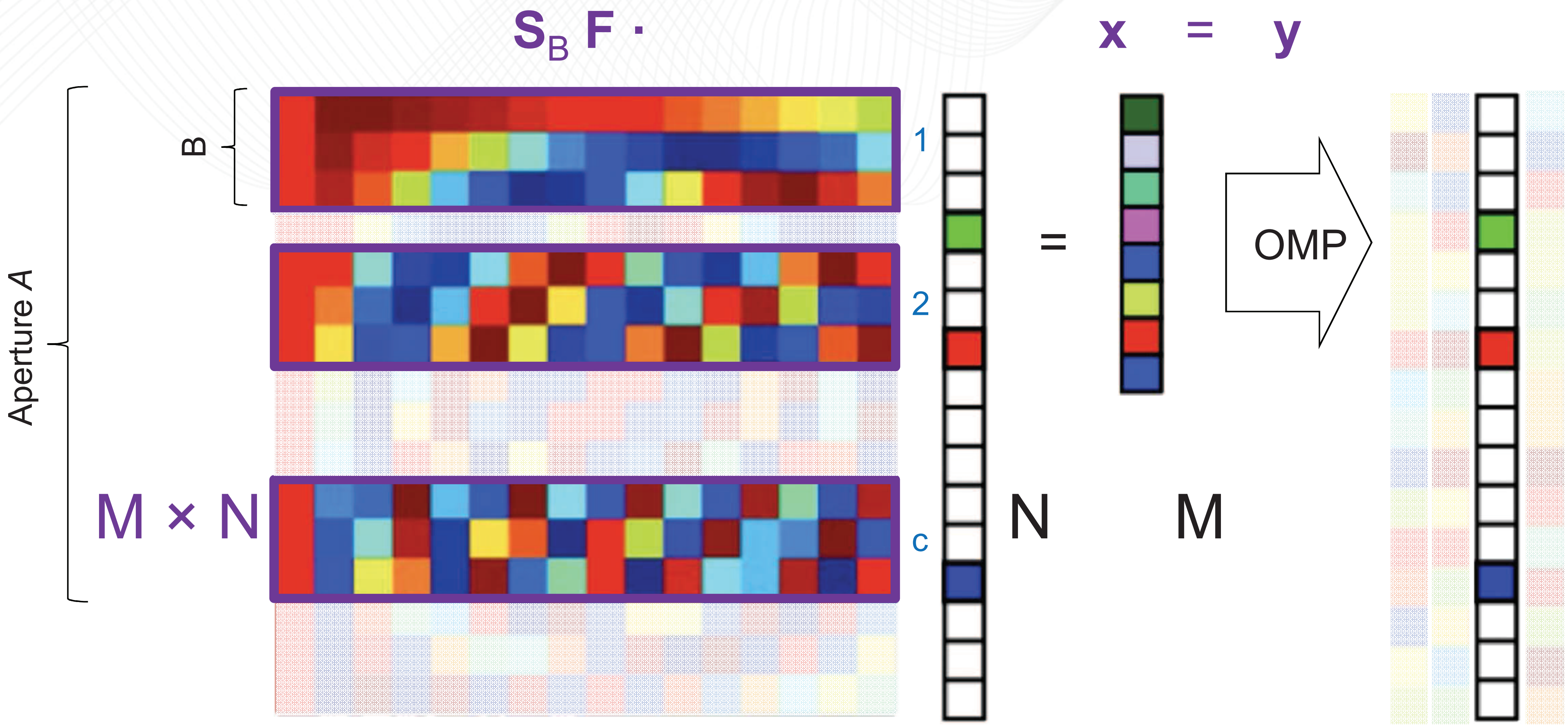
  H. Stahl, J. Mietzner, and R.F.H. Fischer, “A sub-Nyquist radar system based on optimized sensing matrices derived via sparse rulers,” in *Proc. Int. Workshop on Compressed Sensing Theory and its Appl. to Radar, Sonar, and Remote Sensing (CoSeRa’15)*, Pisa, Italy, June 2015

- In practical system sub-sampling would be conducted in analog domain
- Option (b) seems more attractive for hardware implementation, since it can be realized with few bandpass filters (BPFs), e.g. c < 10

- Care must be taken concerning the spacing of the BPFs in frequency domain
- It turns out that the overall bandwidth extent (*aperture A*) has significant impact

Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Idea (cont.)



- $\mathbf{S}_B$: Block selection matrix (M x N), $\Phi := \mathbf{S}_B\,\mathbf{F}$

Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Idea (cont.)

- For good recovery performance, spacing of BPFs should be ‚irregular'
- Baranski et al. proposed corresponding scheme with c=4 BPFs; spacing optimized heuristically

  E. Baransky, G. Itzhak, N. Wagner, I. Shmuel, E. Shoshan, and Y. Eldar, "Sub-Nyquist radar prototype: Hardware and algorithm," *IEEE Trans. On Aerosp. and Electr. Systems*, vol. 50, no. 2, pp. 809–822, Apr. 2014.

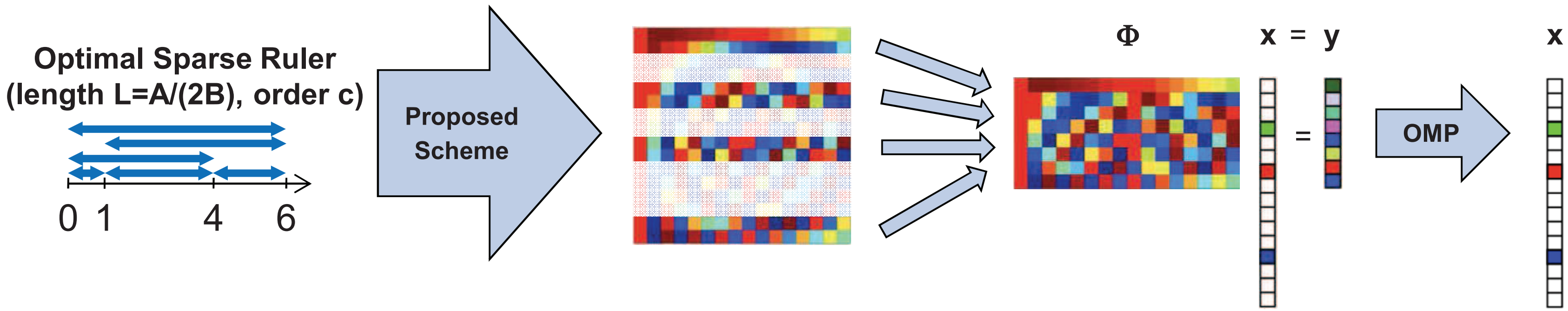
Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Idea (cont.)

- For good recovery performance, spacing of BPFs should be ‚irregular'
- Baranski et al. proposed corresponding scheme with c=4 BPFs; spacing optimized heuristically

  E. Baransky, G. Itzhak, N. Wagner, I. Shmuel, E. Shoshan, and Y. Eldar, "Sub-Nyquist radar prototype: Hardware and algorithm," *IEEE Trans. On Aerosp. and Electr. Systems*, vol. 50, no. 2, pp. 809–822, Apr. 2014.

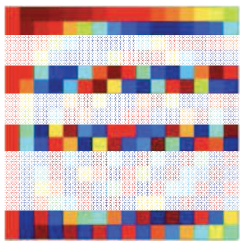
- In CoSeRa 2015 we showed:
  - Placing BPFs according to marks of optimal sparse ruler (OSR) offers good recovery performance for *arbitrary* sub-sampling q = M/N and numbers of BPFs c
  - Superior to random placement of the BPFs

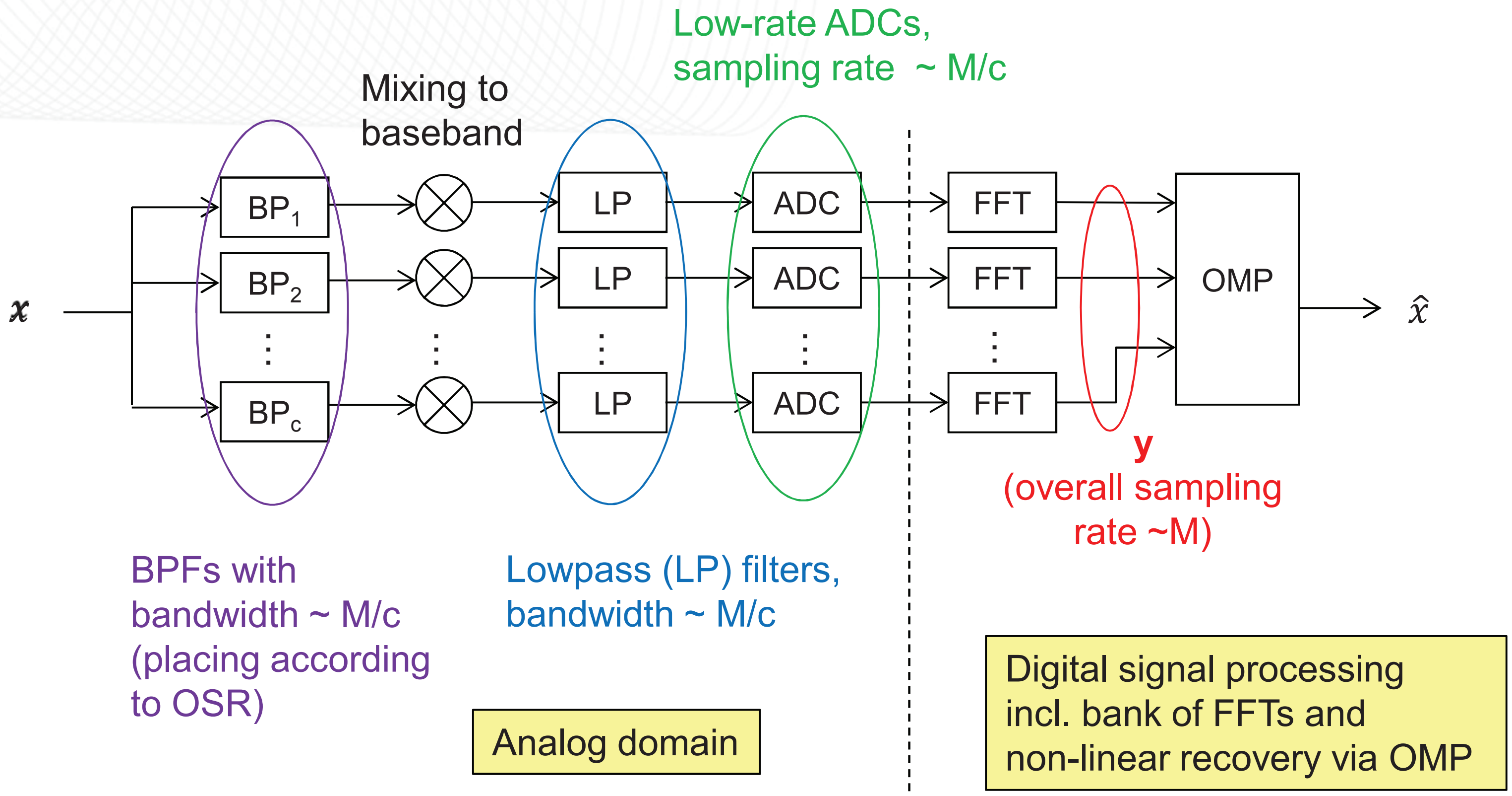


Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Practical Realization using Bandpass Filters

- Analog domain: Sampling via c parallel BPFs





Outline

- Sub-Nyquist Radar Scheme with Generalized Sampling in Fourier domain
  - Optimized Design based on Sparse Rulers
  - Practical Realization using Bandpass Filters
- Performance Analysis based on Simulation Results
  - Perfect Support Reconstruction
  - Relaxed Support Reconstruction
- Application to Real-World Radar Data
  - Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar
  - Reconstruction Results
- Conclusions and Future Work

Performance Analysis based on Simulation Results

Perfect vs. Relaxed Support Reconstruction

- Task: Recover delays (ranges) $t_k \approx n_k \cdot \Delta$ of K point targets based on measurement equation $\mathbf{y} = \Phi \cdot \mathbf{x} = \mathbf{S}_B \cdot \mathbf{F} \cdot \mathbf{x}$

  $\Rightarrow$ Support reconstruction problem (as **x** is approximately sparse)
  (Δ: grid-size that would result from Nyquist sampling, n_k integer = range cell index)

Performance Analysis based on Simulation Results

Perfect vs. Relaxed Support Reconstruction

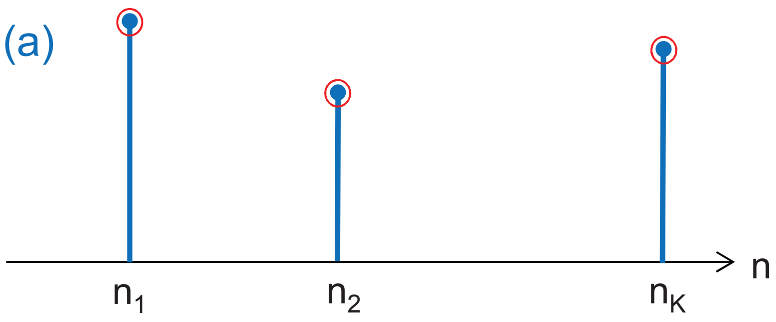
- Task: Recover delays (ranges) $t_k \approx n_k \cdot \Delta$ of K point targets based on measurement equation $\mathbf{y} = \Phi \cdot \mathbf{x} = \mathbf{S}_B \cdot \mathbf{F} \cdot \mathbf{x}$

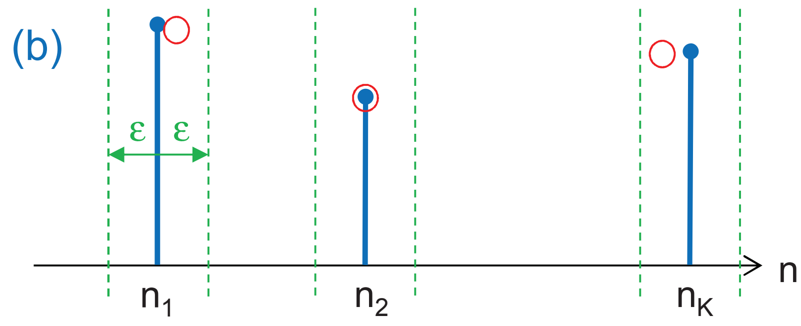
  $\Rightarrow$ Support reconstruction problem (as **x** is approximately sparse)
  (Δ: grid-size that would result from Nyquist sampling, n_k integer = range cell index)

- Two cases considered:

  (a) Perfect support reconstruction $\Rightarrow$ Detections of OMP must exactly fit the range cell indices n_k

  (b) Relaxed support reconstruction $\Rightarrow$ Detections of OMP may be within tolerance window $\{n_k - \varepsilon, \ldots, n_k + \varepsilon\}$
  (ε: denoted as *scope*)





OMP detections

Performance Analysis based on Simulation Results

Perfect Support Reconstruction

- Perfect support reconstruction: *Worst-case coherence* $\mu_{\mathbf{\Phi}}$ of measurement matrix $\mathbf{\Phi}$ known to be a good substitute for resulting probability of detection P_d
- Worst-case coherence is *maximum correlation* $\rho(n,n')$ between any two columns n, n' of $\mathbf{\Phi}$ ($n' \neq n$)
- For considered sub-sampling in Fourier domain it suffices to consider only correlations $\rho(n,1)$, n = 2, ..., N

Performance Analysis based on Simulation Results

Perfect Support Reconstruction

- Perfect support reconstruction: *Worst-case coherence* $\mu_{\boldsymbol{\Phi}}$ of measurement matrix $\boldsymbol{\Phi}$ known to be a good substitute for resulting probability of detection P_d
- Worst-case coherence is *maximum correlation* $\rho(n,n')$ between any two columns n, n' of $\boldsymbol{\Phi}$ ($n' \neq n$)
- For considered sub-sampling in Fourier domain it suffices to consider only correlations $\rho(n,1)$, n = 2, ..., N

- In CoSeRa 2015 we showed: Correlations $\rho(n,1)$ can easily be calculated via DFT of selection vector $\mathbf{s}_B$ (corresponding to matrix $\mathbf{S}_B$)

  $\Rightarrow$ Efficient calculation of $\mu_{\boldsymbol{\Phi}}$ possible, e.g. based on a FFT of $\mathbf{s}_B$

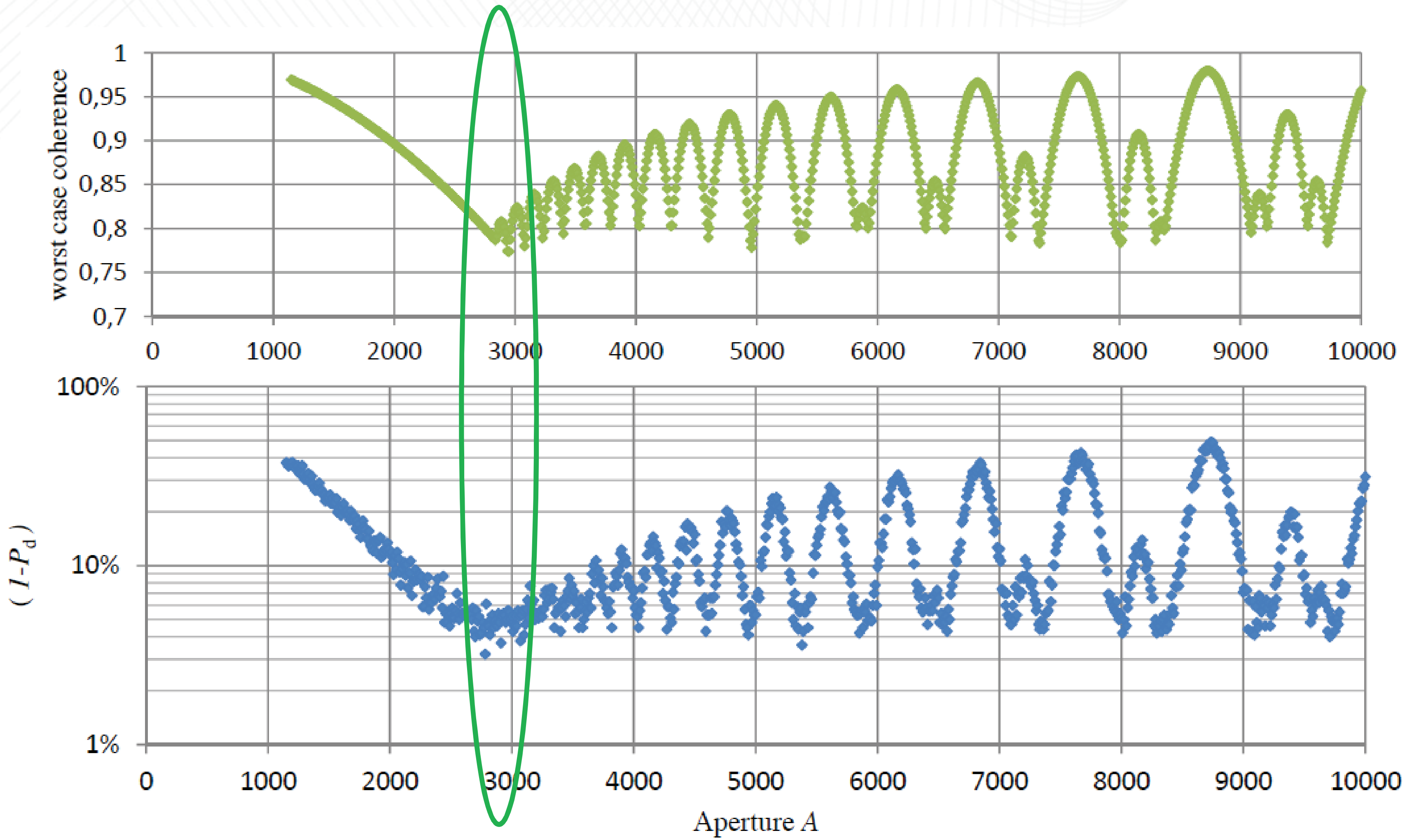
  $\Rightarrow$ Very useful e.g. for optimizing the aperture value A of the sub-Nyquist radar scheme (for given parameters N, M, c, B)

$$\rho_{\boldsymbol{\Phi}}(n,1) \propto ||[\mathbf{F} \cdot \mathbf{s}_B]_n|| \quad \rightarrow \mu_{\boldsymbol{\Phi}} = \max_{n \in \{2,\ldots,N\}} \rho_{\boldsymbol{\Phi}}(n,1)$$

Performance Analysis based on Simulation Results

Perfect Support Reconstruction

- Example: Optimization of aperture value A for c=4 BPFs spaced according to OSR



Sub-sampling factor q = 6.4 % (M = 640, N=10000)

c = 4 BPFs (OSR)

K = 6 point targets

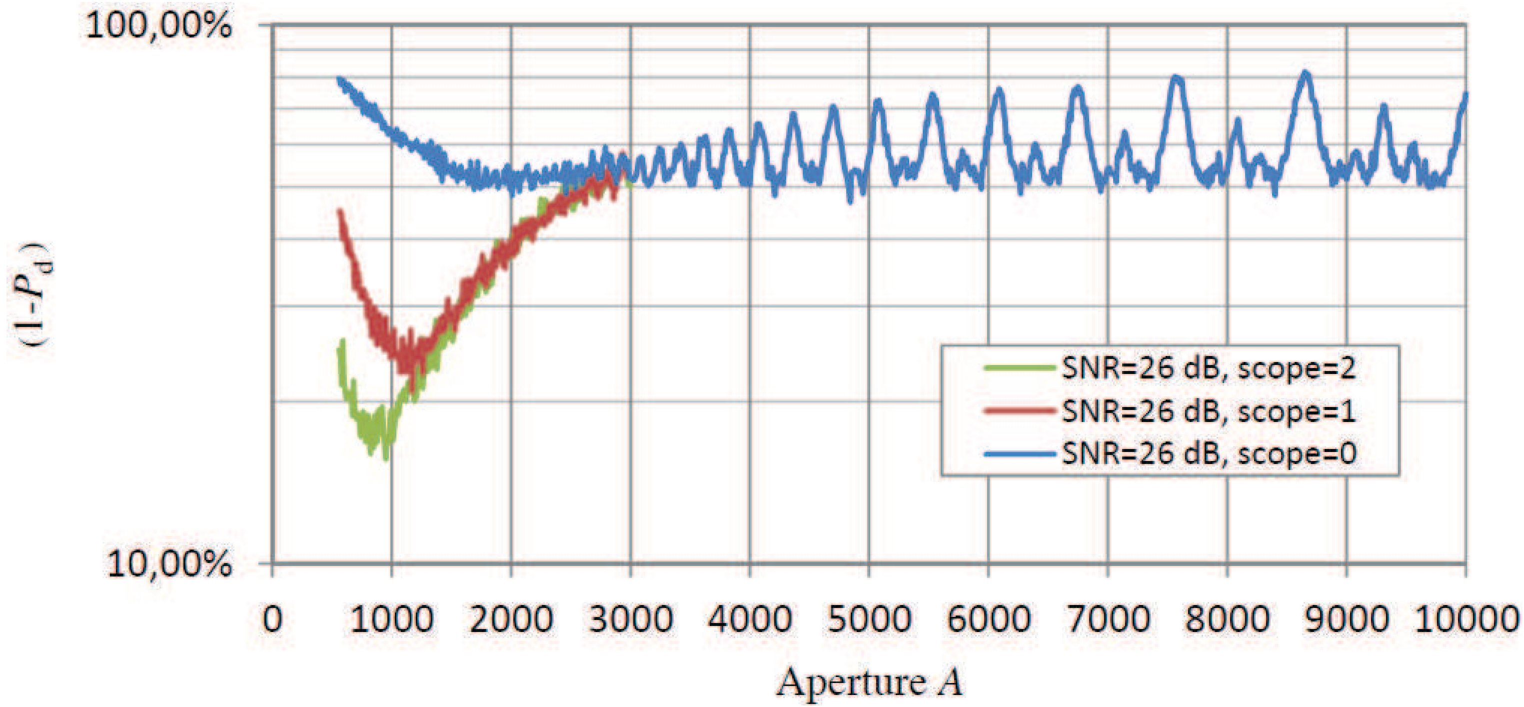
SNR = 26 dB

⇒ Excellent correspondence

Performance Analysis based on Simulation Results

Relaxed Support Reconstruction

- Relaxed support reconstruction ($\varepsilon > 0$): Strong correspondence between worst-case coherence μ_Φ and resulting detection performance does not hold anymore
- In fact, when increasing ε smaller aperture values A become optimal
- Reasonable, as we essentially allow for a coarser range resolution, which can generally be achieved by smaller overall signal bandwidth (corresponding to a smaller aperture value A)



Sub-sampling factor q = 3.2 % (M = 320, N=10000)

c = 4 BPFs (OSR)

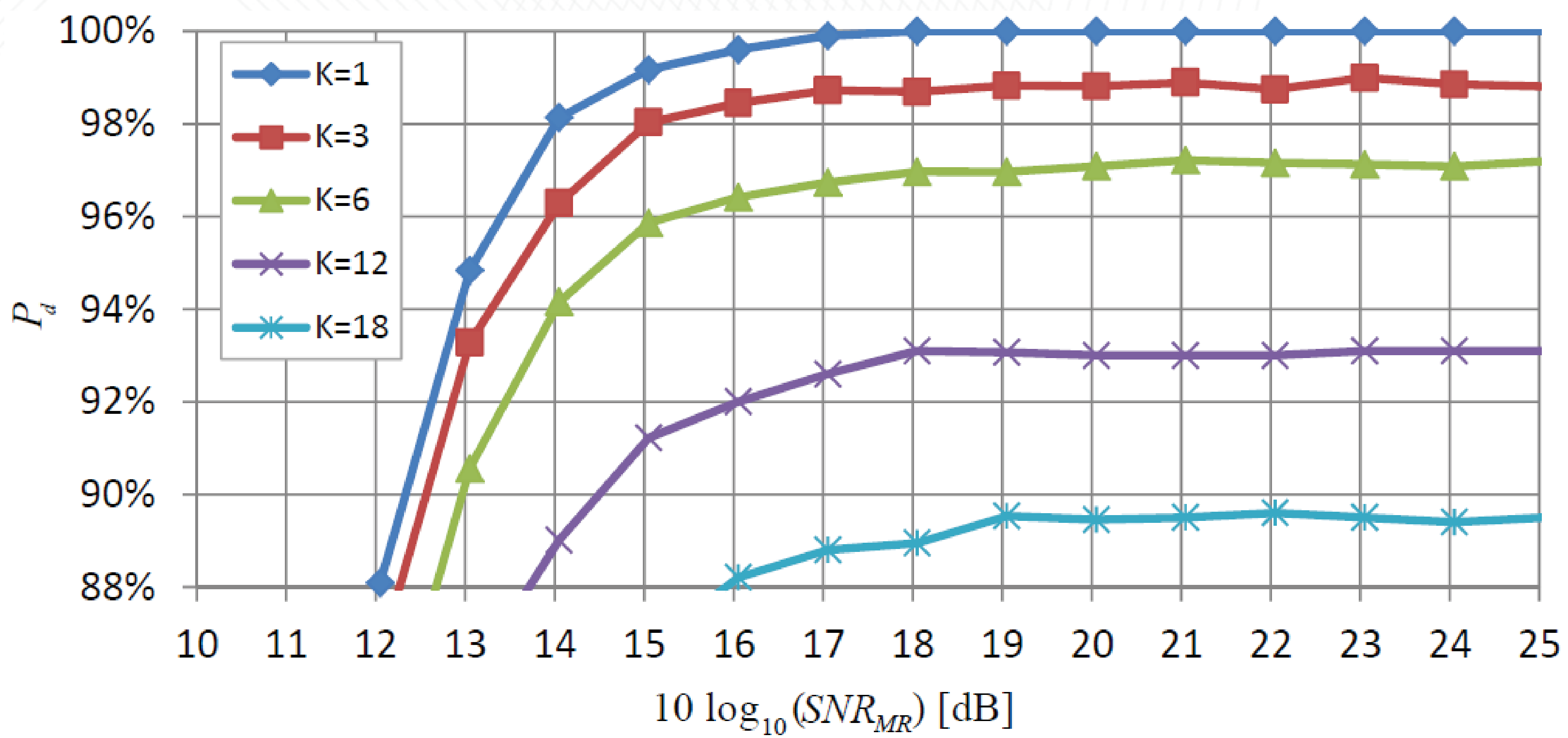
K = 6 point targets

SNR = 26 dB

Performance Analysis based on Simulation Results

Relaxed Support Reconstruction

- Simulation results for detection probability P_d for different numbers of target objects K (optimized aperture A)



Sub-sampling factor q = 3.2 % (M = 320, N=10000)

c = 4 BPFs (OSR)

Scope $\varepsilon = 1$

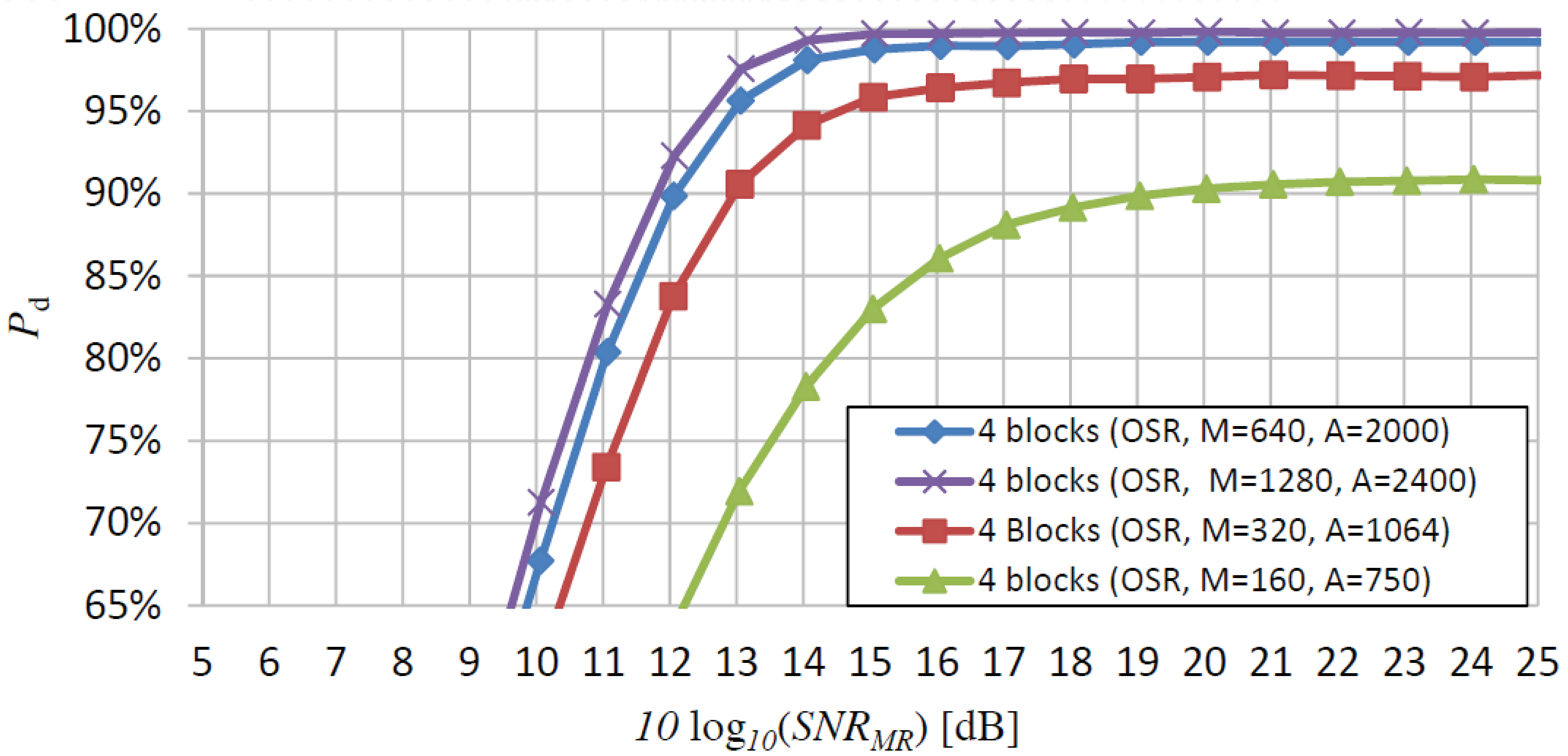
K = 1, 3, 6, 12, 18 point targets

$\Rightarrow$ Even for K=12 targets P_d-value of 93% achieved for sufficiently high SNRs

Performance Analysis based on Simulation Results

Relaxed Support Reconstruction

- Simulation results for detection probability P_d for different sub-sampling factors q (optimized apertures A)



Sub-sampling factor
q = 1.6%, 3.2 %, 6.4%, 12.8%
(N=10000)

c = 4 BPFs (OSR)

Scope ε = 1

K = 6 point targets

⇒ Results show well-known relation between parameters N, M, K (i.e., for good performance N >> M >> K)

Outline

- Sub-Nyquist Radar Scheme with Generalized Sampling in Fourier domain
  - Optimized Design based on Sparse Rulers
  - Practical Realization using Bandpass Filters
- Performance Analysis based on Simulation Results
  - Perfect Support Reconstruction
  - Relaxed Support Reconstruction
- Application to Real-World Radar Data
  - Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar
  - Reconstruction Results
- Conclusions and Future Work

Application to Real-World Radar Data

Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

- Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar
  - Purpose: Capture realistic clutter/ noise scenarios, off-grid effects & physically extended targets

Airbus EBS SmartRadar pod mounted under wing of a Learjet 35

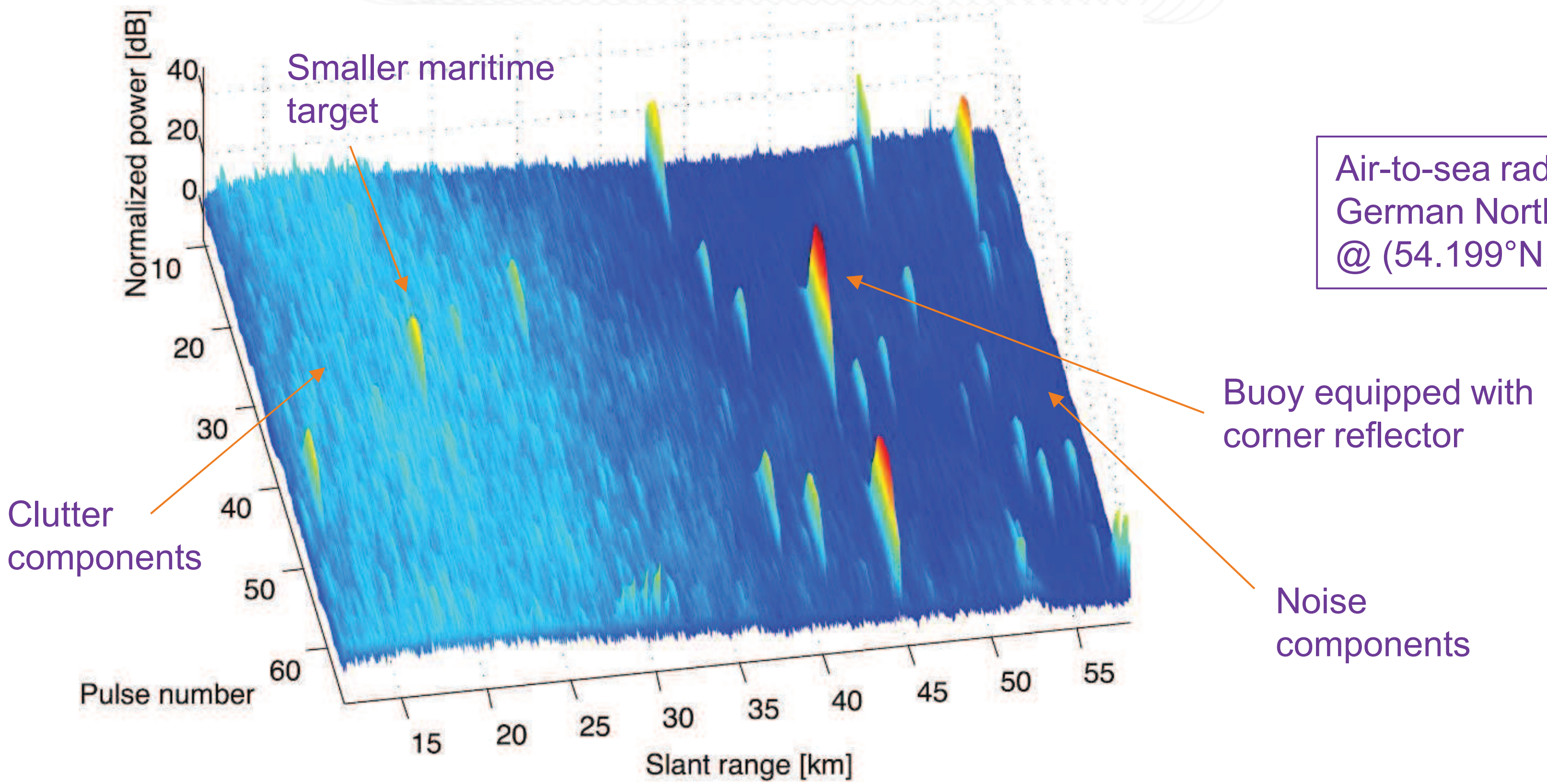
Pod includes radar sensor, AESA antenna, core electronics, on-board processing unit and data recorder

Supported by German MOD

Application to Real-World Radar Data

Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

- Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar
  - Processing done completely in digital domain by sub-sampling high-rate ADC data (retroactively)



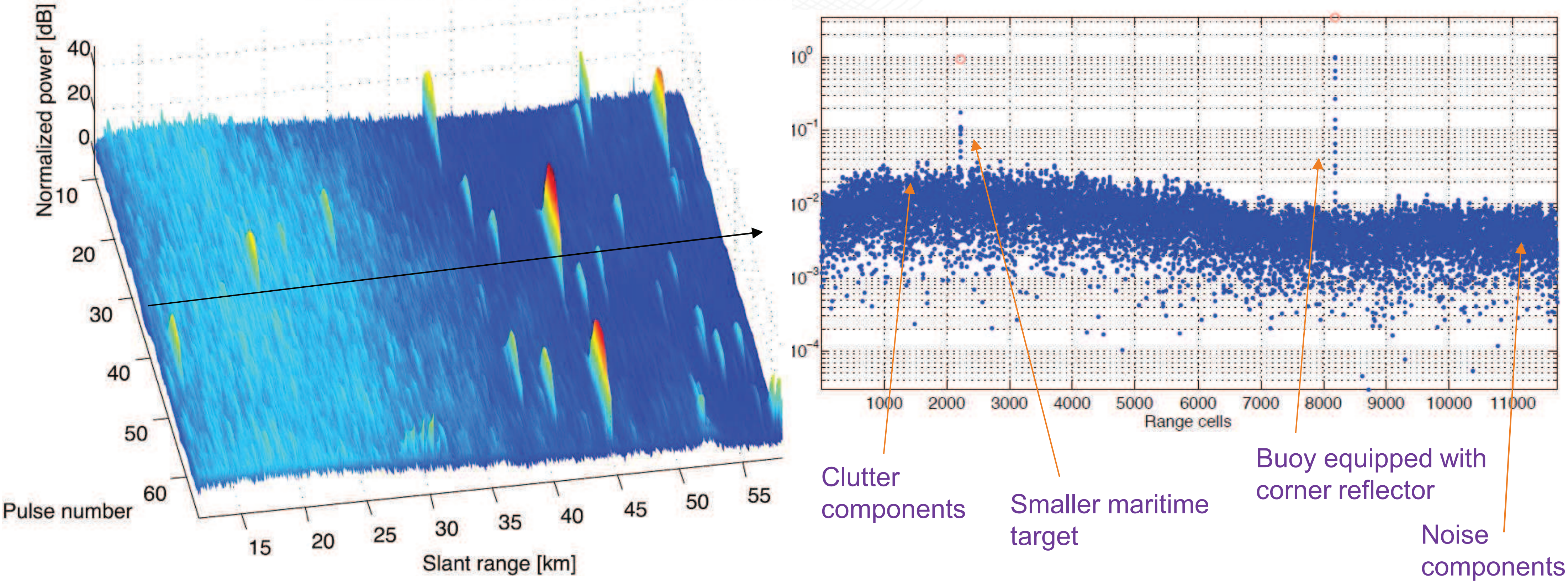
Air-to-sea radar data set recorded above German North Sea on July 4, 2014 @ (54.199°N, 6.757°E, altitude 1400 m)

Application to Real-World Radar Data

Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

- Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar

  → Exemplary reconstruction result (c = 4 BPFs, OSR; sub-sampling factor q = 2.7%)
  o: Target detections from OMP (K = 2 iterations)

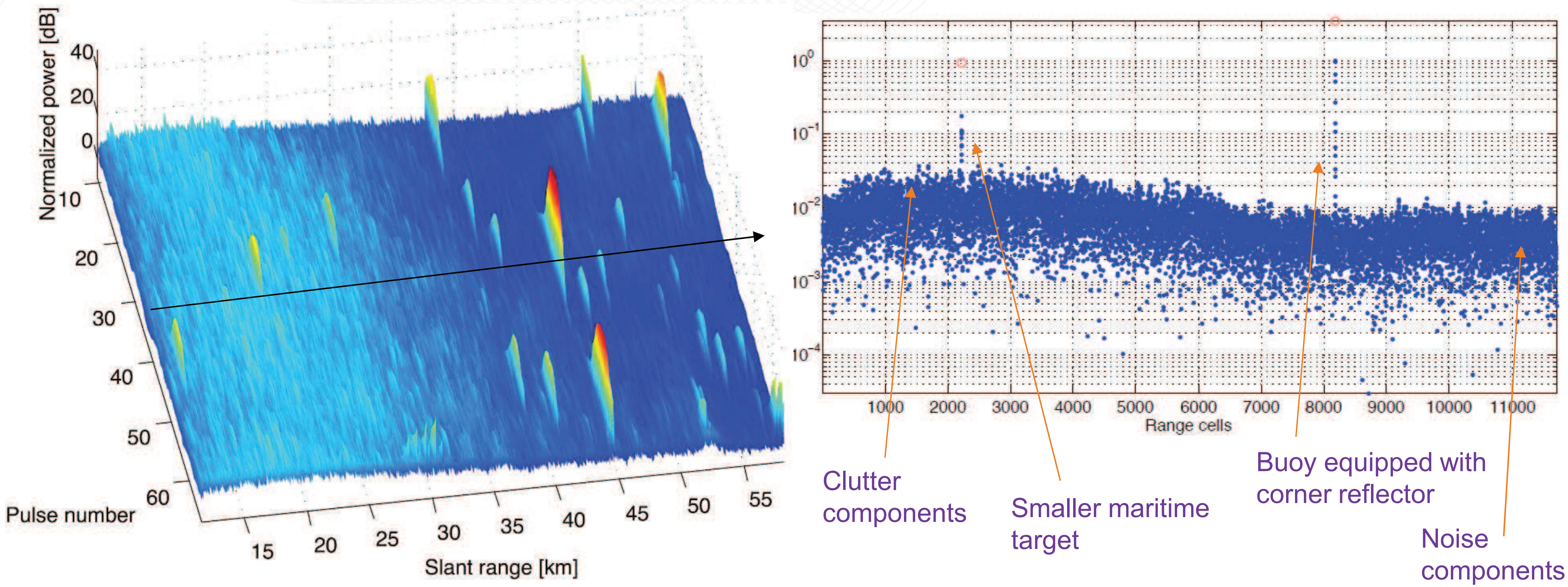


Application to Real-World Radar Data

Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

- Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar

  → Generally, reconstruction results were stable for sufficient SNR/ SCR and target object spacing



Conclusions and Future Work

Summary

- Investigated performance of a practicable sub-Nyquist radar system based on a set of parallel BPFs for extracting range information of sparse target objects
- Positions of BPFs in frequency domain optimized based on OSR approach
- Performance of OMP for non-linear recovery step
  - Perfect range recovery: Direct correspondence between worst-case coherence of measurement matrix and probability of missed detection; useful for optimizing aperture
  - Relaxed range recovery: Direct correspondence does not hold anymore; smaller aperture values tend to be optimal compared to perfect range recovery
- Simulation results: Sub-Nyquist radar system achieves detection probabilities > 90% for various numbers of target objects and sub-sampling factors, as long as SNR is sufficiently high (on the order of 12...20 dB)
- Sub-Nyquist radar scheme successfully applied to real air-to-sea radar data recorded with Airbus EBS SmartRadar pod system

Conclusions and Future Work

Topics for Future Work

- More detailed investigations concerning application of sub-Nyquist radar scheme to real radar data
- New methods for optimizing the aperture value in case of relaxed range recovery (so far done by means of computer search)
- Extension of sub-Nyquist radar scheme to (practically relevant) scenario where number of target objects is not known a-priori