Sparse signal separation and imaging in Synthetic Aperture Radar

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Joint work with

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Talk Outline

- Exploiting sparse and structured representations
  - Undersampling/compressed sensing
  - Blind Deconvolution
  - Signal Separation
- Applications of sparsity in SAR
  - Low Frequency SAR
  - Range Correction and Autofocus
  - Ground Moving Target Decomposition
- Data, Sparsity and Computation

Sparse Representations and Decompositions

Sparsity and Compressed Sensing

Signal Model:

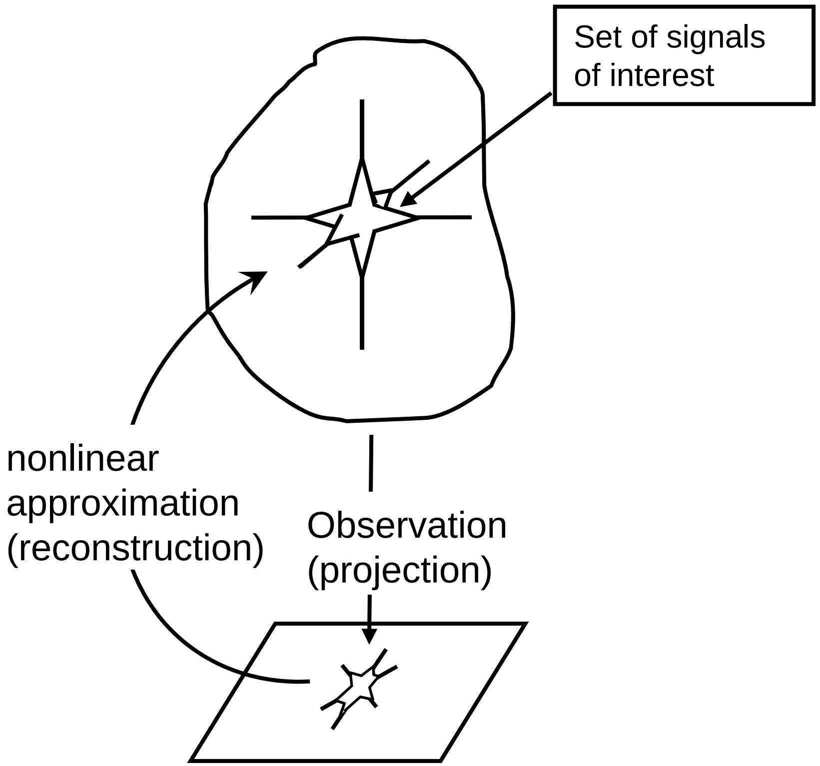
(Approximate) k-sparse signal model

Encoder:

Generalized sampling (typically random projection) that hopefully “preserves” information.

Decoder:

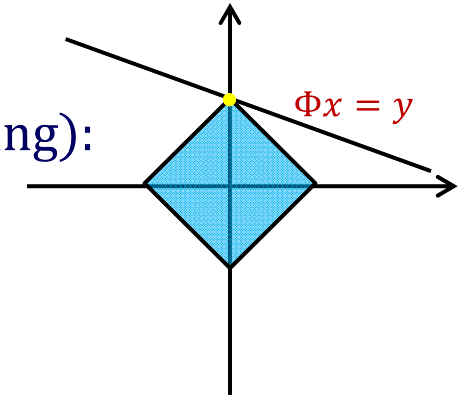
Nonlinear mapping to invert the linear projection on the signal set, e.g. L1, OMP, IHT, Message Passing, etc.



Generic CS

Generic reconstruction algorithm:

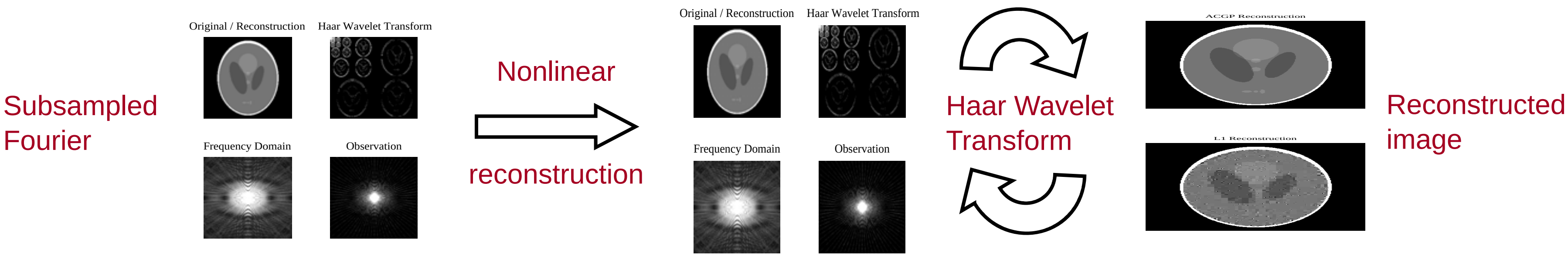
Relaxation: replace l_0 with l_1 (c.f. Iterative Soft Thresholding):



$$\hat{x} = \underset{x}{\operatorname{argmin}} \ \|x\|_1 \text{ subject to } \Phi x = y$$

Theorem: RIP $\Longrightarrow$ guaranteed sparse recovery

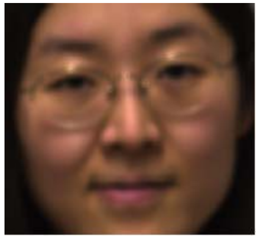
+ many others: IHT, OMP, CoSAMP, AMP, etc…



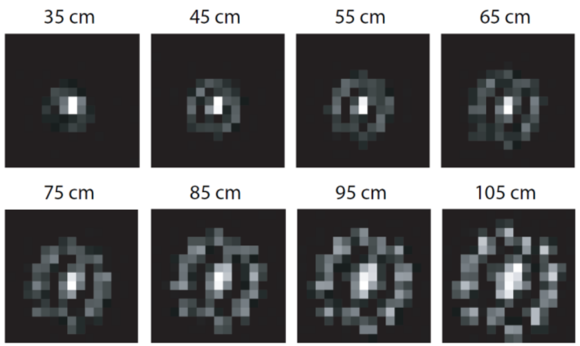
Sparsity based Deconvolution/Deblurring

[Levin et al. “Image and Depth from a Conventional Camera with a Coded Aperture” SIGGRAPH 07]

Sparsity based blind deconvolution/deblurring…



and depth through focusing



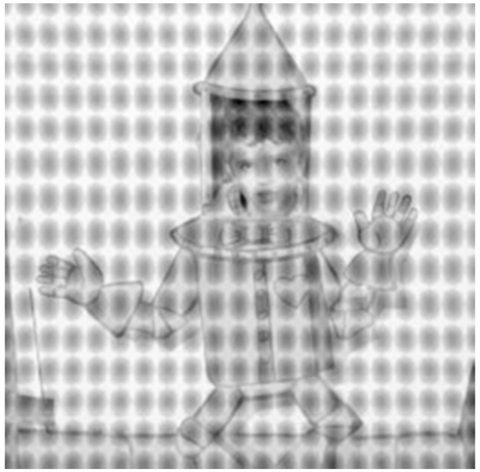
Blur kernels as a function of depth

Estimated depth map

Sparsity and Separating Decompositions

[J.-L. Starck, M. Elad, and D.L. Donoho, "Redundant Multiscale Transforms and their Application for Morphological Component Analysis", 2004]

Sparse representations can enable a meaningful decomposition through morphological differences…



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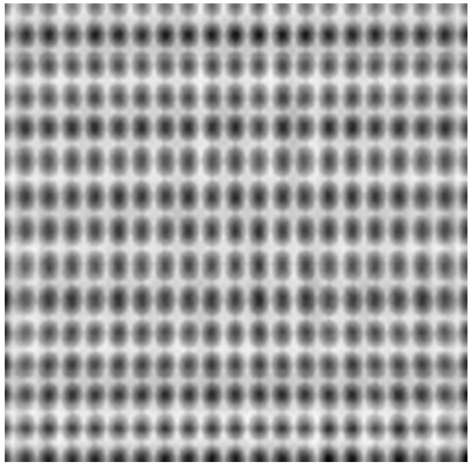


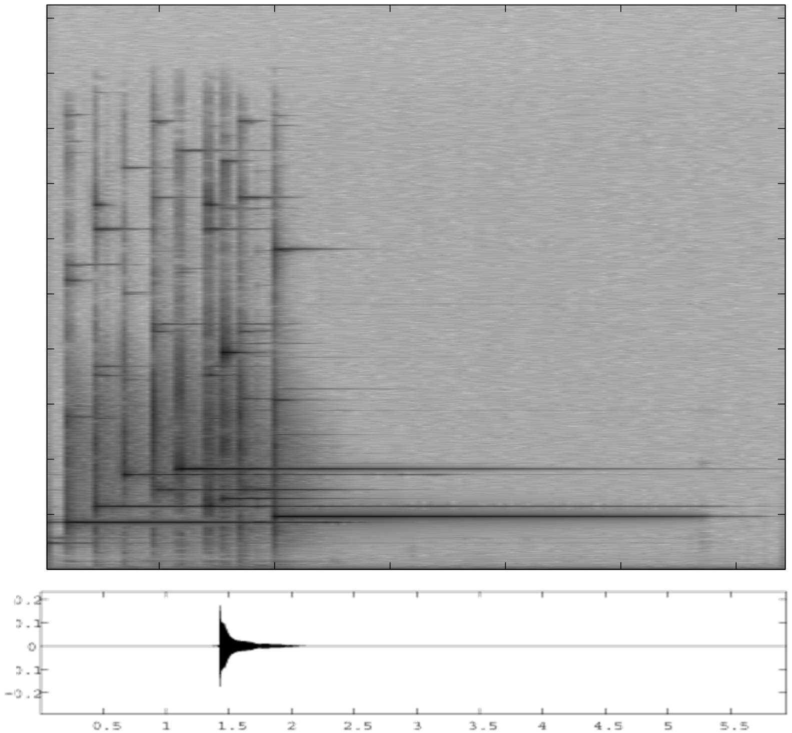
curvelets **Global DCT**

$$\{\hat{\alpha}_0, \hat{\alpha}_1\} = \underset{\alpha_0,\alpha_1}{\operatorname{argmin}} \; \|C\alpha_0\|_1 + \|D\alpha_1\|_1, \text{ s.t. } \alpha_0 + \alpha_1 = x$$

Sparsity and Separating Decompositions

[D. & Daudet "Sparse Audio Representations using the MCLT" 2006]

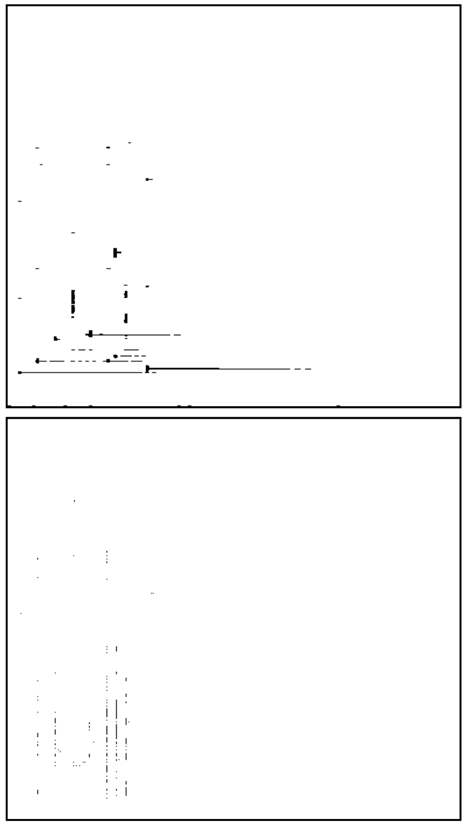
Sparse decomposition into Dual-resolution components, enables separation of individual notes



good frequency representation

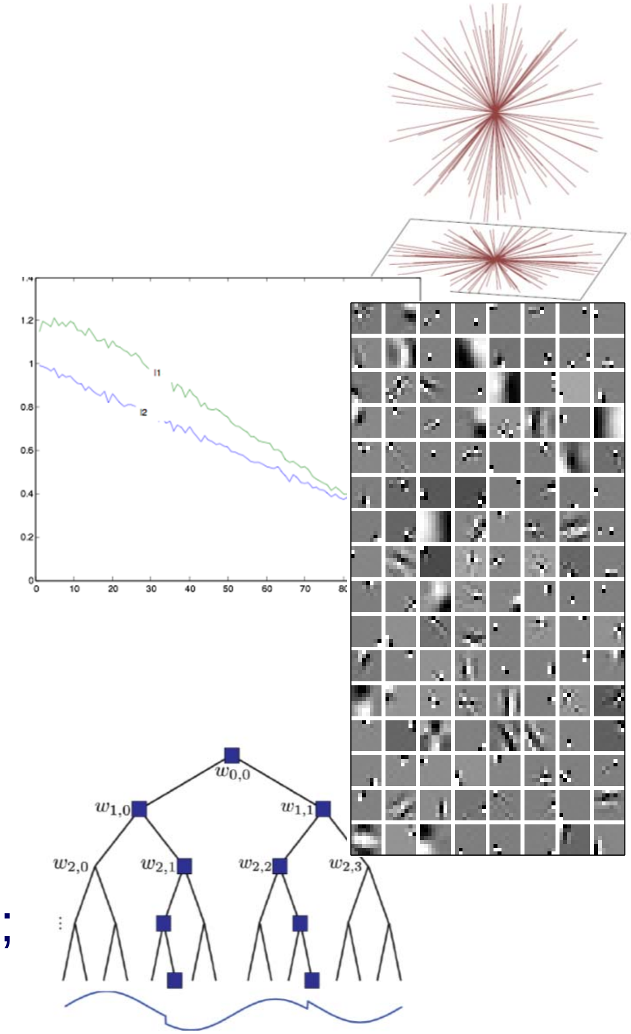
+

good time representation



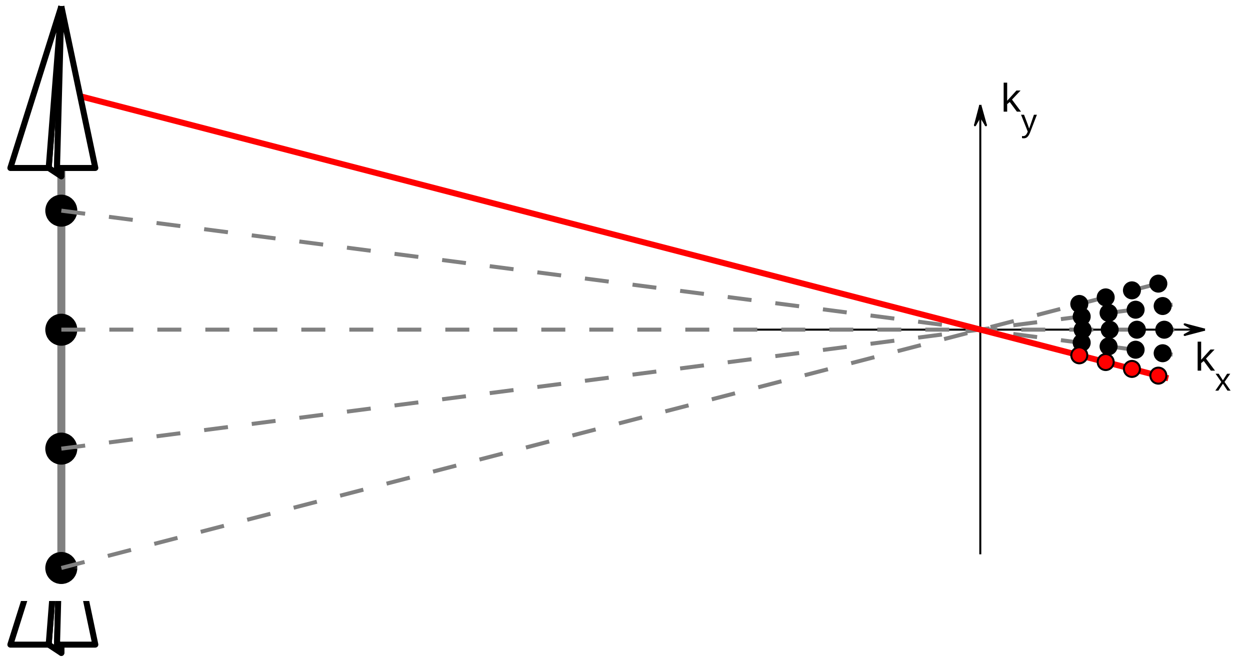
New directions & challenges in sparsity & CS

- Fundamental Statistical bounds;
- Better algorithms: structured, Bayesian, message passing;
- Data driven representations;
- Continuous/off-the grid models/multi-resolution imaging;
- Blind deconvolution/calibration;
- Sparse signal separation;
- Compressed detection/signal processing
- Hardware/computationally efficient solutions;



Applications in Synthetic Aperture Radar

SAR acquisition



SAR acquisition can be thought of as approximately sampling in k-space

SAR System model

Idealized dechirped SAR phase history model:

$$\mathbf{Y} = \Phi_F(\mathbf{X}) = \left\{ \sum_{k=1}^{K} \sum_{l=1}^{L} \mathbf{X}_{kl} \exp\left(\frac{-j4\pi f_m u_{kl}(\tau_n)}{c} \right) \right\}_{m,n}$$

where

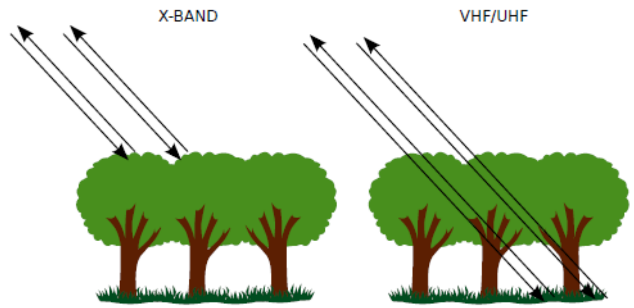
- $\mathbf{X}$ is the scene reflectivities (discretized);
- τ_n is the time of the nth pulse;
- $u_{kl}(\tau_n)$ is the distance from platform to target relative to scene centre (can incorporate velocities, DEM, etc.);
- f_m denotes the range frequencies, $m = 1, \dots, M$ associated with the dechirping process.

Airborne Low Frequency SAR

Low Freq. Synthetic Aperture Radar

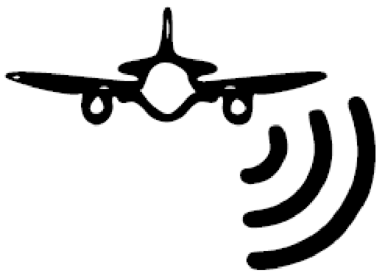
Why image with UHF/VHF?

- Foliage penetration (FoPEN) Radar
- Ground penetration Radar (GPR)

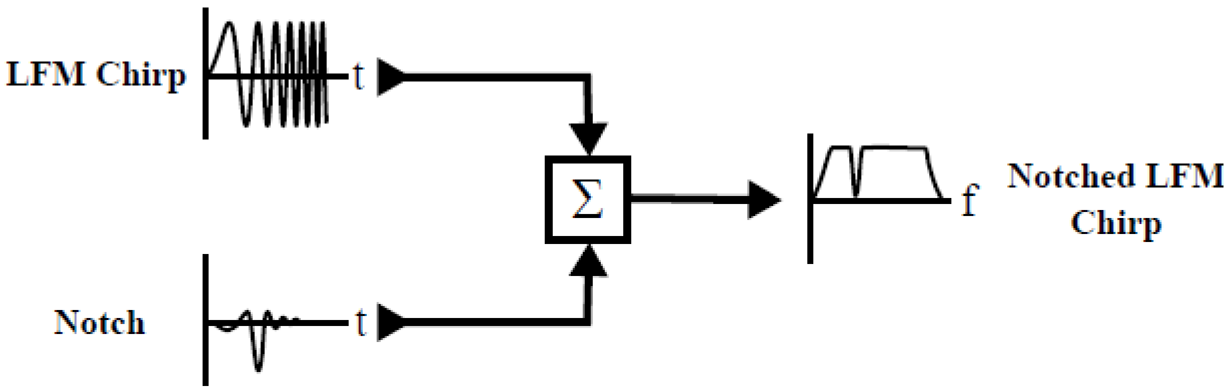


Major Issues:

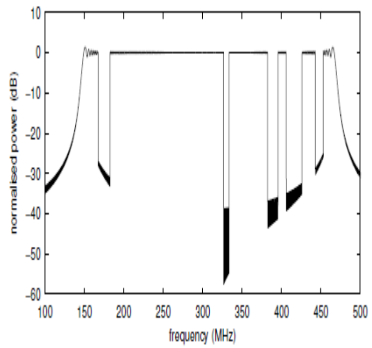
1. Interference between SAR systems and radio, television and communications systems.
2. Radio Frequency Interference (RFI)
3. Calibration/autofocus

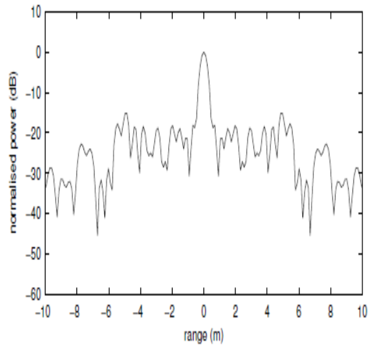


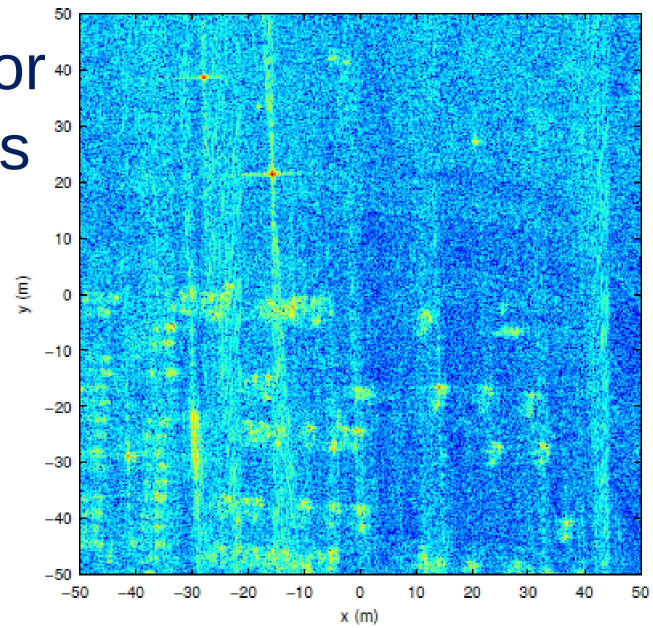
Notched LFM on Transmit



Traditional imaging techniques lead to poor image formation generating high sidelobes due to missing data







Standard Back projection

Sparsity in SAR images

Interaction of Reflectors in a Range Cell

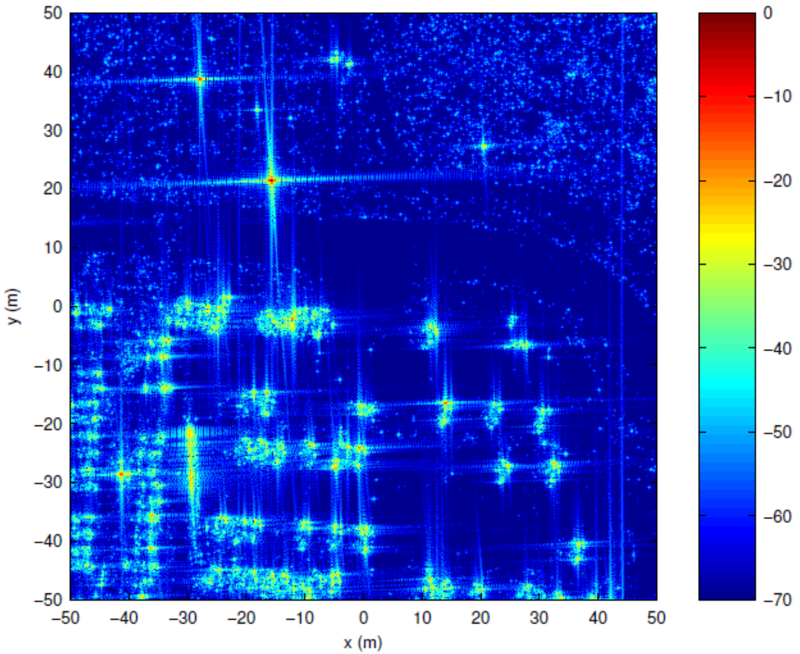
- Random interference: Speckle dominates images due to many random reflectors in a range cell - not compressible.
- Coherent interference: Coherent reflectors (often targets of interest) localized high intensity reflectors - compressible in spatial domain.

Compressed Sensing SAR Image Formation

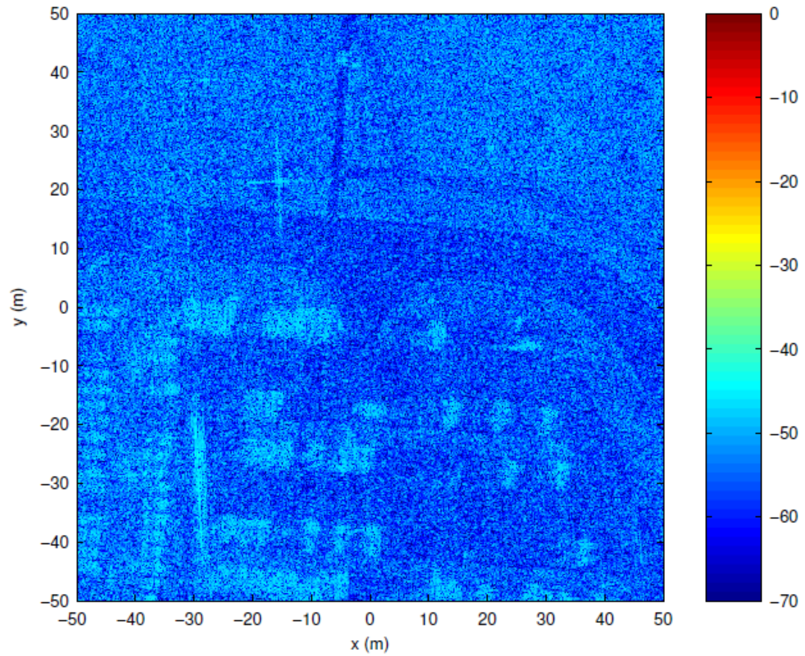
Decompose image into compressible and uncompressible components…

$$\hat{\mathbf{X}}_s = \underset{\mathbf{X}_s}{\operatorname{argmin}} ||\mathbf{X}_s||_1 \quad s.t.\ \|\mathbf{Y} - \Phi_F(\mathbf{X}_s)\|_F \le \epsilon$$

$$\hat{\mathbf{X}}_{bg} = \underset{\mathbf{X}_{bg}}{\operatorname{argmin}} \left\| \mathbf{Y} - \Phi_F\left(\hat{\mathbf{X}}_s + \mathbf{X}_{bg}\right) \right\|_F$$



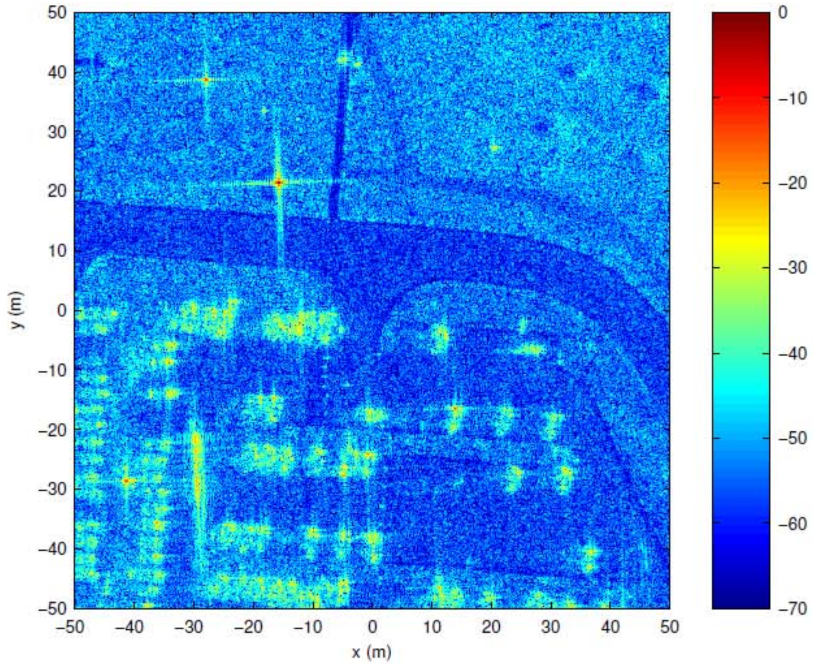
Bright Targets



Background Speckle

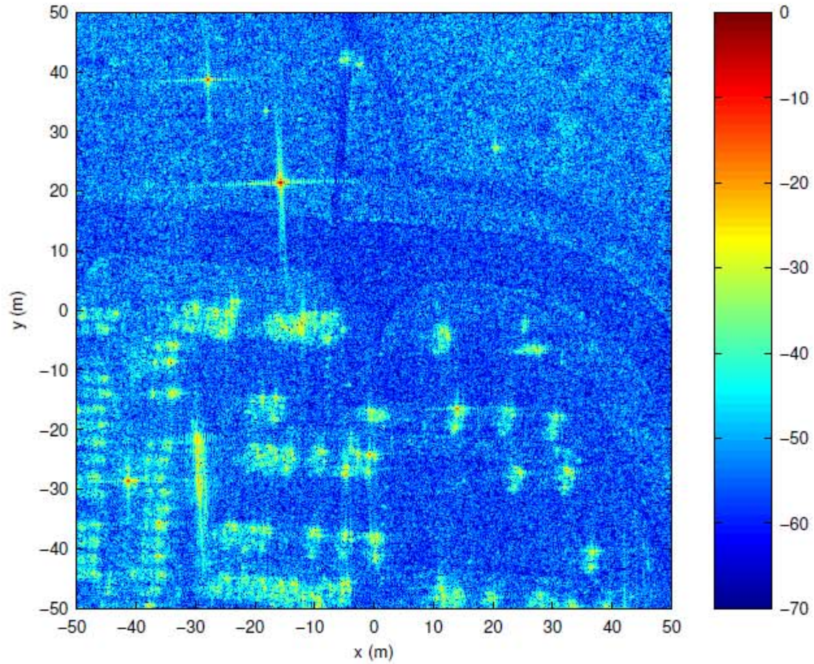
Compressed Sensing Image Formation

- Significant improvement in imaging of bright targets!
- Degradation in background speckle compared with fully sampled image



Fully Sampled Image Formation

Standard Image Formation

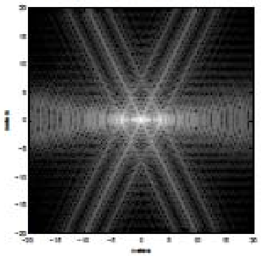


Combined CS Reconstructions

Radio Frequency Interference

Additional RFI can be detected in dead time between pulses and the RFI statistics estimated.

Filtered PSF

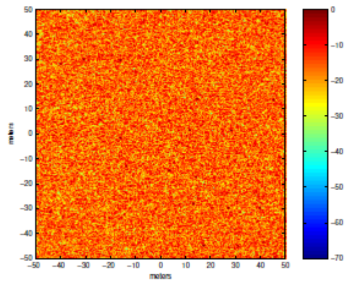


Traditional solution is to filter out radar returns

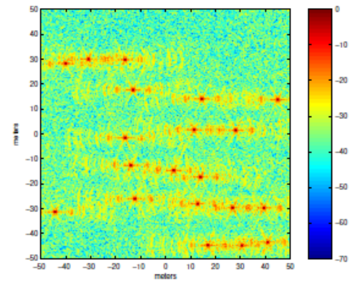
– introduces large sidelobes

Instead incorporate RFI suppression through weighted fidelity term:

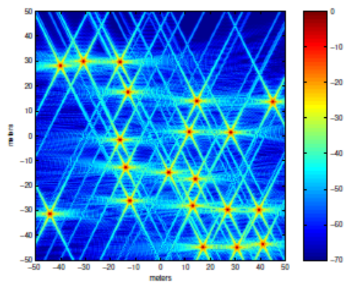
$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\operatorname{argmin}} \, ||\mathbf{X}||_1$$
$$s.t. \; ||\mathbf{Y} - \Phi_{\mathrm{F}}(\mathbf{X})||_{Q_N^{-1}} \le \epsilon$$



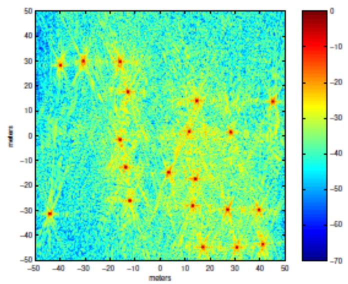
Filtered back-projection



Wiener filtered followed by filtered back-projection



RFI-aware sparse image formation



Range compression RFI-aware sparse image formation

where Q_N is dechirped/deskewed the RFI covariance matrix (approximated well using a diagonal matrix) [Kelly et al 2013]

Autofocus

Inaccuracies in propagation delay estimates introduce unknown phase errors, ϕ_{ϵ_n}. These defocus targets and degrade reconstruction.

For small delay errors, $\epsilon_n \leq \lambda/8c$ (equiv. range errors $\delta R_n \leq \lambda/16$):

$$\phi_{\epsilon_n} \approx \omega_0 \epsilon_n - \alpha \epsilon_n^2$$

where

- ϵ_n is the delay error at the *n*th transmit pulse;
- ω_0 is the carrier frequency;
- α is the chirp rate;

The adjusted model approximates as:

$$\mathbf{Y} = \Phi_F(\mathbf{X})\,\mathrm{diag}\{e^{j\phi}\}$$

Classical autofocus, e.g. Phase Gradient Autofocus, assumes far field, fully determined and separable - not appropriate for undersampled data.

Autofocus

For undersampled SAR a better solution is:

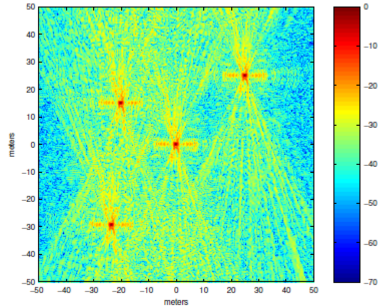
$$\underset{\mathbf{X},\mathbf{d}}{\text{minimise}} \ \|\mathbf{X}\|_1$$

$$\text{such that: } \|\mathbf{Y}\,\mathrm{diag}\{\mathbf{d}_n\} - \Phi_F(\mathbf{X})\|_F \leq \sigma$$

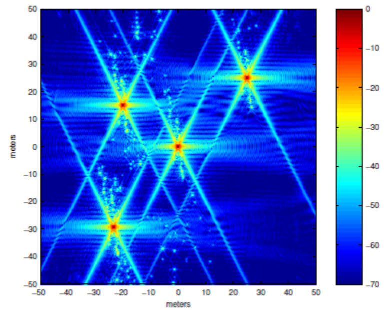
$$|\mathbf{d}_n| = 1, n = 1, \ldots, N$$

- Fast block-relaxation algorithms exist [Kelly et al 2012/14] (virtually no additional cost)
- No far field/ small aperture assumptions

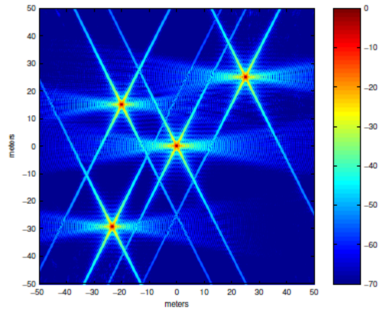
However no theoretical guarantees



Backprojection



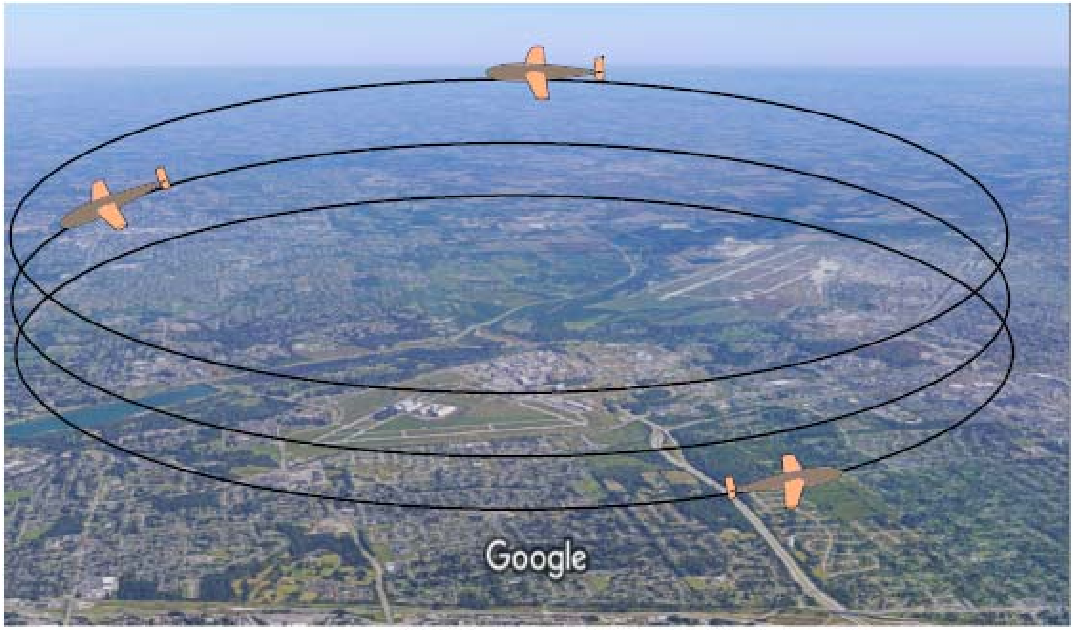
Sparse recon.



Sparse recon. + Autofocus

Multi-Pass Autofocus

Multi-pass Autofocus



Imagery ©2016 Google, Landsat, NOAA, Map data ©2016 Google 500 m

Multiple flight passes offer opportunity for limited elevation information:

- Interferometric SAR
- TomoSAR
- Compressive Volumetric SAR

However first need to be able to coherently combine different passes

Multi-pass Autofocus

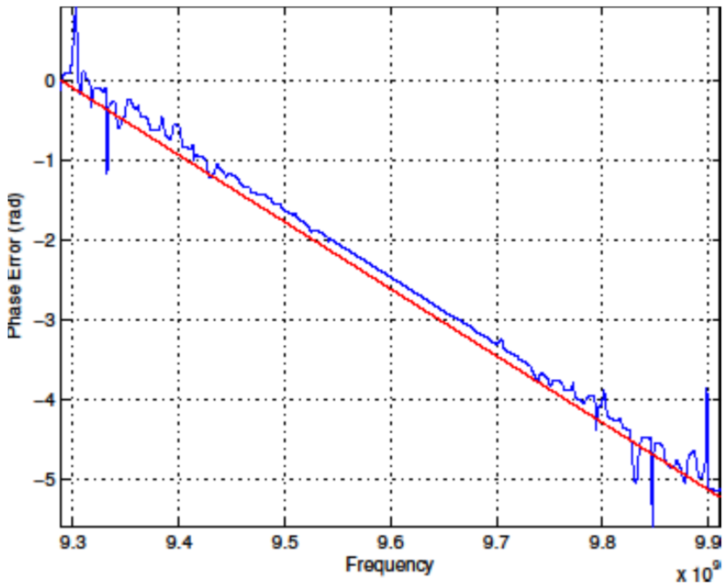
Coherently combining multipass data may result in relative delay errors that are too large for classical autofocus techniques: $\epsilon_n > \lambda/8c$

An improved approximation is:

$$\mathbf{Y}_{\mathrm{m,n}} = \Phi_F(\mathbf{X}) \times \exp\left\{-j\left(2\pi f_m \epsilon_n + \frac{\alpha\epsilon_n^2}{2}\right)\right\}$$

Note the extra phase term.

Phase error is now a linear function of range frequency, f_m.



Multi-pass Autofocus

Extended Autofocus Algorithm can be viewed as a Structured Phase Retrieval Problem. Proposed Algorithm, inspired from Gershberg-Saxton algorithm: alternate between enforcing sparsity and phase constrained data fidelity.

Iterate the following steps:

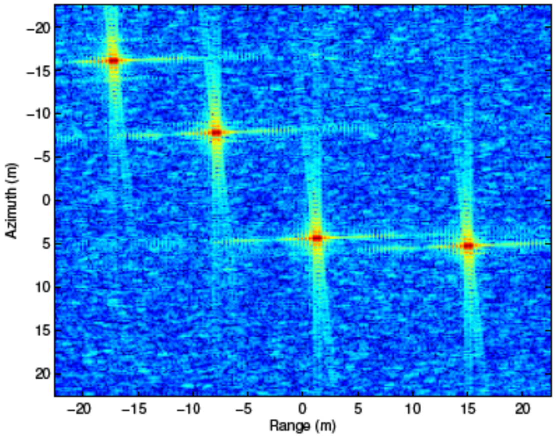
1. Element-wise Soft Thresholding: $\mathbf{X}^{[k]} = S_\lambda\left(\Phi_F^H(\Gamma_\epsilon^{[k]} \odot \mathbf{Y})\right)$
2. Estimate phase errors: $\epsilon^{[k+1]} = \underset{\epsilon}{\operatorname{argmin}} \left\| \Gamma_\epsilon \odot \mathbf{Y} - \mathbf{X}^{[k]} \right\|_F$

where

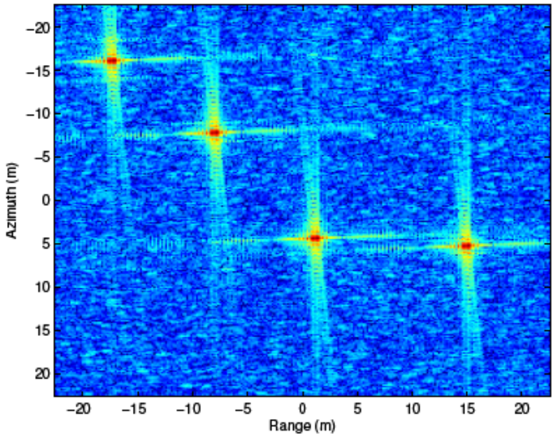
- ϵ is the vector of delay errors;
- Γ_ϵ encodes the phase errors as a linear function of range frequency;
- $\odot$ denotes elementwise multiplications

Multi-Pass Autofocus Algorithm

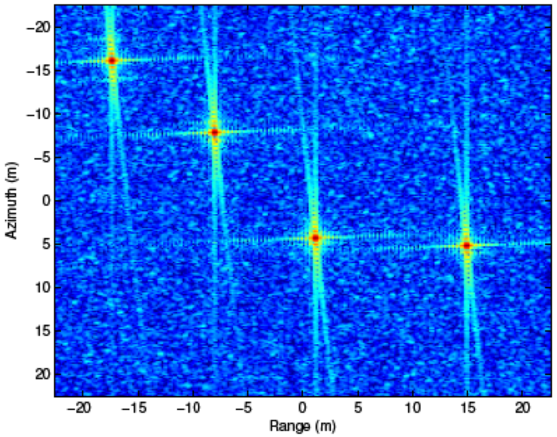
For 2 passes with a 20cm relative range error in X-band ($\lambda \approx 30cm$) Phase Gradient Autofocus (PGA) no longer works while proposed technique does.



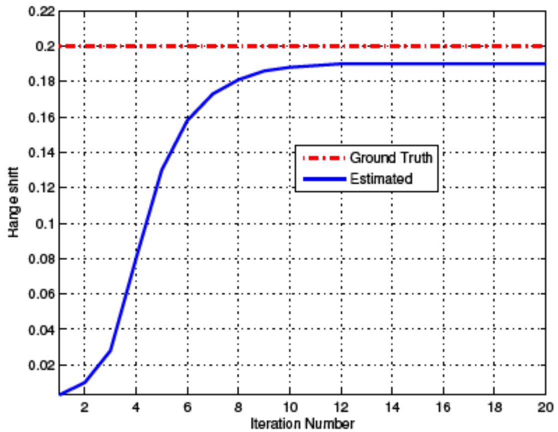
Original phase history



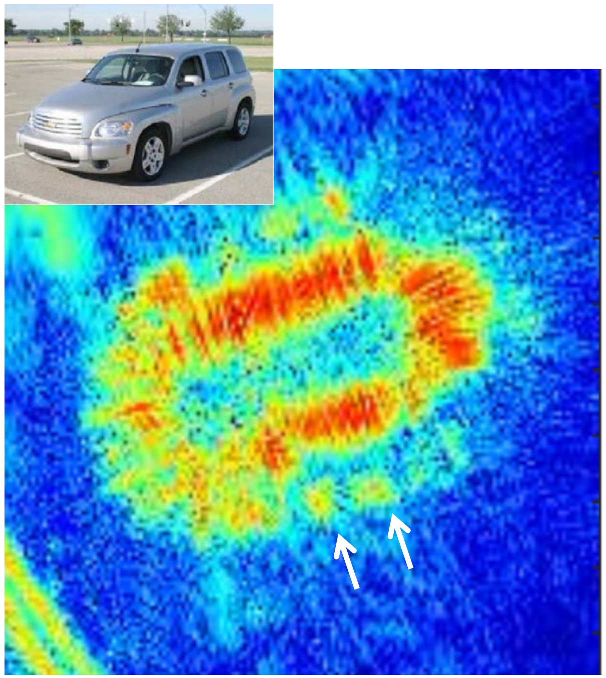
PGA corrected phase history



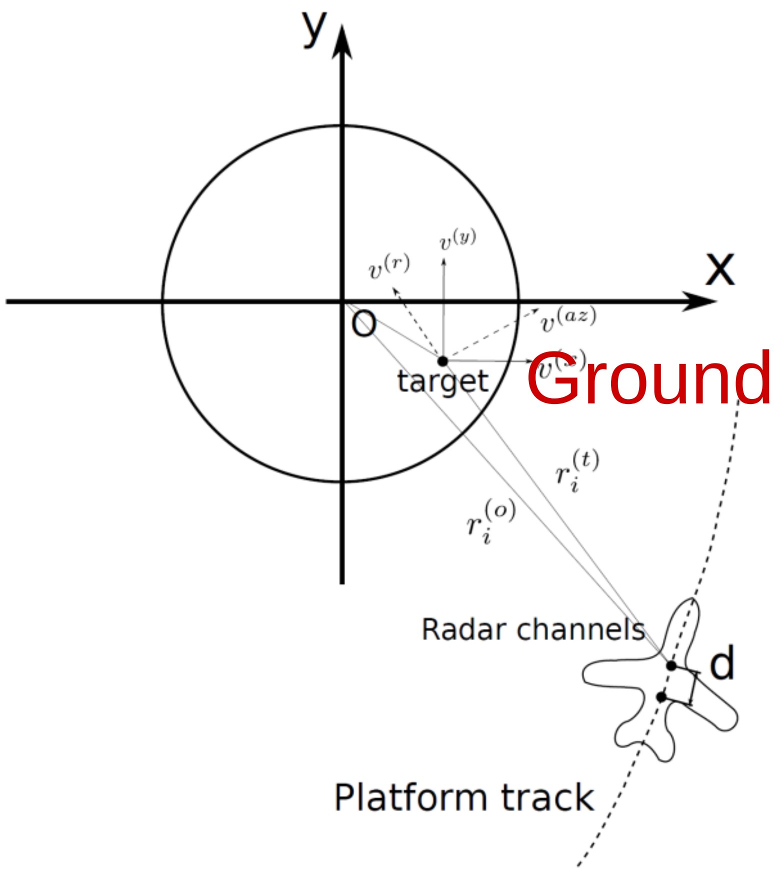
Proposed algorithm



Convergence



2-pass reconstruction of car (GOTCHA data) manages to resolve car pillars (arrows)



Ground Moving Target Decomposition

Exploiting Sparsity in SAR+GMTI

Conventional SAR imaging assumes a static scene.

Dynamic targets therefore appear displaced and defocussed.

Multiple channels can be used to identify moving targets using phase differences across the different channels (e.g. DPCA, ATI, etc.)

Sparsity based Dynamic Imaging

1. Image Formation with full dynamic-static decomposition using sparsity:

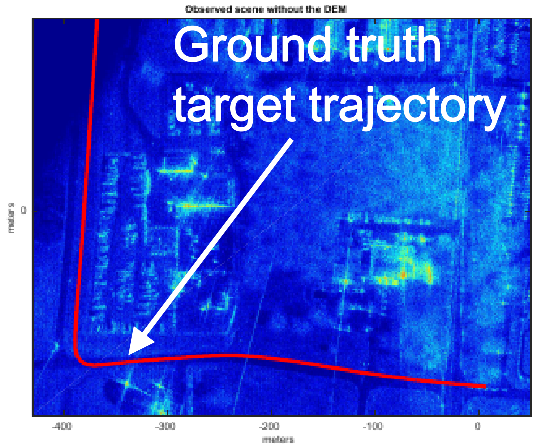
$$\mathbf{X} = \mathbf{X_s} + \mathbf{X_d}$$

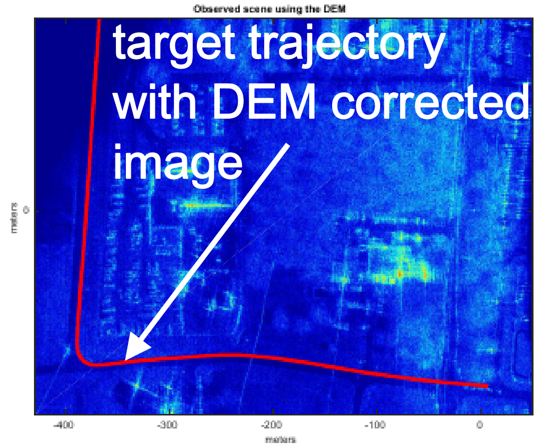
(without displacement correction or refocussing)

2. Full target velocity estimation through sparsity-based refocussing

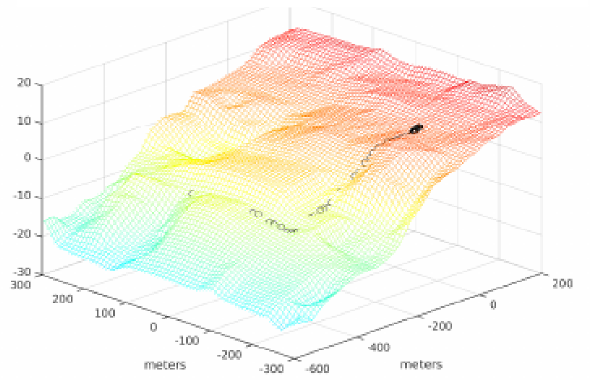
Including Elevation Information

First we need to do some pre-processing... Unfortunately there is no digital elevation map (we got ours from US Geological Survey)





Digital Elevation Map



Exploiting Sparsity in SAR+GMTI

Step one: Image Formation with Dynamic-Static Decomposition:

Proposed model:

$$\underset{\mathbf{X}_s,\mathbf{X}_d,\mathbf{P}}{\text{minimise}} \;\; \frac{1}{2}\sum_i \left\|\widetilde{\mathbf{Y}}_i - \Phi_F^0\left(\mathbf{X}_s + \mathbf{X}_d \odot \mathbf{P}^{i-1}\right)\right\|_F^2$$

$$\text{such that:} \quad \|\mathbf{X}_d\|_0 \le s \qquad (1)$$

$$|\mathbf{P}_{kl}| = 1, k = 1, \dots, K, l = 1, \dots, L$$

$$\text{supp}(\mathbf{X}_d) = \text{supp}(\mathbf{P} - 1)$$

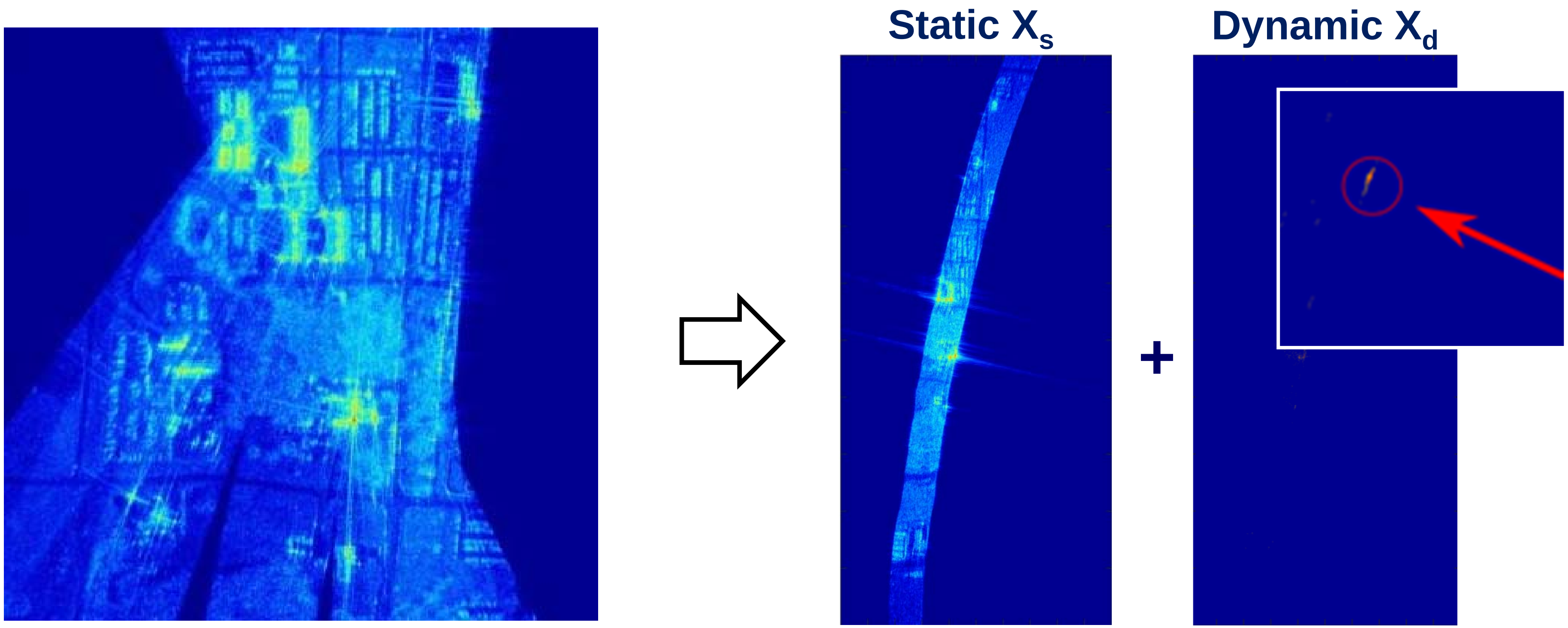
$\widetilde{\mathbf{Y}}_i$ is the balanced phase history for the ith channel;

$\mathbf{X}_s$ and $\mathbf{X}_d$ are static and dynamic (without displacement correction) components,

$\mathbf{P}$ is the phase correction matrix for the dynamic component $\mathbf{X}_d$ and Φ_F^0 is the forward model assuming zero velocity

Exploiting Sparsity in SAR+GMTI

"Morphological Decomposition" into static and dynamic components (using small sub-apertures)…



Radial velocity and target displacement can now be estimated from phase correction matrix, **P**.

Exploiting Sparsity in SAR+GMTI

Step 2: Full target velocity estimation using sparsity-based refocussing

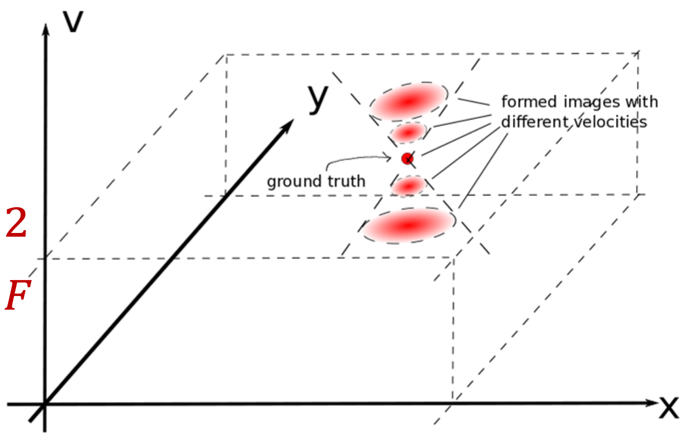
(imaging with the correct velocity will produce a sparser image)

$$\min_{\mathbf{v}} \left\| X_{\mathrm{Target}}(\mathbf{v}) \right\|_1$$

such that

$$X_{\mathrm{Target}}(\mathbf{v}) = \min_{\mathbf{X}} \left\| \Phi_F^0(\mathbf{X}_d) - \Phi_F^V(\mathbf{X}) \right\|_F^2$$

$$\mathbf{v} = \Upsilon(v^{(z)}, v^{(r)})$$

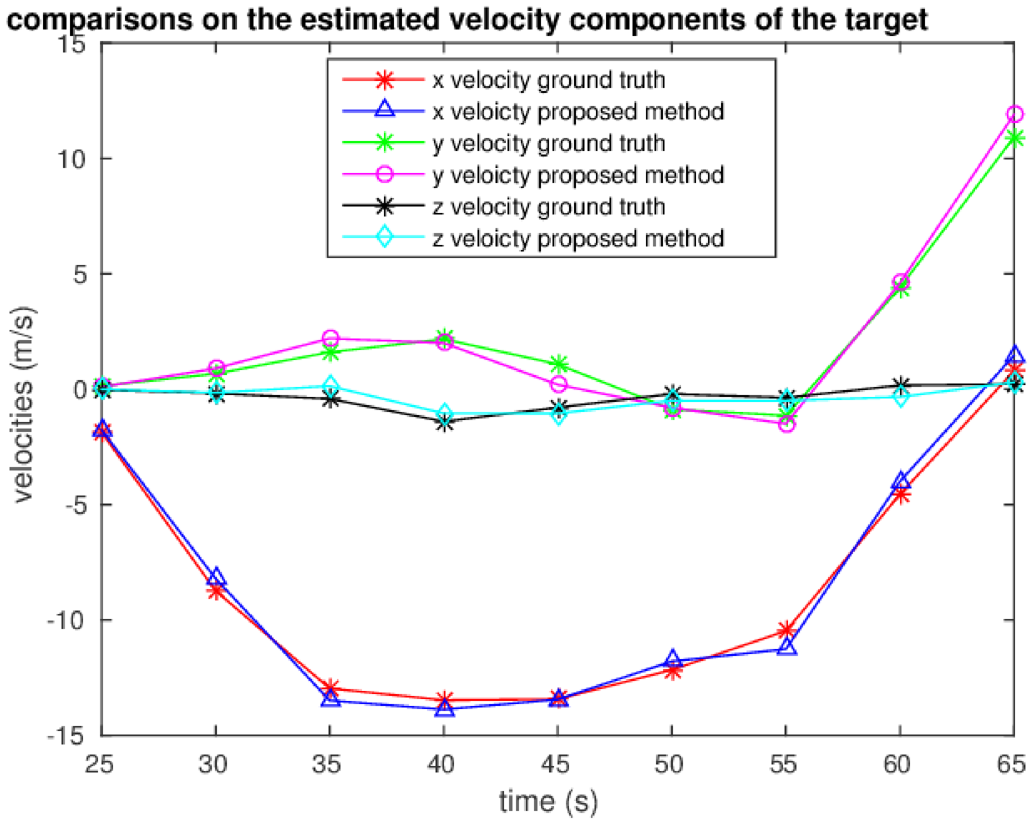


where

- $\mathbf{X}_d$ is the dynamic component restricted to target neighbourhood
- Φ_F^V is the forward model with the corrected radial velocity;
- Υ enforces consistency with radial velocity, $v^{(r)}$, and target movement on the DEM

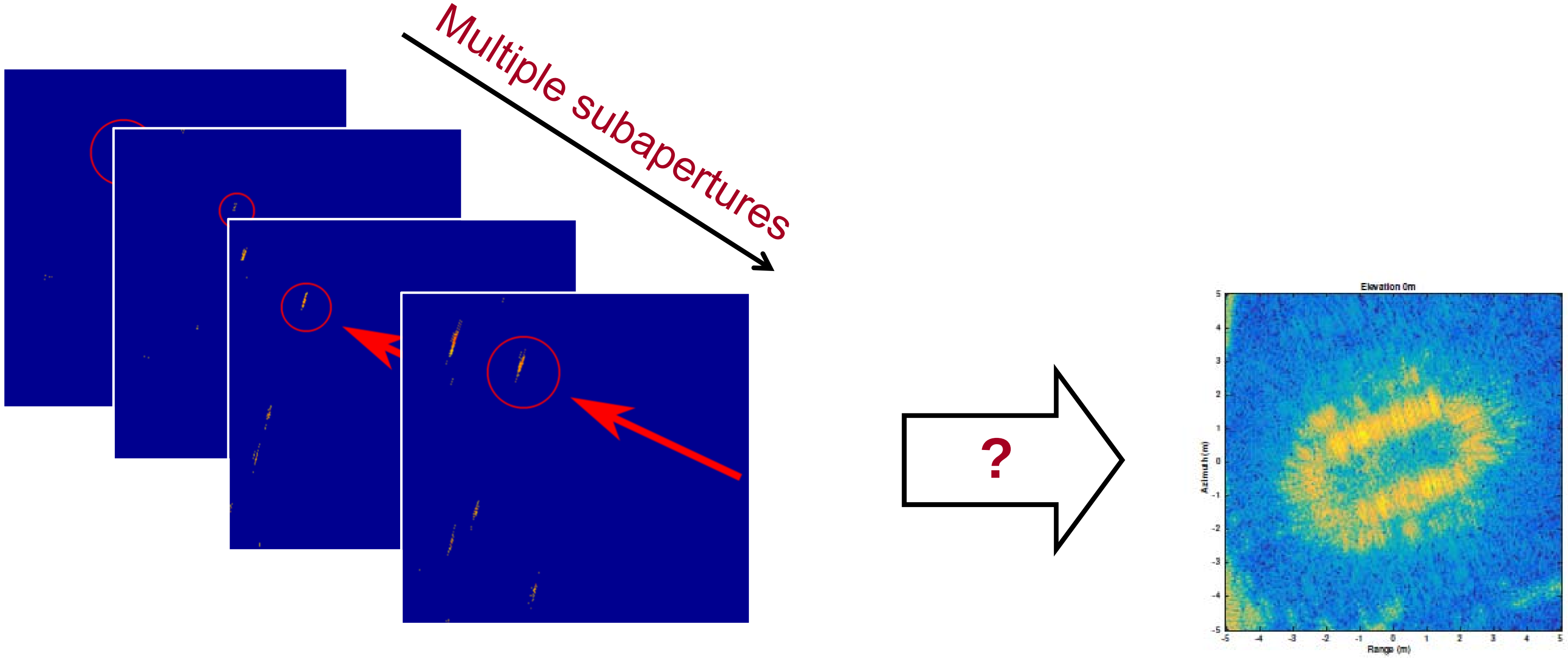
Target Velocity Estimates

x-,y-,z- velocities all accurately estimated from SAR data



Open Challenge…

Given a dynamic decomposition we should now be able to form a large aperture (high resolution) image of the moving target using velocity estimation and displacement correction (Inverse SAR)….



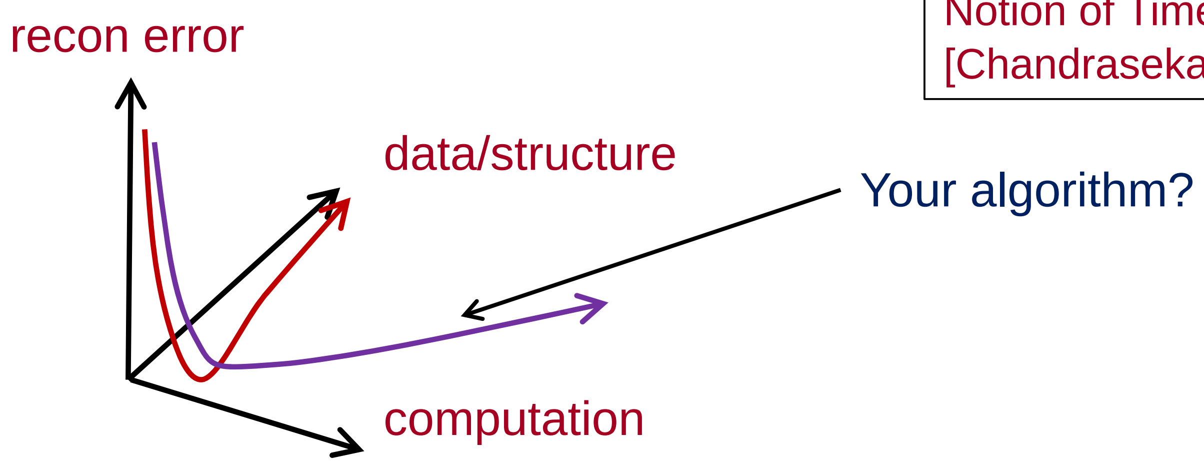
Data, Computation and Sparsity

Computation and Inverse Problems

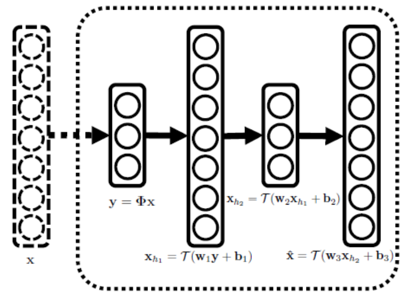
Traditionally attention has focused on estimation accuracy with little attention to computation.

What do we actually want?

Notion of Time-Data Complexity [Chandrasekaran & Jordan 13]



- Iterative vs non-iterative (Deep NN reconstruction?)?
- Randomized vs deterministic?
- What are the appropriate computational models?



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Now recruiting…

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