COMPRESSIVE SENSING THEORY AND THE REAL WORLD

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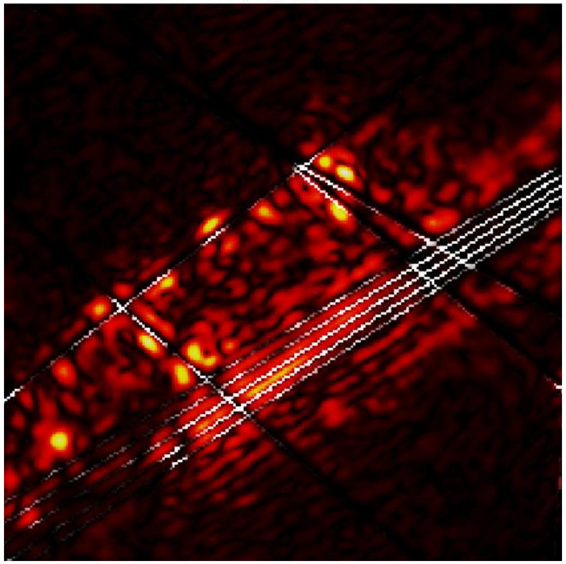
- General about sensing
- (Interference and deviations from sparsity)
- Continuous recovery
- Random (?) projections
- Large scenes, large rawdata
- Non-linear sensing
- Higher level information retrieval



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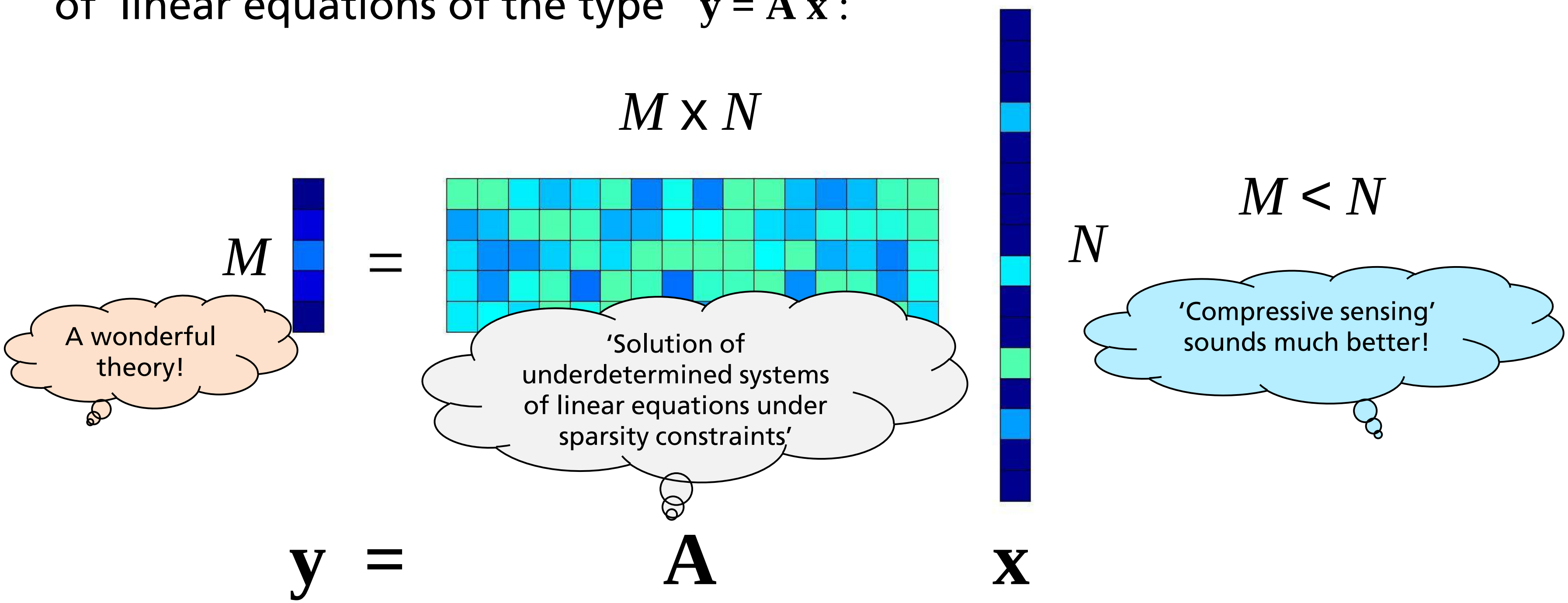
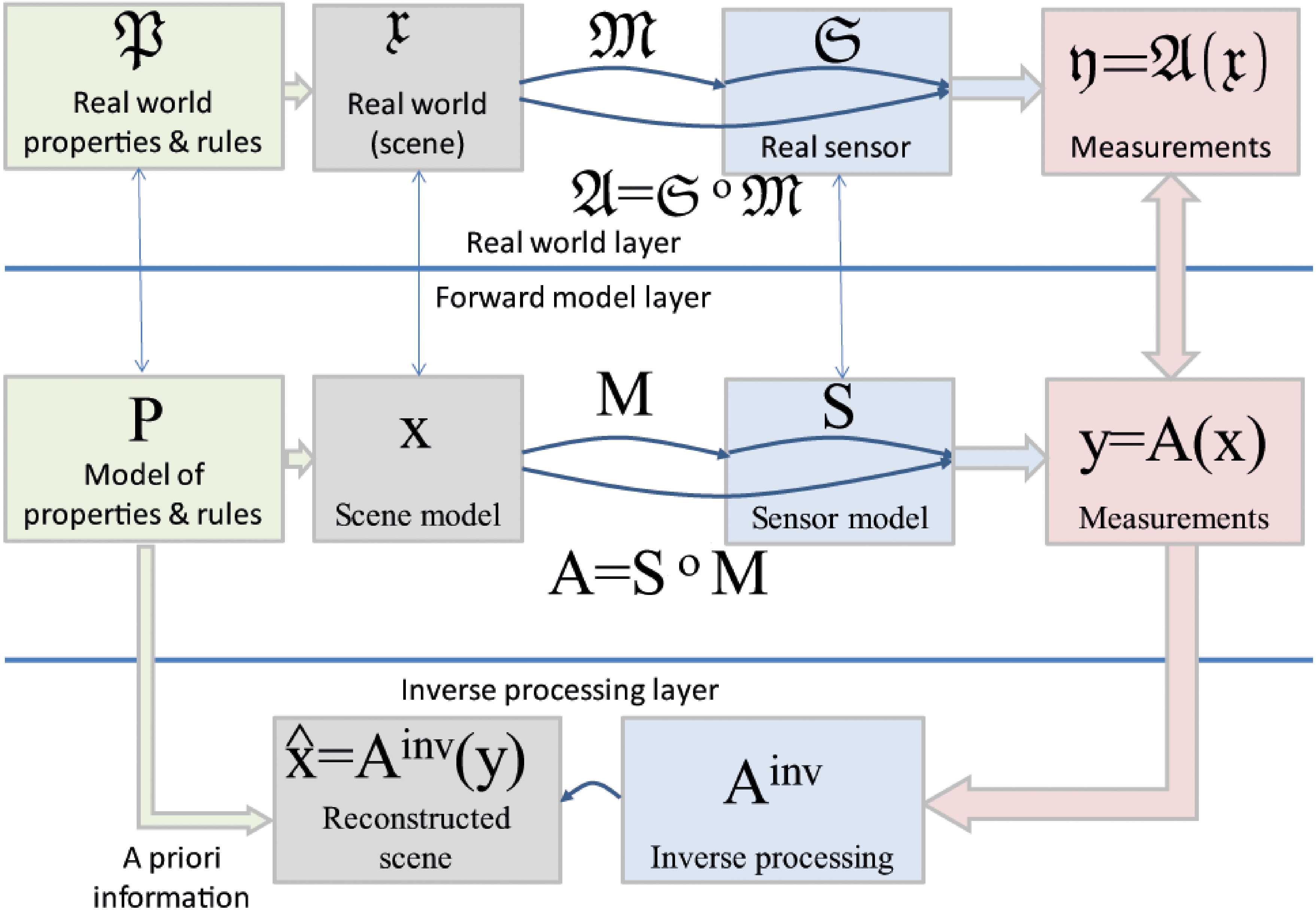
CS – A MATHEMATICAL TOOL

Solution of underdetermined systems of linear equations ...

- Compressive sensing techniques generally deal underdetermined systems of linear equations of the type $\mathbf{y} = \mathbf{A}\,\mathbf{x}$:



- Under certain conditions on **A** there is an unique solution for S-sparse **x**.



𝔓
Real world properties & rules
𝔵
Real world (scene)
𝔐
𝔖
Real sensor
𝔶=𝔄(𝔵)
Measurements
𝔄=𝔖∘𝔐
Real world layer
Forward model layer
P
Model of properties & rules
x
Scene model
M
S
Sensor model
y=A(x)
Measurements
A=S∘M
Inverse processing layer
x̂=A^inv(y)
Reconstructed scene
A^inv
Inverse processing
A priori information

APPLICATION OF CS TO REAL WORLD SENSORS (RADAR)
Questions to be asked

- What are the reasons to apply CS to a special radar task?
  - Sparing samples? (temporal, spatial, ...)
  - Better performance? (super resolution, image quality, additional information,...)
- Power budget preserved?
- Computing effort?
- Robustness, stability?
- Guarantee to fulfill the specs?

CHALLENGE I: INTERFERENCE, DEVIATIONS FROM SPARSITY

The concept of 'compressible signals' , i.e. deviations from exact sparsity, and the robustness against interference have been treated adequately in the mathematical theory.

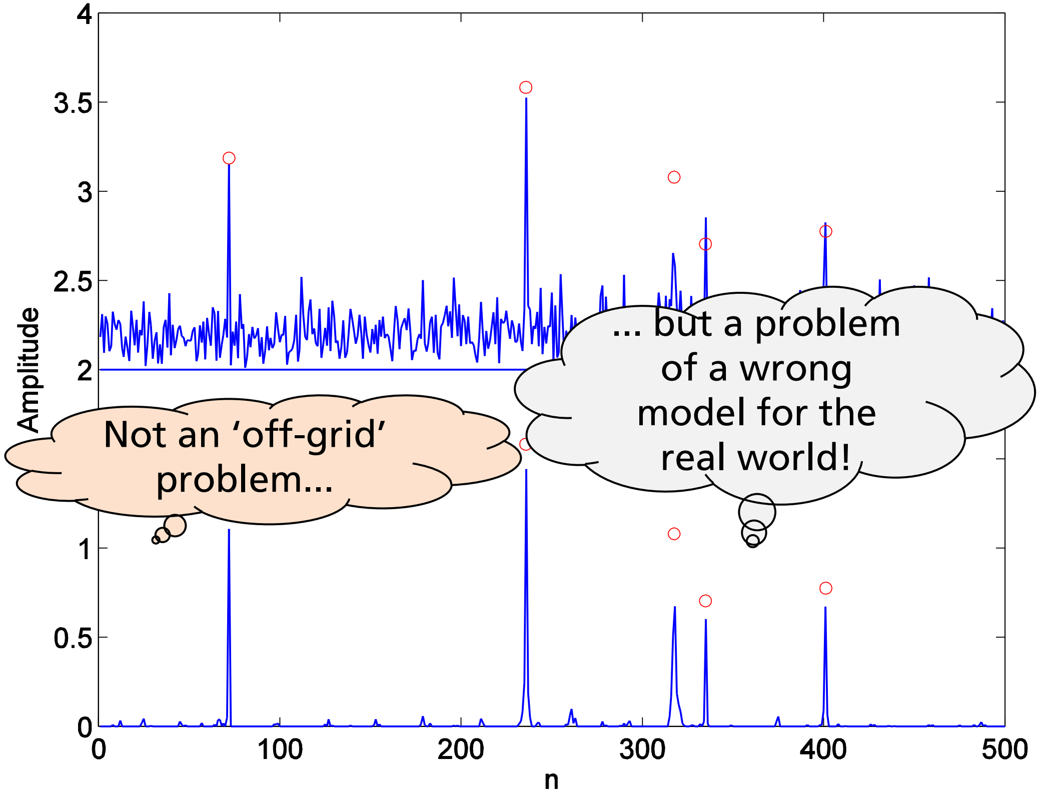
✓

CHALLENGE II: CONTINUOUS RECOVERY

A lot of papers on the ‘off-grid problem’ or ‘continuous sparse recovery’ have been published. Nevertheless …

CHALLENGE II: CONTINUOUS RECOVERY
Simulated scene with points between the grids

- The scene model for CS is in principle discrete and finite, the real world is continuous. Prob(Position at a grid point) = 0.



Positions uniformly randomly chosen.

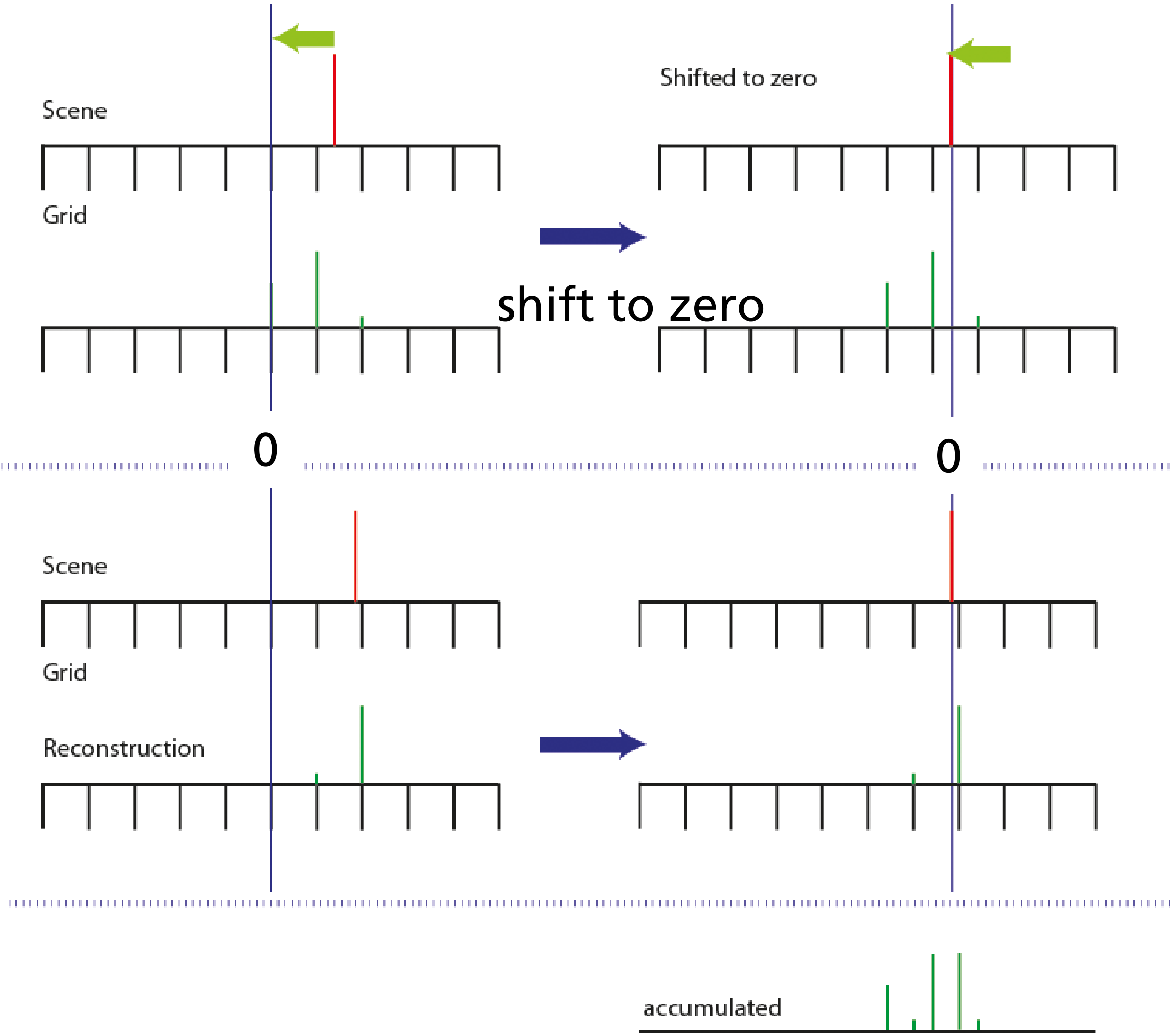
Top: reconstruction with matched filter.
Bottom: reconstruction with ℓ_1-minimization.

The markers indicate the true values.

N = 500, M = 100, S = 5, noise = -30dB, partial Fourier matrix.

CHALLENGE II: CONTINUOUS RECOVERY

Hidden sidelobes

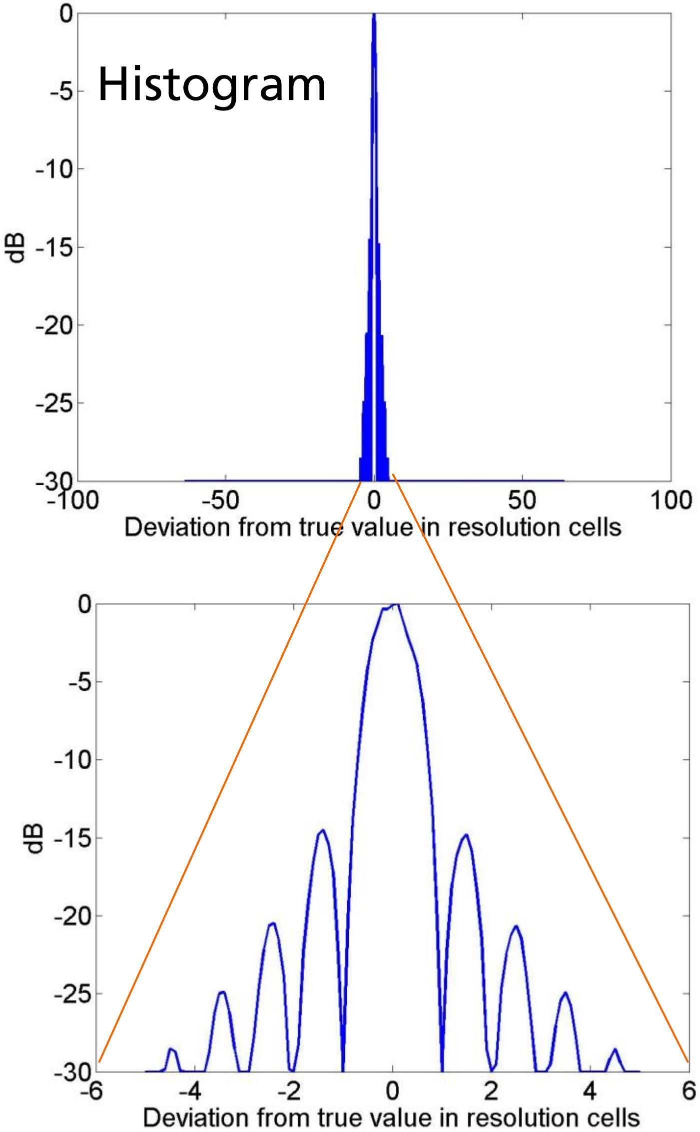


true positions

grid

accumulate and build histogram

CHALLENGE II: CONTINUOUS RECOVERY
Hidden sidelobes



Monte Carlo Simulation

using *spgl1* for sparse recovery

N = 128;

M = 50;

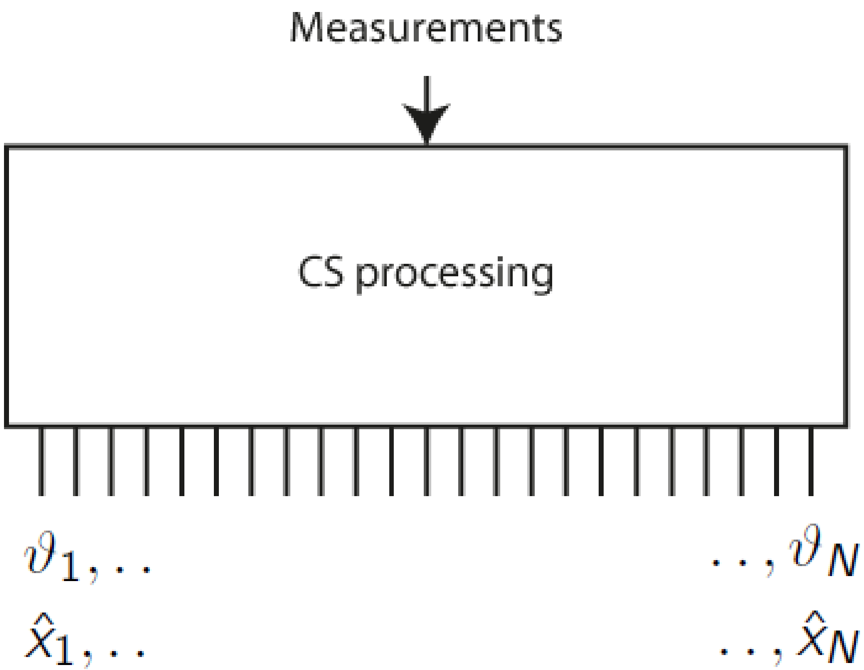
Noise -30 dB;

Number of iterations = 10 000;

CHALLENGE II: CONTINUOUS RECOVERY

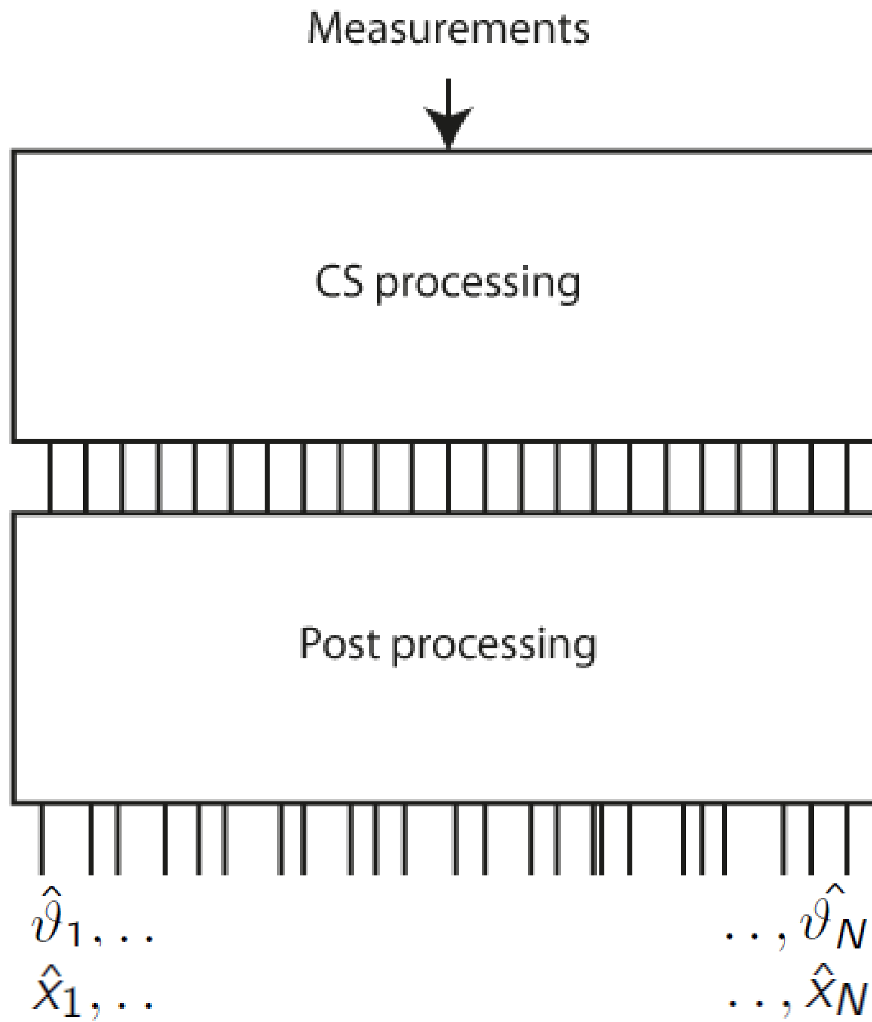
$$x(\vartheta) = \sum_{s=1}^{S} x_s \delta(\vartheta - \vartheta_s)$$

Model and approaches



CS-Estimator of a spiky scene

$$\begin{pmatrix} \hat{x}_1 \\ \hat{\vartheta}_1 \end{pmatrix}, \ldots, \begin{pmatrix} \hat{x}_N \\ \hat{\vartheta}_N \end{pmatrix}$$



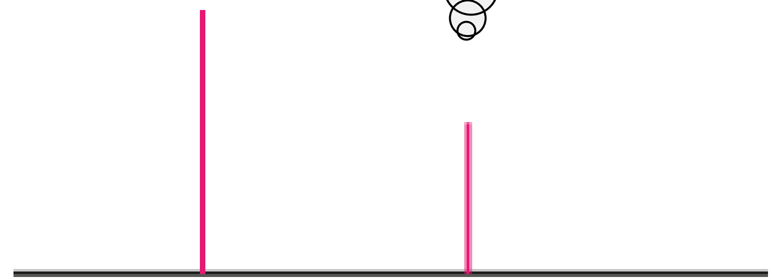
CHALLENGE II: CONTINUOUS RECOVERY

How to measure the performance?

How to measure the reconstruction quality?

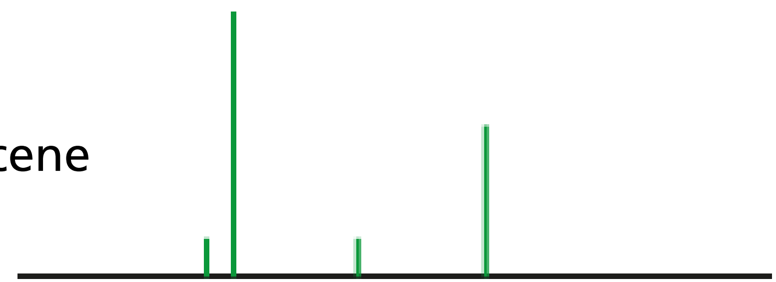
$$x(\vartheta) = \sum_{s=1}^{S} x_s \delta(\vartheta - \vartheta_s)$$

'spiky' scene



$$\hat{x}(\vartheta) = \sum_{s'=1}^{S'} \hat{x}_s \delta(\vartheta - \hat{\vartheta}_{s'})$$

estimated scene
('spiky' too)



$\hat{\vartheta}_{s'}$ *may coincide with the grid points*

CHALLENGE II: CONTINUOUS RECOVERY

A new performance measure (to be discussed!)

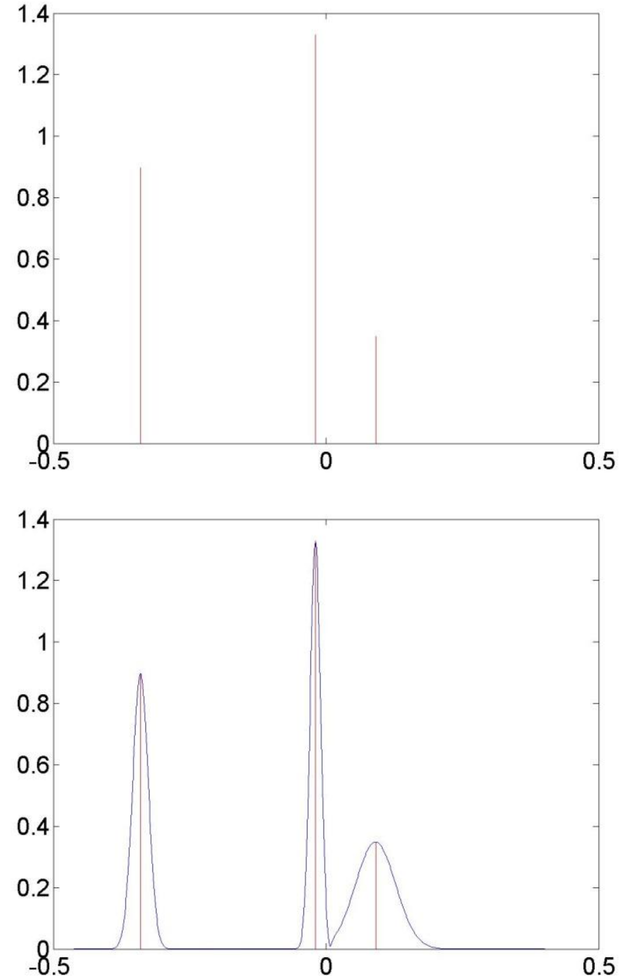
$$x(\vartheta) = \sum_{s=1}^{S} x_s \delta(\vartheta - \vartheta_s)$$ Scene

$$\tilde{x}(\vartheta) = \sum_{s=1}^{S} x_s w_s(\vartheta - \vartheta_s)$$

w_s is a window centered at 0 e.g.

$$w_s(\vartheta) = \exp\left\{-\frac{\vartheta^2}{\sigma_s^2}\right\}$$

where σ_s^2 corresponds to the CRB

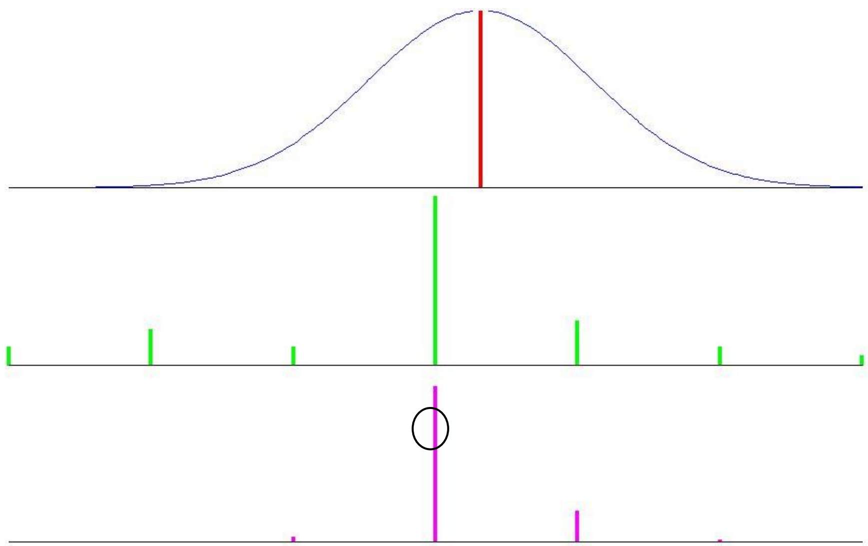


CHALLENGE II: CONTINUOUS RECOVERY

A new performance measure (to be discussed!)

Let $\mathbf{x}$ be the S-dimensional vector composed of the x_s.
For each $s = 1 \ldots S$ find a 'partner' $\tilde{x}_s$ and arrange them to a vector $\tilde{\mathbf{x}}$. The remaining error is defined as $\epsilon = \|\mathbf{x} - \tilde{\mathbf{x}}\|_2$.
The partner to x_s is found as follows

$$\rho(s) = \underset{s'}{\operatorname{argmax}}\{\Re\{x_s w_s(\vartheta_s - \hat{\vartheta}_{s'})\hat{x}_{s'}^*\}\} \qquad \tilde{\mathbf{x}}_s := \hat{x}_{\rho(s)} w_s(\vartheta_s - \hat{\vartheta}_{\rho(s)})$$



(1) Scene point with window

$$x_s w_s(\vartheta - \vartheta_s)$$

(2) Estimated scene

$$\hat{x}(\vartheta) = \sum_{s'=1}^{S'} \hat{x}_s \delta(\vartheta - \hat{\vartheta}_{s'})$$

(3) Real part of product of (1) and (2)*

CHALLENGE II: CONTINUOUS RECOVERY

Ways to overcome the grid bondage

1. Refinement of grid
2. Gradient based
3. Adaptive raster points

Some papers about off-grid and gridless CS

[4] A. Panahi and M. Viberg, Gridless compressive sensing, in: *2014 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 3385–3389, May 2014.

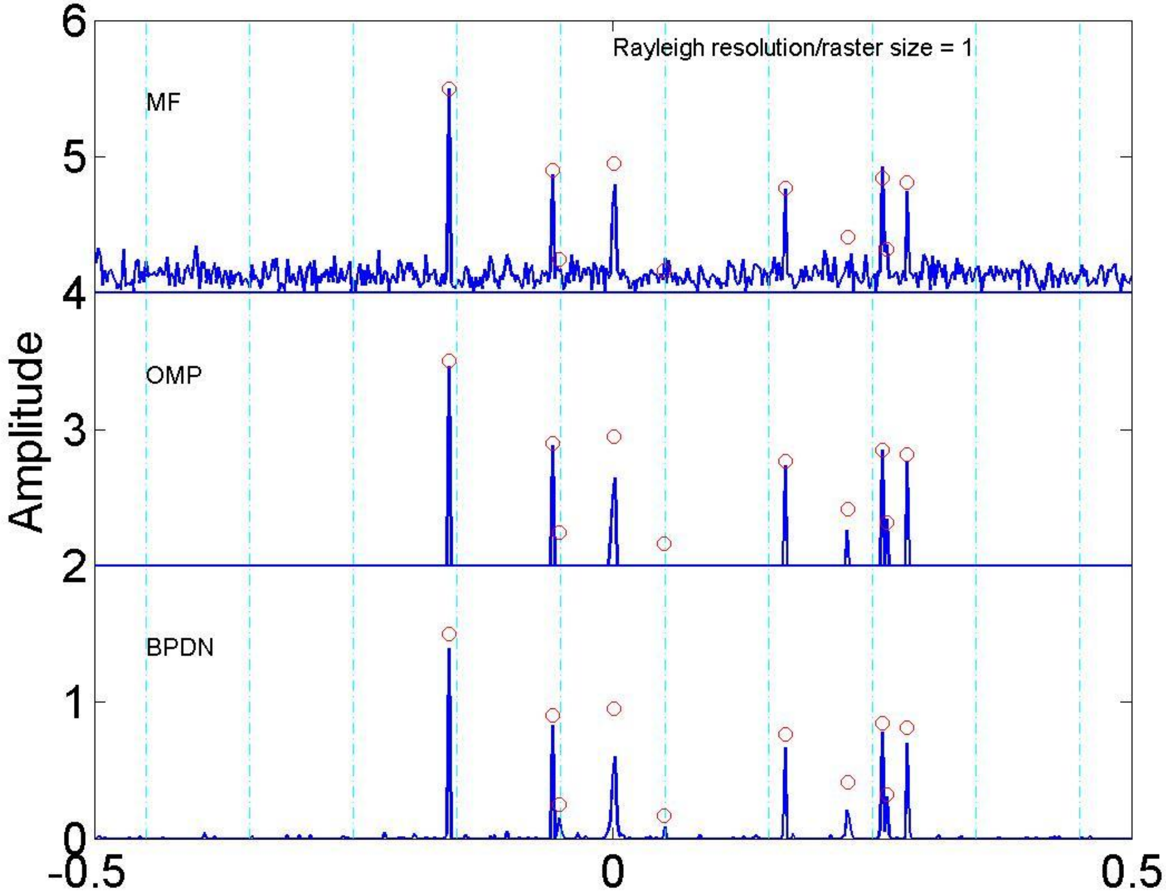
[5] B. Qian, H. Wen, H. Kuoye, W. Yanping, L. Yun and T. Weixian, Off-grid effect free imaging method based on improved OMP approach for DLLA 3D SAR, in: *IET International Radar Conference 2015*, pp. 1–4, Oct 2015.

[6] O. Teke, A. C. Gurbuz and O. Arikan, Sparse delay-Doppler image reconstruction under off-grid problem, in: *2014 IEEE 8th Sensor Array and Multichannel Signal Processing Workshop (SAM)*, pp. 409–412, June 2014.

CHALLENGE II: CONTINUOUS RECOVERY

1. Approach: Refinement of grid

Computed with 'spgl1'



Resolution cell / raster =1.0

N0 = 500   # resolution cells

N = N0 ... 5000

Oversampling = 1 ... 10

M = 200

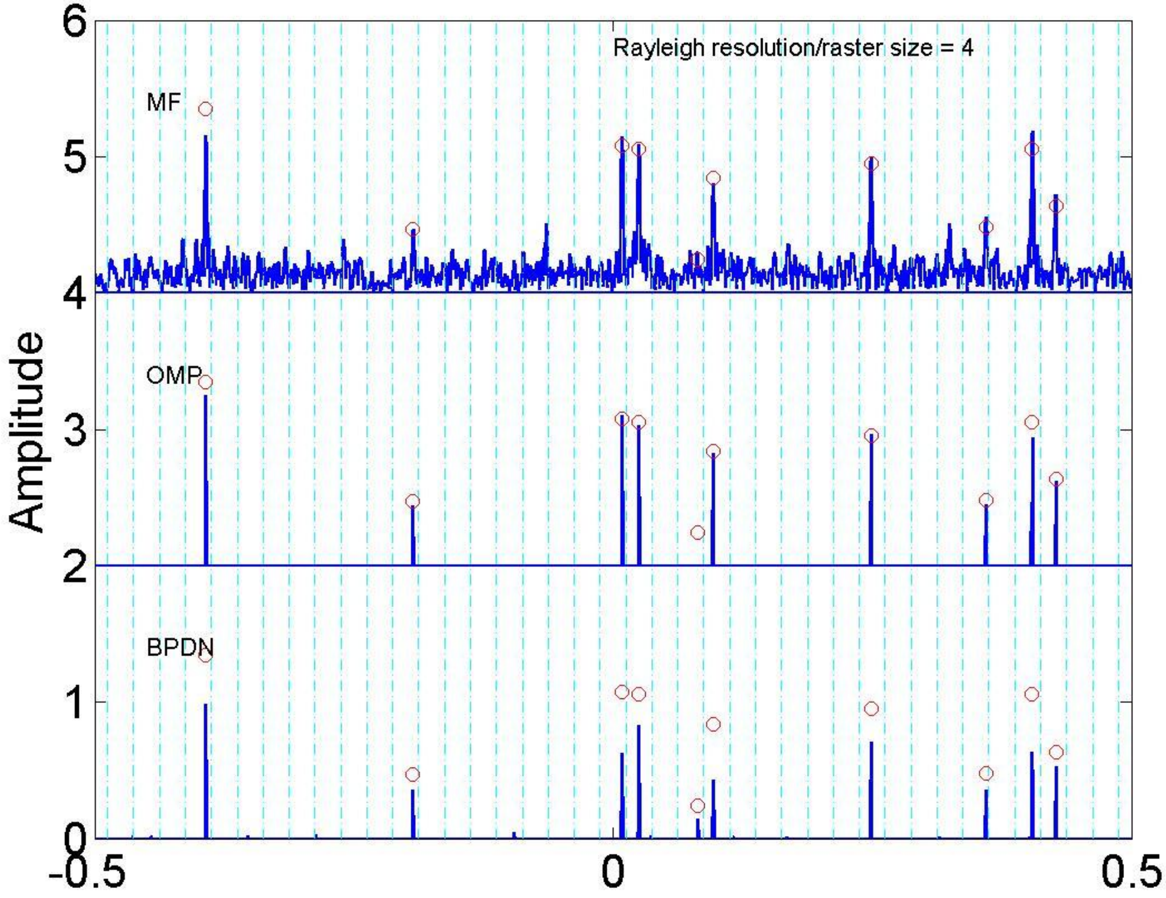
S = 10

Simulations = 400

dBnoise=-25;

CHALLENGE II: CONTINUOUS RECOVERY
1. Approach: Refinement of grid

Computed with 'spgl1'



Resolution cell / raster = 4.0

N0 = 500 # resolution cells

N = N0 ... 5000

Oversampling = 1 ... 10

M = 200

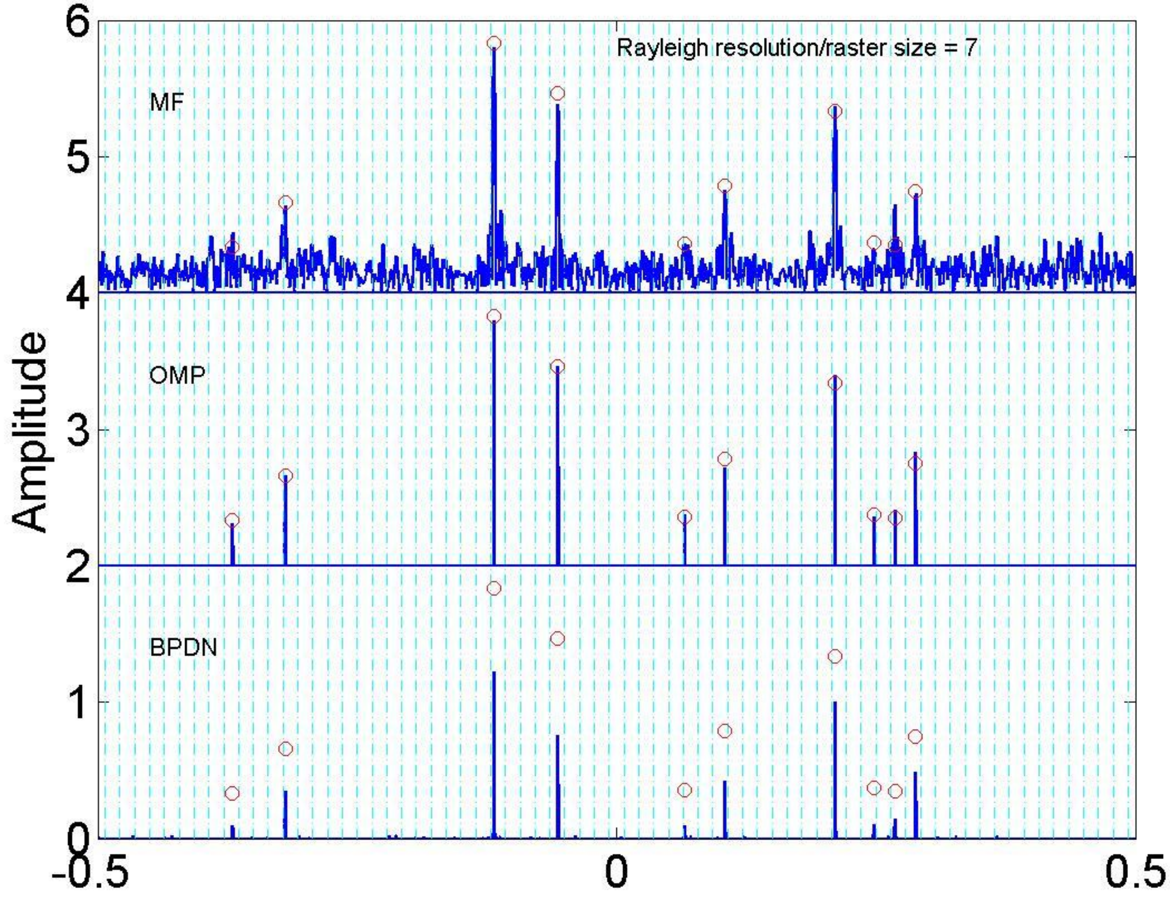
S = 10

Simulations = 400

dBnoise=-25;

CHALLENGE II: CONTINUOUS RECOVERY
1. Approach: Refinement of grid

Computed with ‘spgl1’



Resolution cell / raster = 7.0

N0 = 500 # resolution cells

N = N0 ... 5000

Oversampling = 1 ... 10

M = 200

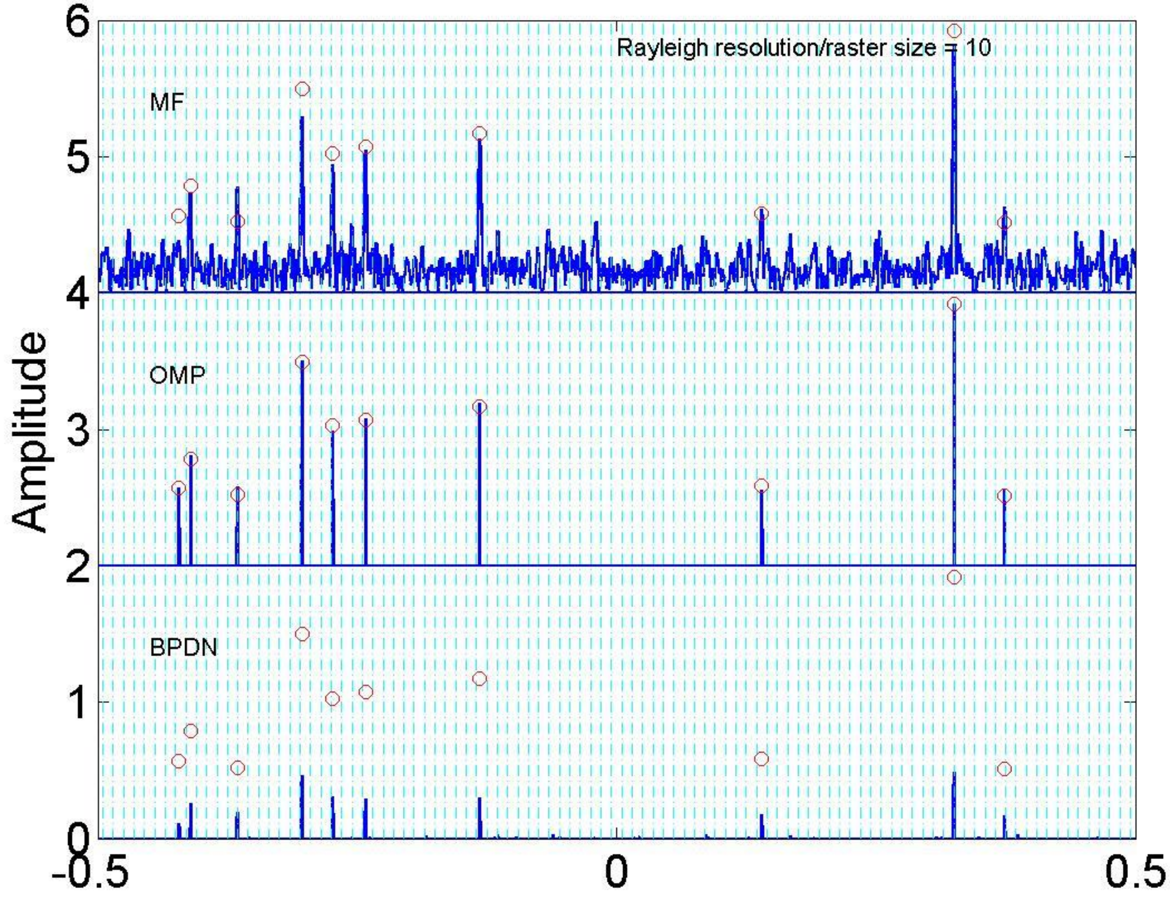
S = 10

Simulations = 400

dBnoise=-25;

CHALLENGE II: CONTINUOUS RECOVERY
1. Approach: Refinement of grid

Computed with 'spgl1'



Resolution cell / raster = 10.0

N0 = 500 # resolution cells

N = N0 ... 5000

Oversampling = 1 ... 10

M = 200

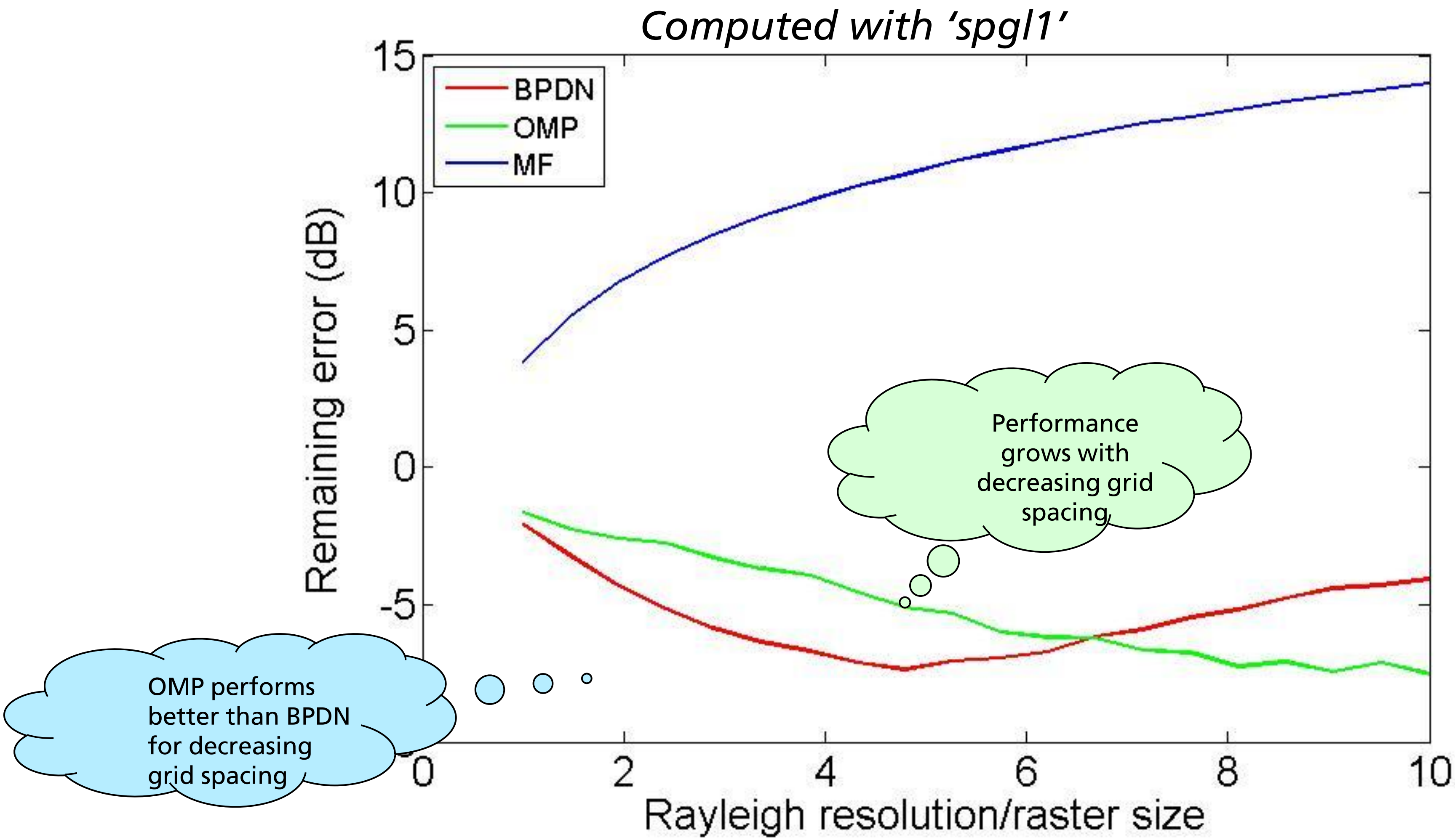
S = 10

Simulations = 400

dBnoise=-25;

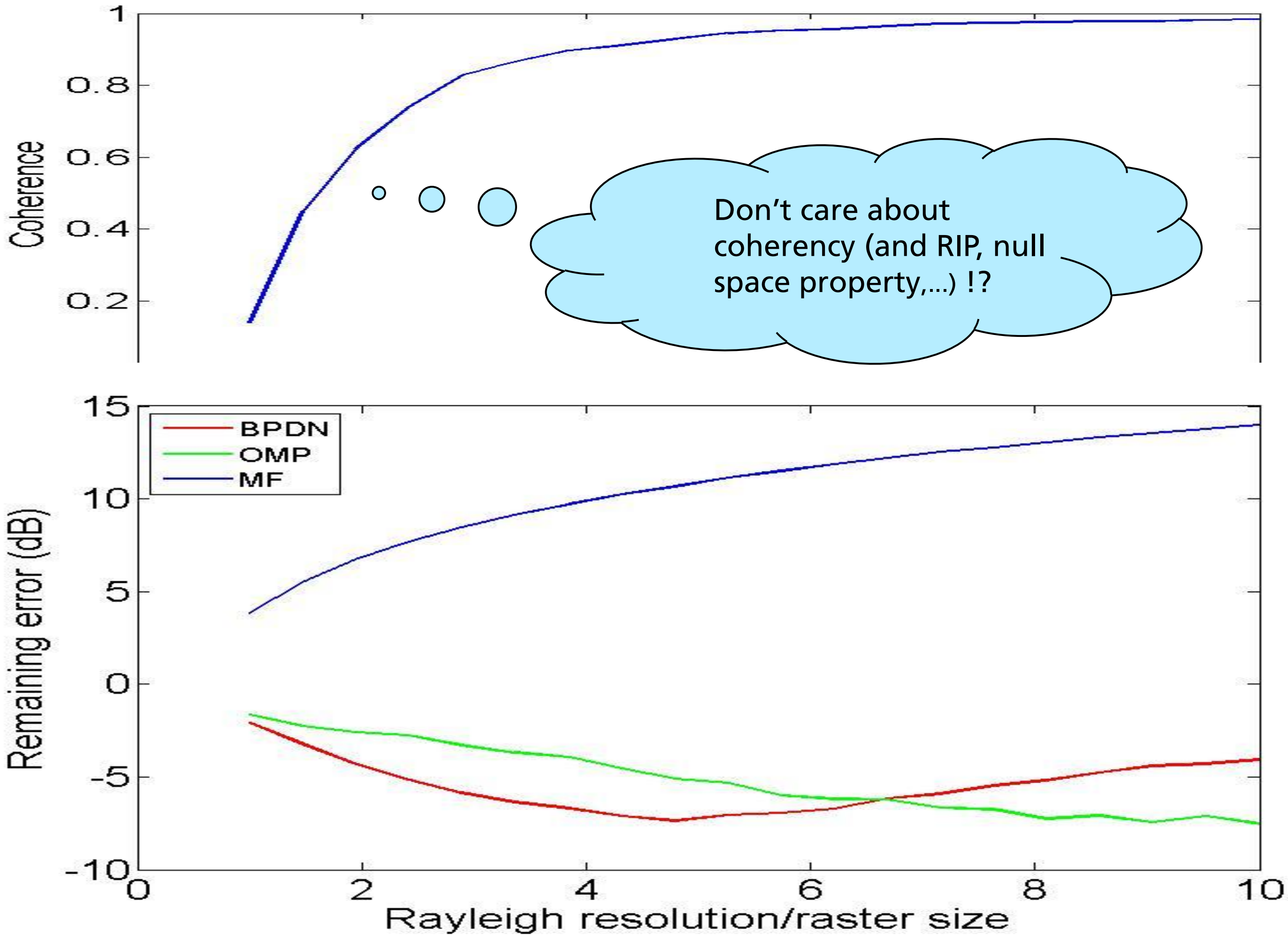
CHALLENGE II: CONTINUOUS RECOVERY

Performance measure for decreasing grid spacing



CHALLENGE II: CONTINUOUS RECOVERY

Performance measure for decreasing grid spacing



CHALLENGE II: CONTINUOUS RECOVERY

2. Approach: Add Taylor components

$$\begin{aligned}
\mathbf{y} &= \sum_{s=1}^{S} w_s \mathbf{s}(\vartheta_s) + \mathbf{n} \\
&= \sum_{s=1}^{S} w_s \mathbf{s}(\overline{\vartheta_s} + \Delta\vartheta_2) + \mathbf{n} \\
&= \sum_{s=1}^{S} w_s \left[\mathbf{s}(\overline{\vartheta_s}) + \Delta\vartheta_s \mathbf{s}_\vartheta(\overline{\vartheta_s}) + \frac{1}{2}\Delta\vartheta^2 \mathbf{s}_{\vartheta\vartheta}(\overline{\vartheta_s}) + \ldots \right] + \mathbf{n} \\
&\approx \sum_{s=1}^{S} w_s \mathbf{s}(\overline{\vartheta_s}) + w_s \Delta\vartheta_s \mathbf{s}(\overline{\vartheta_s}) + \mathbf{n} \qquad \text{Linear approximation} \\
&= \sum_{s=1}^{S} w_s \mathbf{s}(\overline{\vartheta_s}) + z_s \mathbf{s}(\overline{\vartheta_s}) + \mathbf{n}
\end{aligned}$$

Addition of new columns (gradients) to the sensing matrix

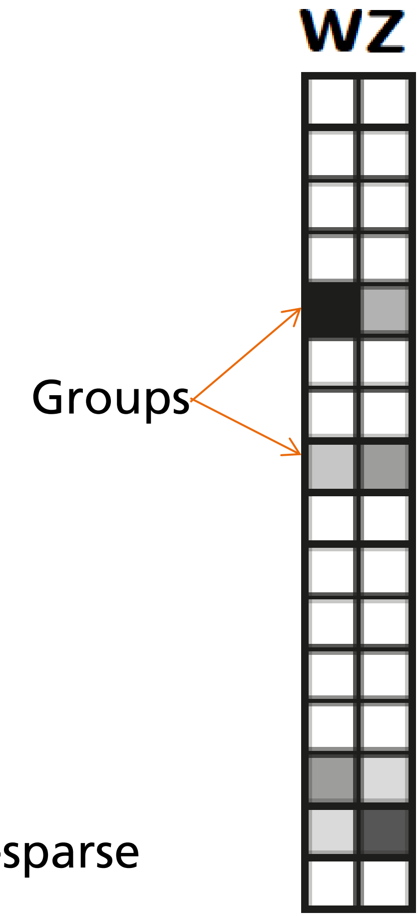
with $z_s = w_s \Delta\vartheta_s$

CHALLENGE II: CONTINUOUS RECOVERY

2. Approach: Add Taylor components

$$\begin{aligned} \mathbf{y} &= \mathbf{B}\mathbf{w} + \mathbf{C}\mathbf{z} + \mathbf{n} \\ &= \mathbf{A}\mathbf{x} + \mathbf{n} \end{aligned}$$

$$\text{with} \quad \mathbf{A} = (\mathbf{B} \quad \mathbf{C}), \quad \mathbf{x} = \begin{pmatrix} \mathbf{w} \\ \mathbf{z} \end{pmatrix}$$



$$\begin{aligned} \Delta\hat{\vartheta}_s &= \Re\left\{ \frac{\hat{z}_s}{\hat{w}_s} \right\} \\ \hat{\vartheta}_s &= \overline{\vartheta}_s + \Delta\hat{\vartheta}_s \end{aligned}$$

A case for block-sparse recovery!

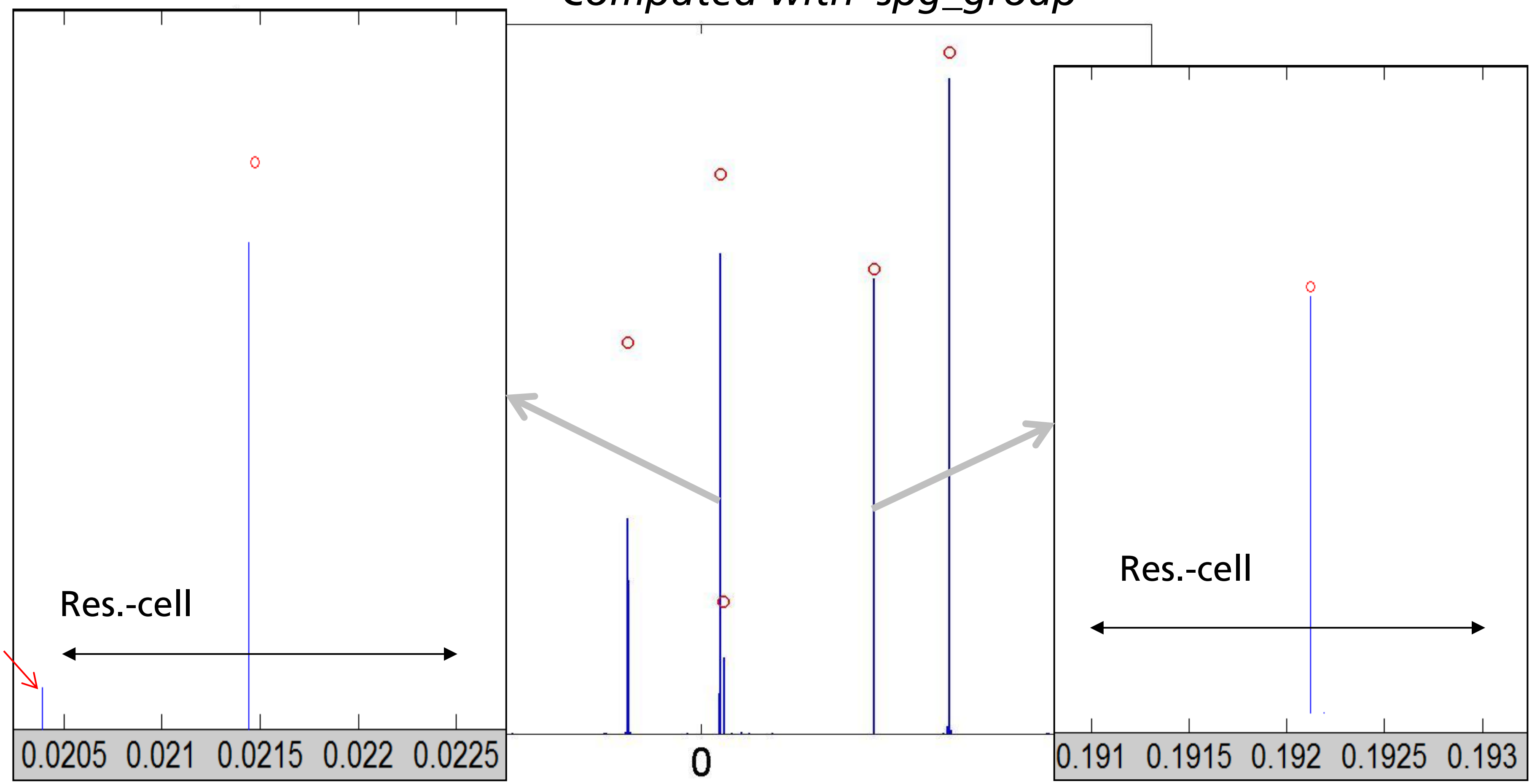
C. Ekanadham, D. Tranchina and E. P. Simoncelli, "Recovery of Sparse Translation-Invariant Signals With Continuous Basis Pursuit, 2011

CHALLENGE II: CONTINUOUS RECOVERY

2. Approach: Add Taylor components

Coherence of the combined sensing matrix ≈ 0.6

Computed with 'spg_group'



Reconstruction with mixed $\ell 1$-$\ell 2$-minimization

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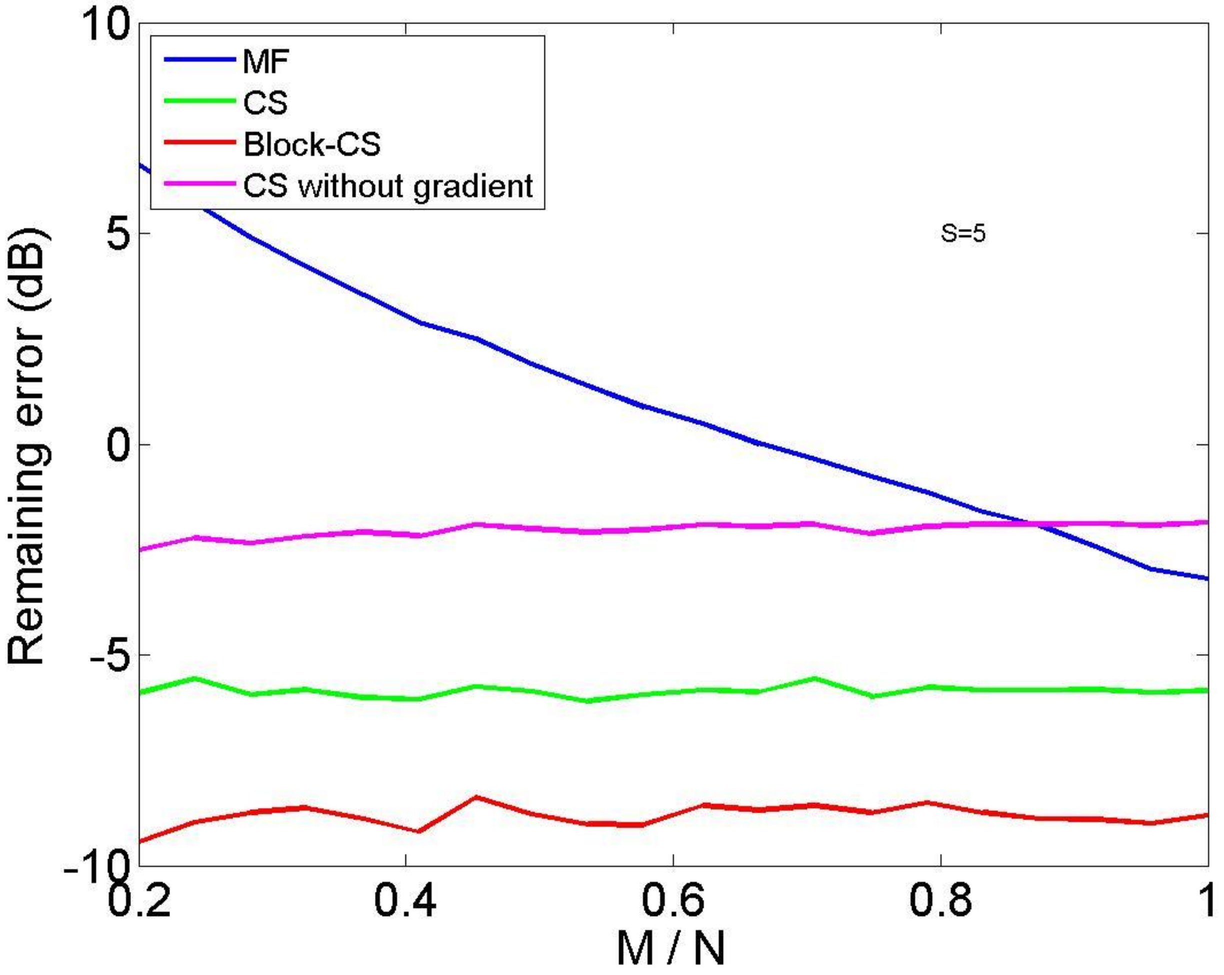
CHALLENGE II: CONTINUOUS RECOVERY

2. Approach: Add Taylor components, performance



N = 500

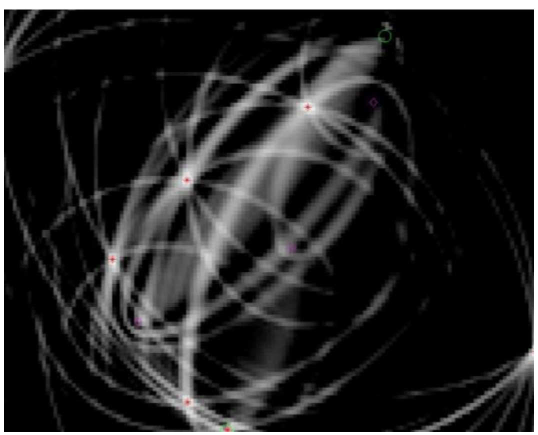
S = 5

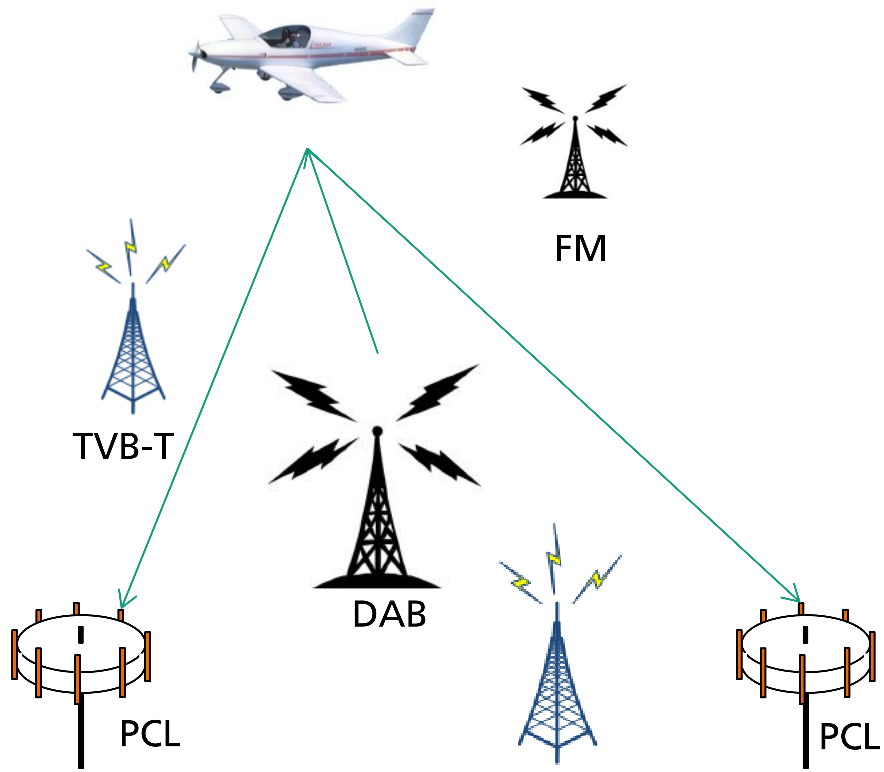
Noise = -25 dB

CHALLENGE II: CONTINUOUS RECOVERY

3. Approach: Adaptive grid

- Iteration:
  - The grid points are shifted according to the actual estimation of displacements
  - Re-applied sparse recovery
  - Next iteration
- Especially interesting for target tracking
- Application example: Passive radar network (PCL), Block-sparse recovery

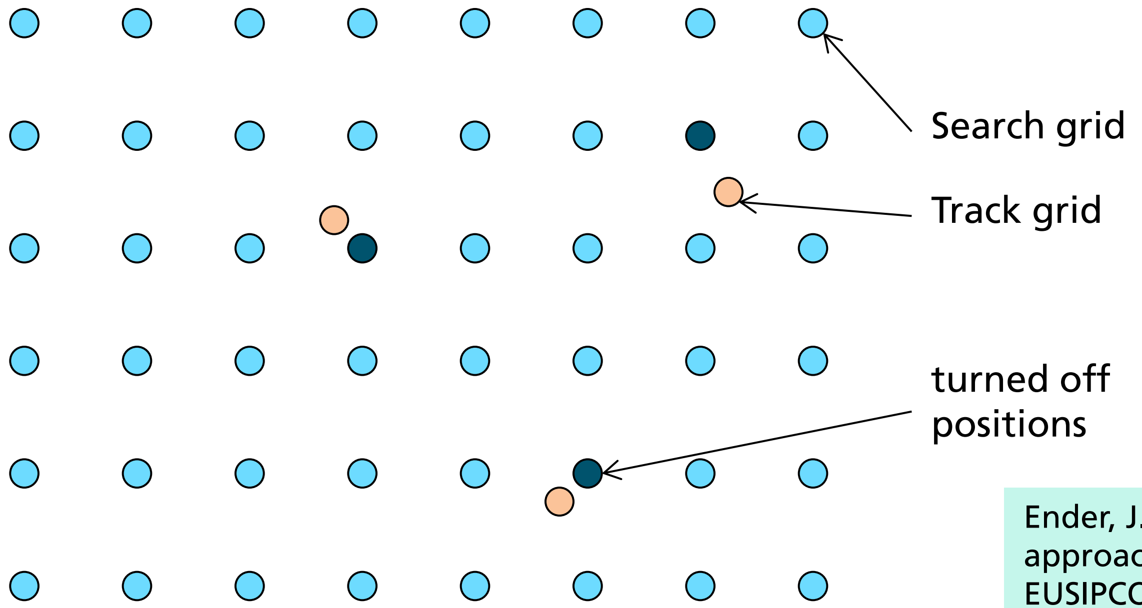




CHALLENGE II: CONTINUOUS RECOVERY

3. Approach: Adaptive grid

- Fixed search grid for the detection of new airplanes
- Dynamic track grid for tracked airplanes, basis for evaluating the remainders by projection
- Fine estimate of positions (here obtained by a second order Taylor approximation) can be integrated into the BOMP iteration

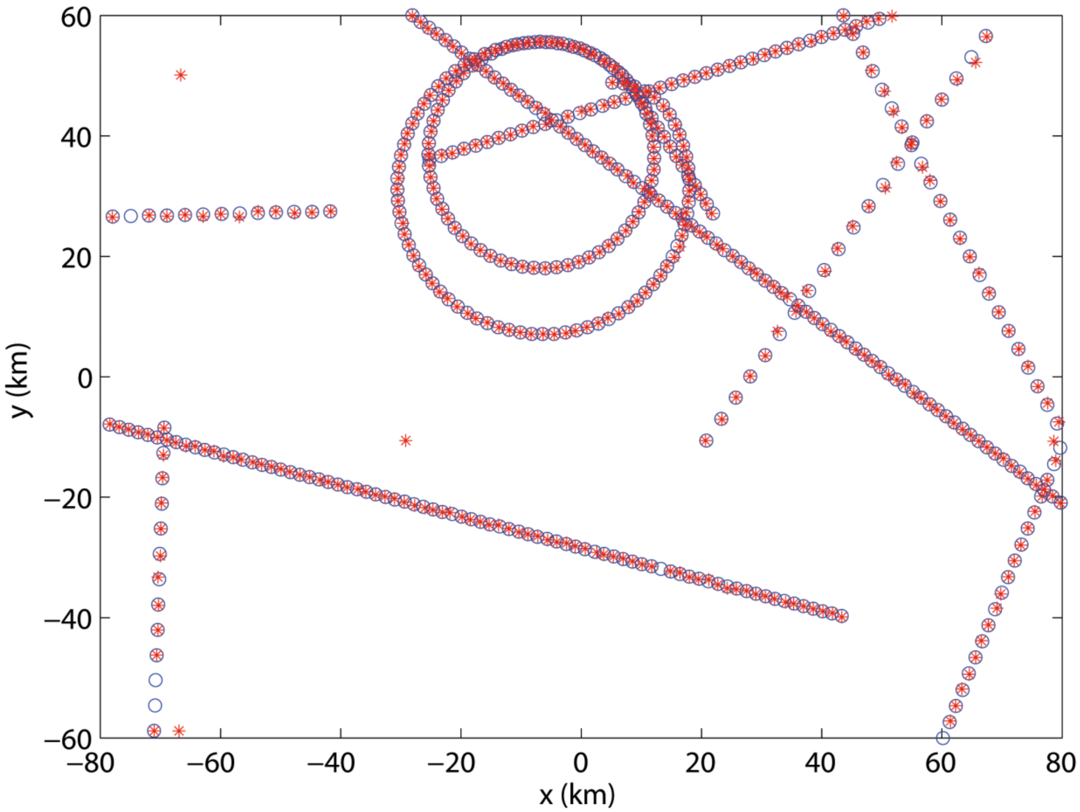


Ender, J.H.G., "A compressive sensing approach to the fusion of PCL sensors," EUSIPCO 2013

CHALLENGE II: CONTINUOUS RECOVERY

3. Approach: Adaptive grid, tracking

True positions of airplanes marked by blue circles



f0	400	MHz
Lambda0	0.75	m
Bandwidth	0.07	MHz
Resolution	2.142	m
NTx	3	
NRx	6	
Narray	2	
L (number sensors)	6	
Nk	94	
Mtotal	1692	
Image	151 x 114	
N	17214	
SNR	20 dB	
Simmax	500	

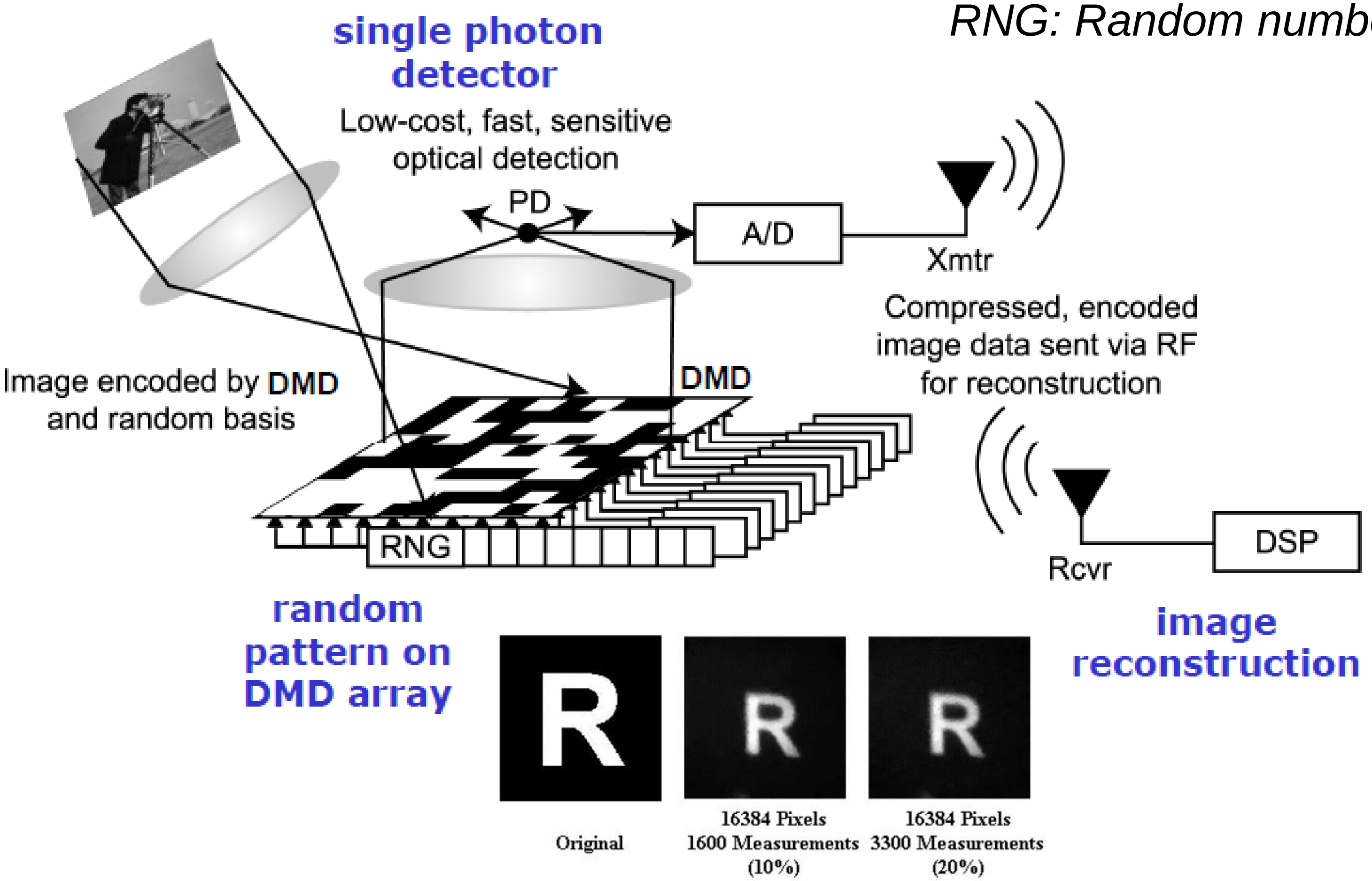
Computed with BOMP

Challenge III: RANDOM (?) PROJECTION

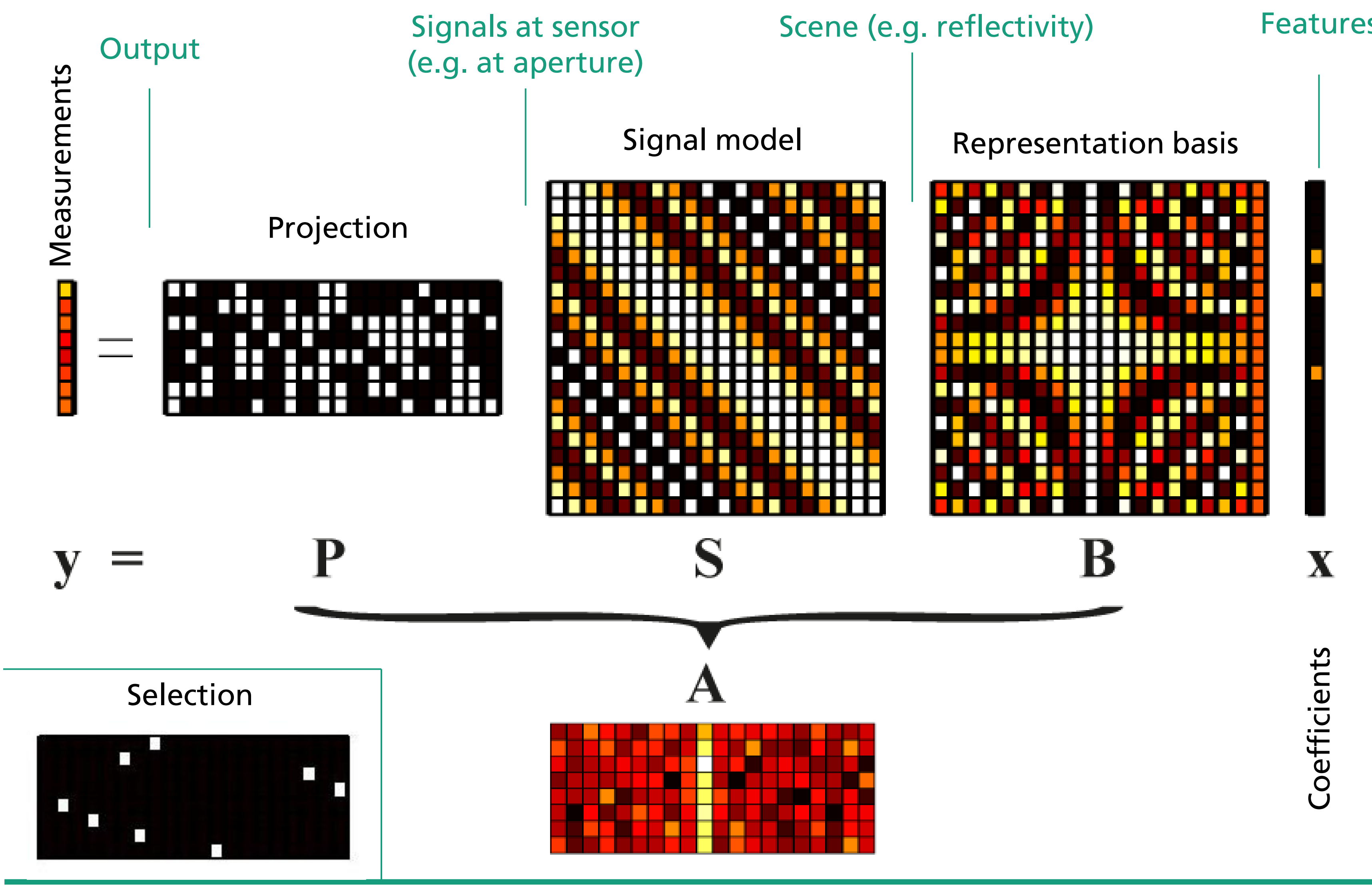
CHALLENGE III: RANDOM (?) PROJECTION

Famous example: Rice Univ single-pixel camera

DMD: Digital micromirror device
RNG: Random number generator

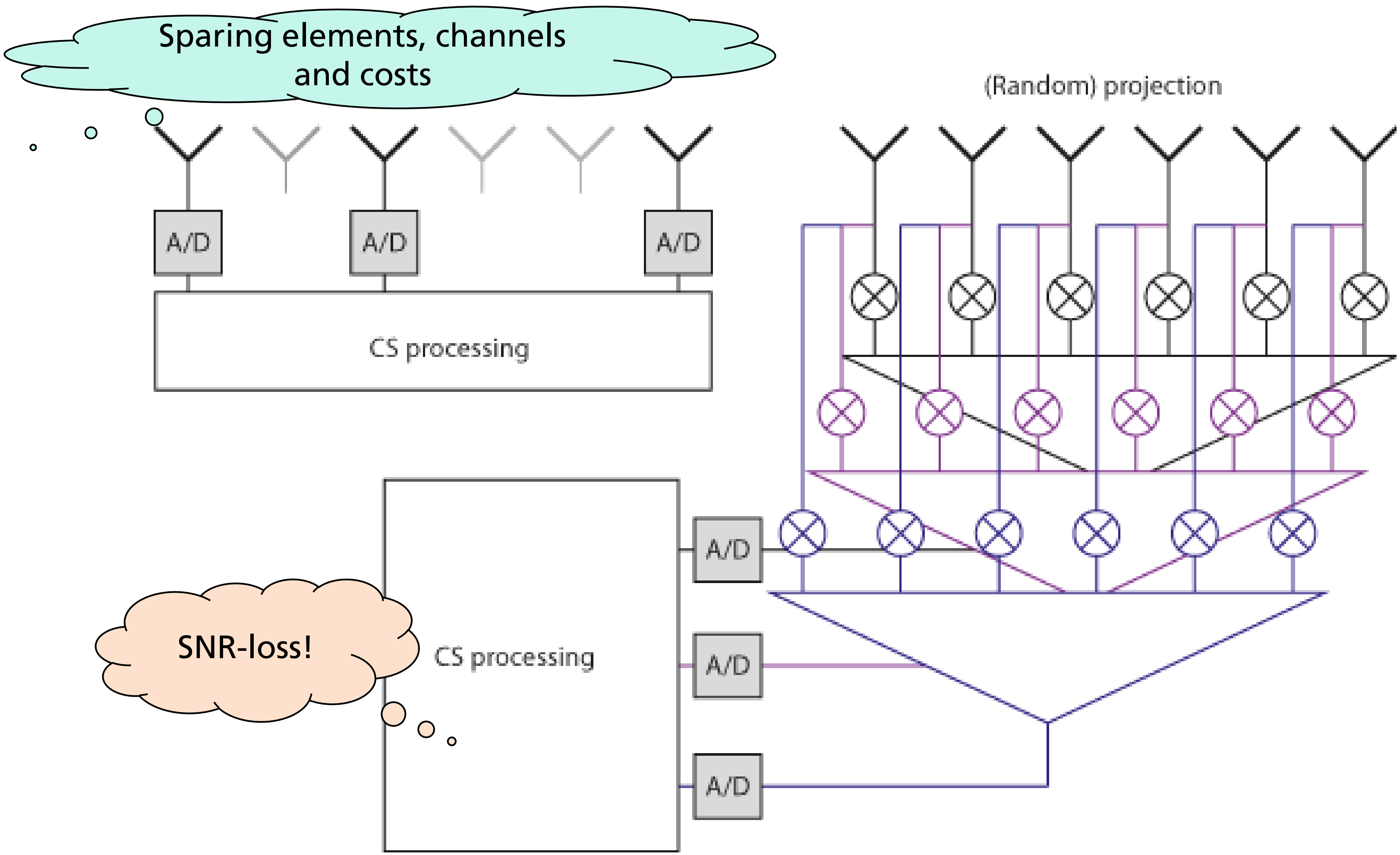


CHALLENGE III: RANDOM (?) PROJECTION



CHALLENGE III: RANDOM PROJECTION (?)

Applied to array antennas



THEOREMS FOR SPARSE RECONSTRUCTION
Random selection of sensing waveforms

THEOREM of Candés and Romberg

Let an S-sparse scene ρ with the coefficient vector $\mathbf{x}$ be given with respect to an orthonormal representation basis $\mathbf{B}$. With uniform probability let M sensor waveforms be drawn from the orthonormal sensing basis $\mathbf{S}$, forming the thinned sensing matrix $\tilde{\mathbf{S}}$ and $\mathbf{A} = \tilde{\mathbf{S}}\mathbf{B}$. Then the solution of the basis pursuit with $\mathbf{y} = \mathbf{A}\mathbf{x}$ is exact with probability $P \geq 1 - \epsilon$, if the following condition holds:

$$M \geq C\mu^2(\mathbf{S}, \mathbf{B}) S \ln(N/\epsilon)$$

with an appropriate constant C.

CHALLENGE III: RANDOM PROJECTION (?)
Why to use random projections / selections?

- Just to be compatible to the theorems?

Deterministic projections / selections?

1. Heuristic choice
2. Optimum sparse ruler
3. Random search

Some papers about deterministic dimension reduction

[1] A. Cohen, W. Dahmen and R. Devore, Compressed sensing and best k-term approximation, *J. Amer. Math. Soc* (2009), 211–231.

[2] R. A. DeVore, Deterministic constructions of compressed sensing matrices, *Journal of Complexity* **23** (2007), 918 – 925, Festschrift for the 60th Birthday of Henryk WoÅºniakowski.

[3] M. A. Herman and T. Strohmer, High-Resolution Radar via Compressed Sensing, *IEEE Transactions on Signal Processing* **57** (2009), 2275–2284.

CHALLENGE III: RANDOM PROJECTION (?)

Naive guess of a deterministic thinning

Computed with 'spgl1'

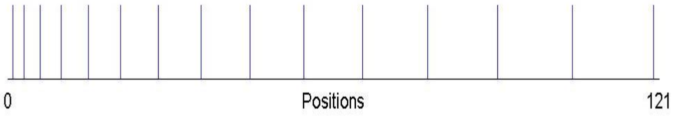
<u>Example</u>

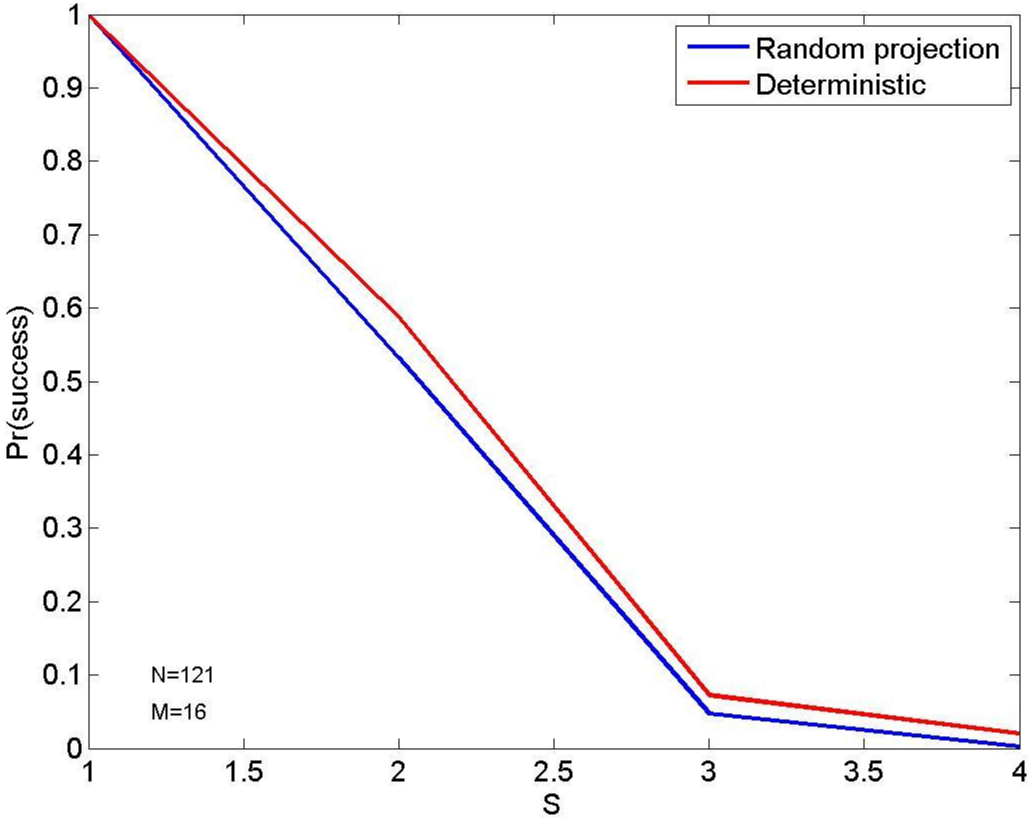
Positions =

[0	1	3	6
10	15	21	28
36	45	55	66
78	91	105	120]

M = 16

N = 121





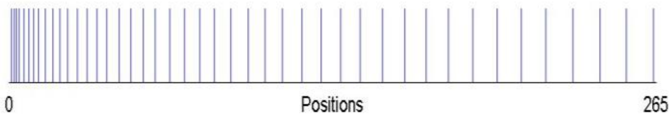
CHALLENGE III: RANDOM PROJECTION (?)
Naive guess of a deterministic thinning

Example

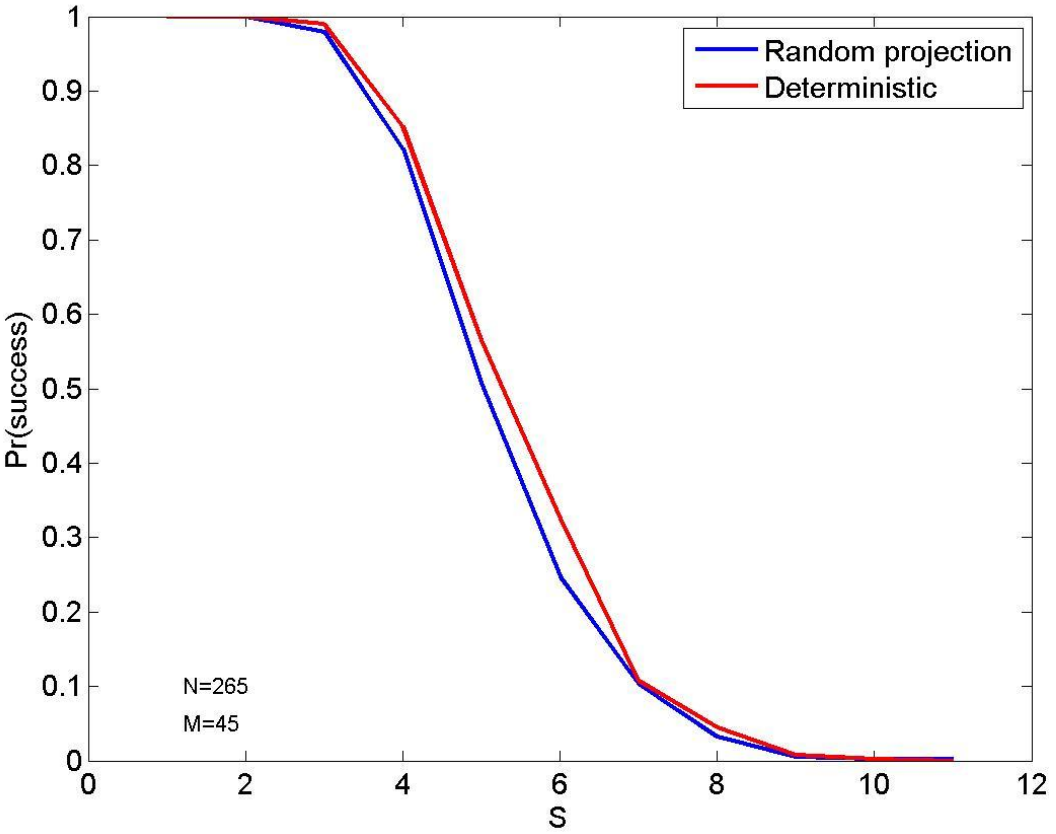
Positions =

[0 1 2 3 4 6 8
10 12 15 18 21 24
28 32 36 40 45 50
55 60 66 72 78 84
91 98 105 112 120
128 136 144 153 162
171180 190 200 210
220 231 242 253 264]

M = 45



Computed with 'spgl1'



CHALLENGE III: RANDOM PROJECTION (?)
Optimum sparse ruler

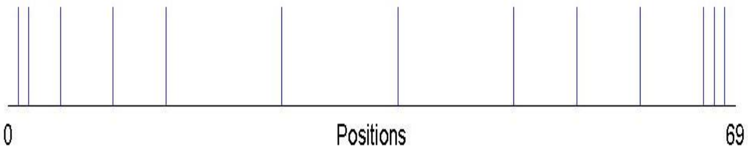
Example

Positions =

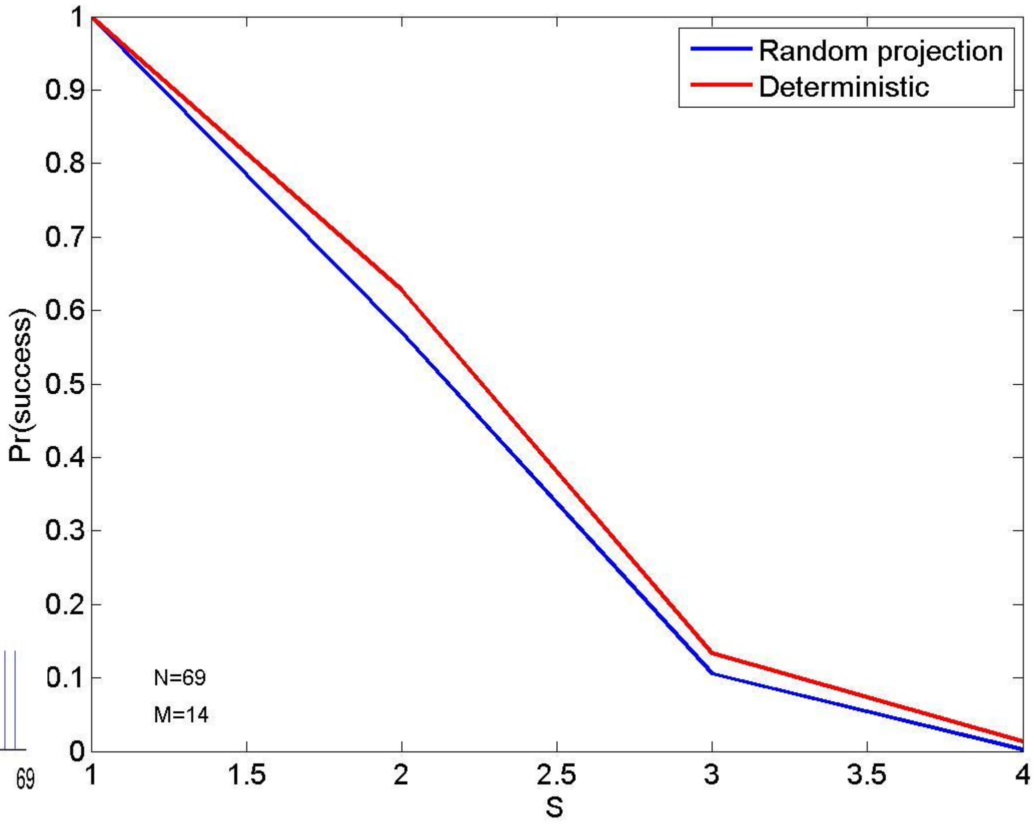
[0, 1, 2, 5, 10, 15, 26, 37, 48, 54, 60, 66, 67, 68]

M = 14

N = 69



Computed with 'spgl1'



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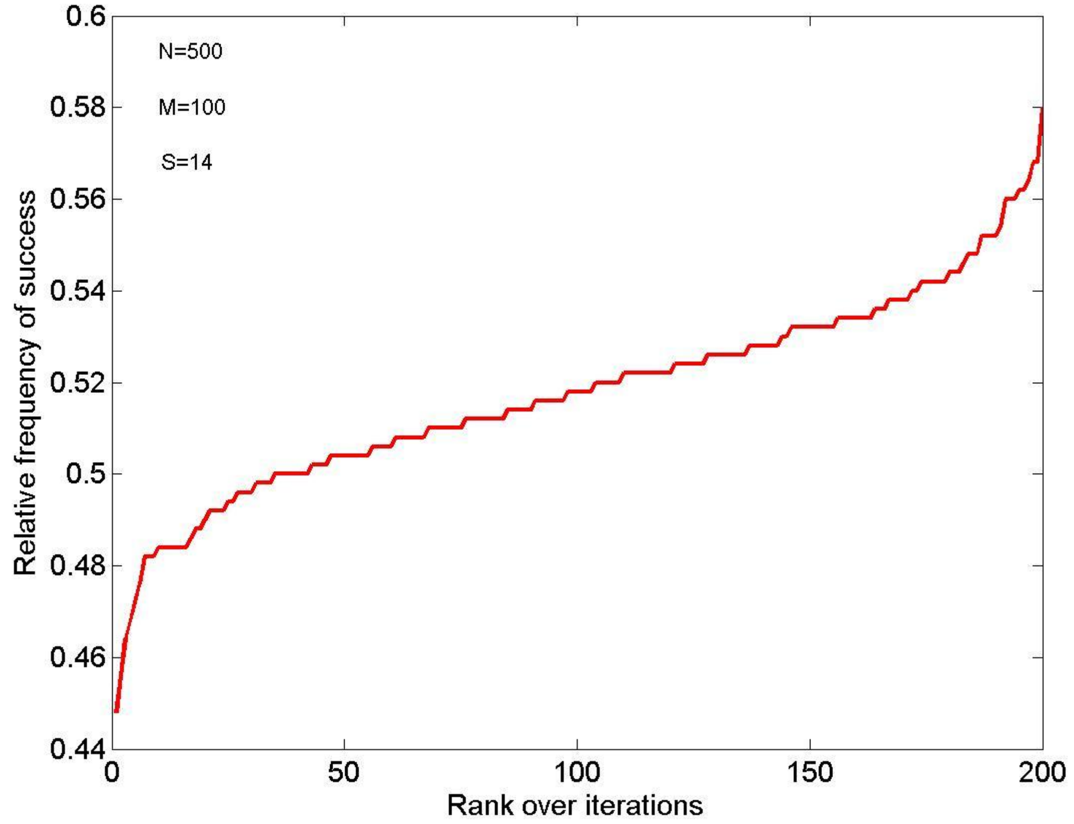
CHALLENGE III: RANDOM PROJECTION (?)

Random search for optimum selection

- 200 random selections
- For each selection
  - Simulation of 200 scenes and reconstructions
- Selection with maximum probability of success fixed
- Further 400 simulations of scenes and reconstructions
- Comparison with non-optimized random selection



CHALLENGE III: RANDOM PROJECTION (?)

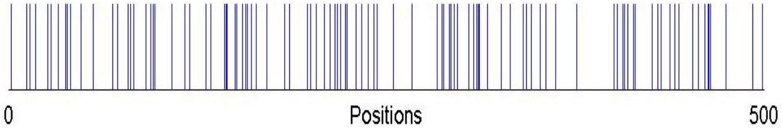
Optimized random selection

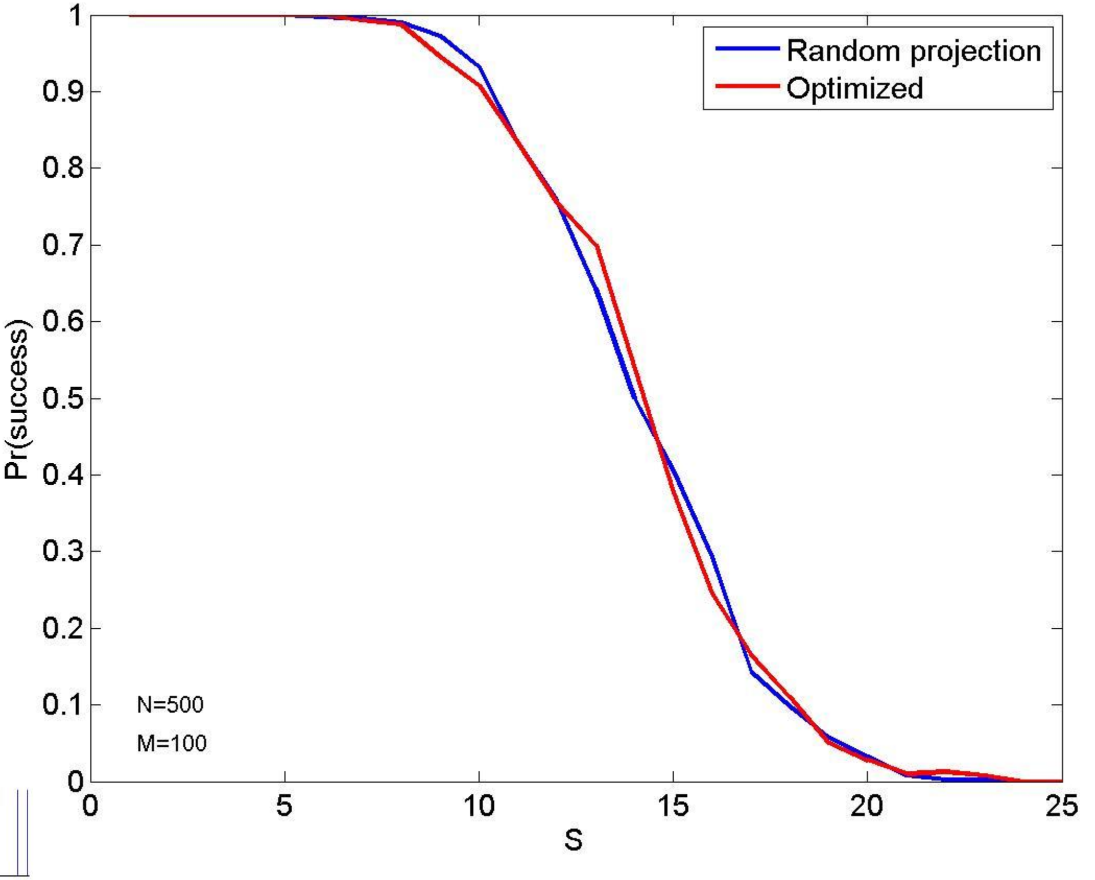
Example

M = 100

N = 500

optimized for S=14





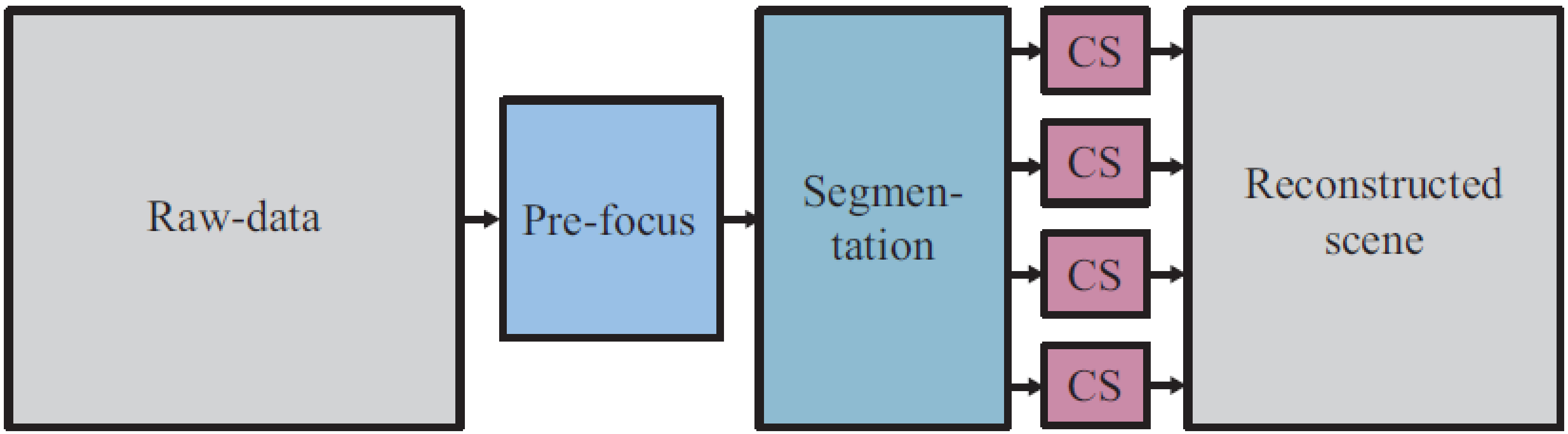
Challenge IV: LARGE SCENES / RAWDATA

For radar techniques as SAR the sensing matrices are for common scene sizes much too large to apply CS algorithms.

We propose a mosaicing technique based on 'pre-focus'

CHALLENGE IV: LARGE SCENES / RAWDATA

Principle of pre-focusing / mosaicing



See also:

S. Qin, Y. D. Zhang, Q. Wu and M. G. Amin, "Large-scale sparse reconstruction through partitioned compressive sensing," 2014 19th International Conference on Digital Signal Processing, Hong Kong, 2014, pp. 837-840.

CHALLENGE IV: LARGE SCENES / RAWDATA

Principle of pre-focusing / mosaicing

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e}$$

Original large-sized linear equation system

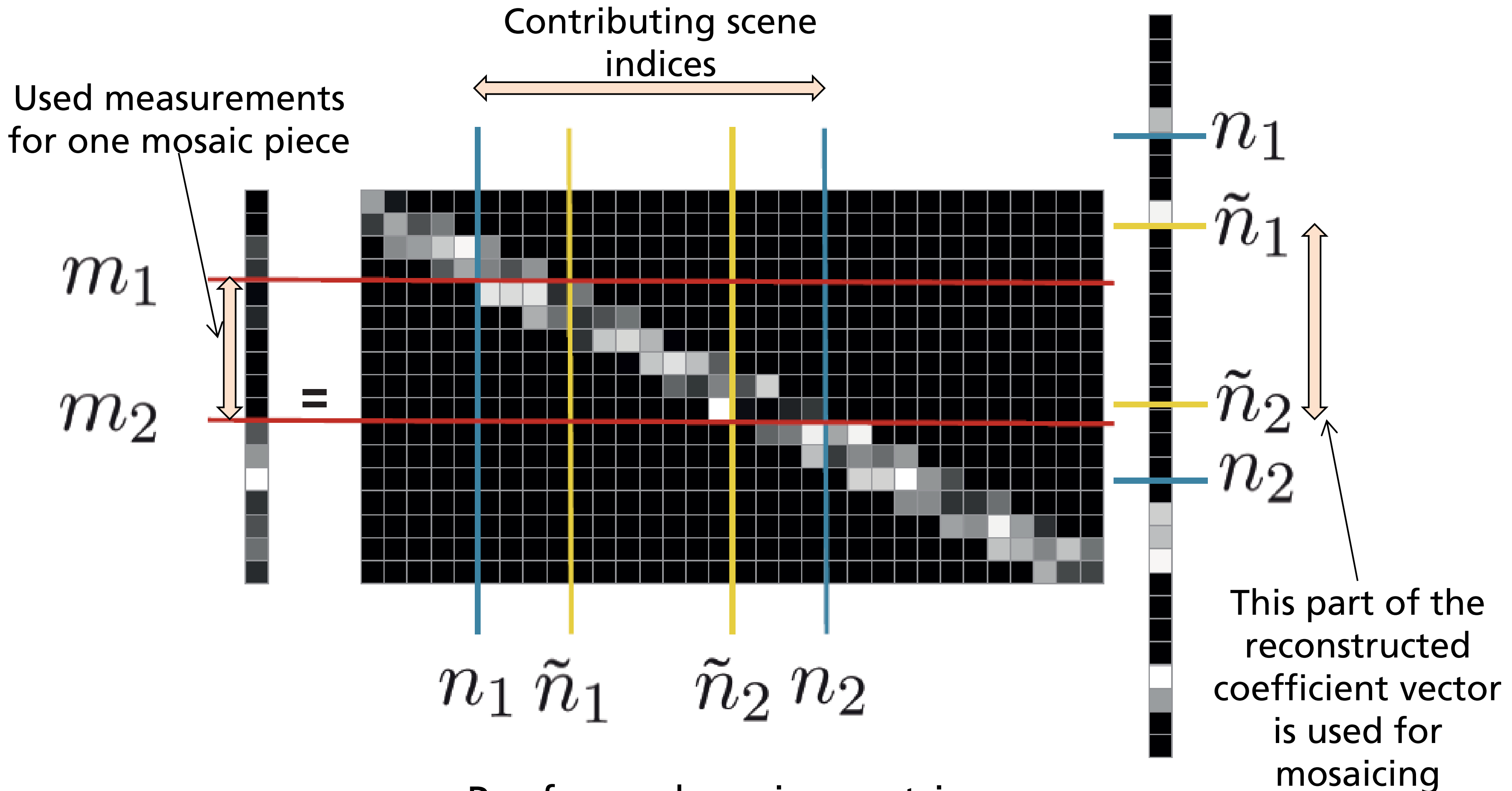
$$\begin{aligned} \widetilde{\mathbf{y}} &= \mathbf{P}\mathbf{y} \\ &= \mathbf{P}\mathbf{A}\mathbf{x} + \mathbf{P}\mathbf{e} \\ &= \widetilde{\mathbf{A}}\mathbf{x} + \widetilde{\mathbf{e}}. \end{aligned}$$

Application of a pre-focus operator

The aim is now to choose **P** in such a way that $\widetilde{\mathbf{A}}$ obtains the form of a band matrix.

CHALLENGE IV: LARGE SCENES / RAWDATA

Principle of pre-focusing / mosaicing

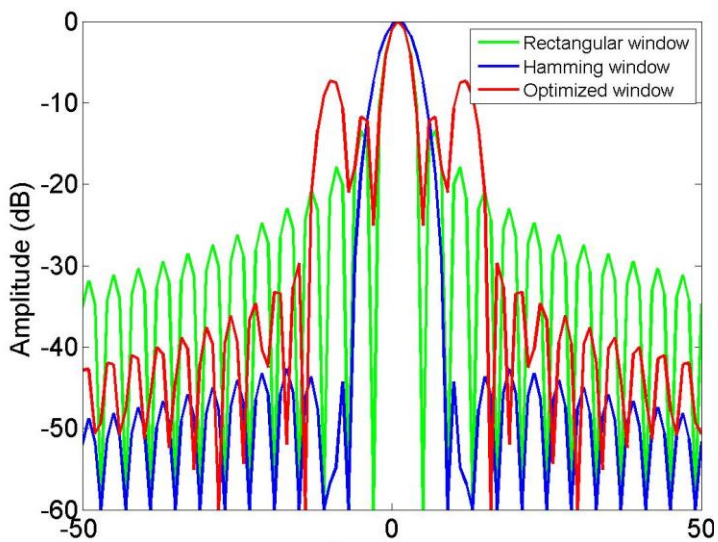


Pre-focused sensing matrix

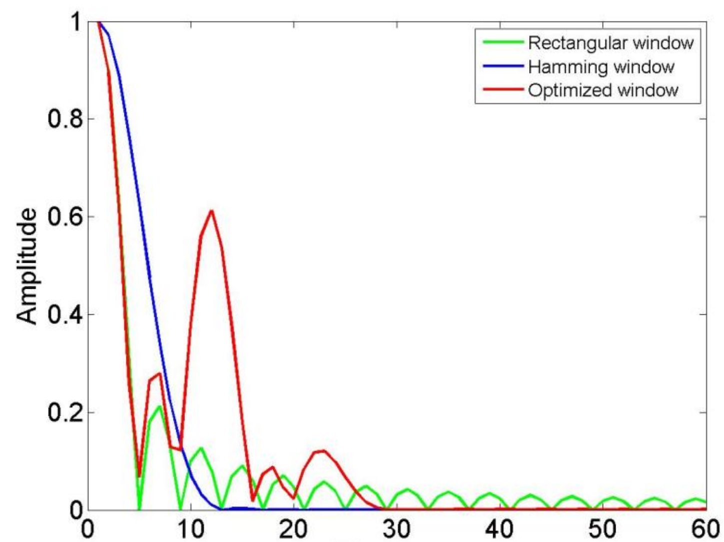
CHALLENGE IV: LARGE SCENES / RAWDATA

Principle of pre-focusing / mosaicing (1D)

- To achieve approximately the form of a band-matrix, the data in the (spatial) frequency domain are pre-focused by a low-pass.
- This has to have very low sidelobes, and an adequate passband interval large enough to preserve enough information.



Transfer functions



Point spread functions

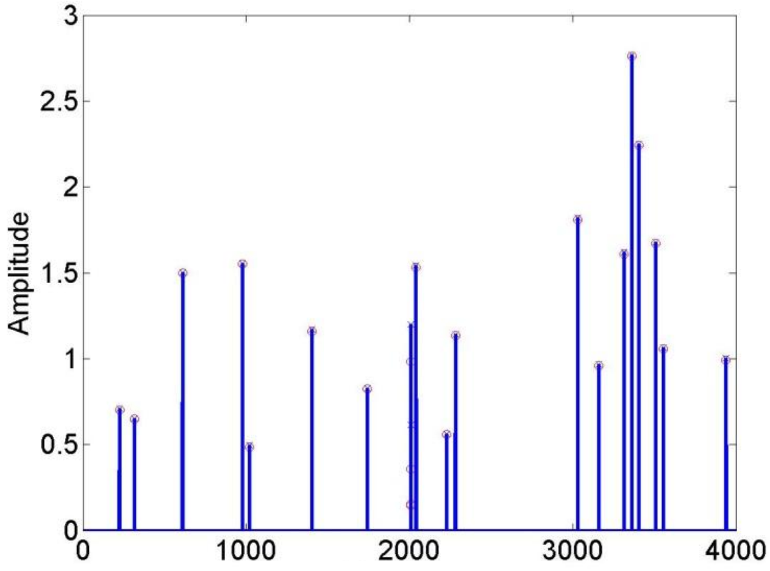
CHALLENGE IV: LARGE SCENES / RAWDATA

Principle of pre-focusing / mosaicing (1D)

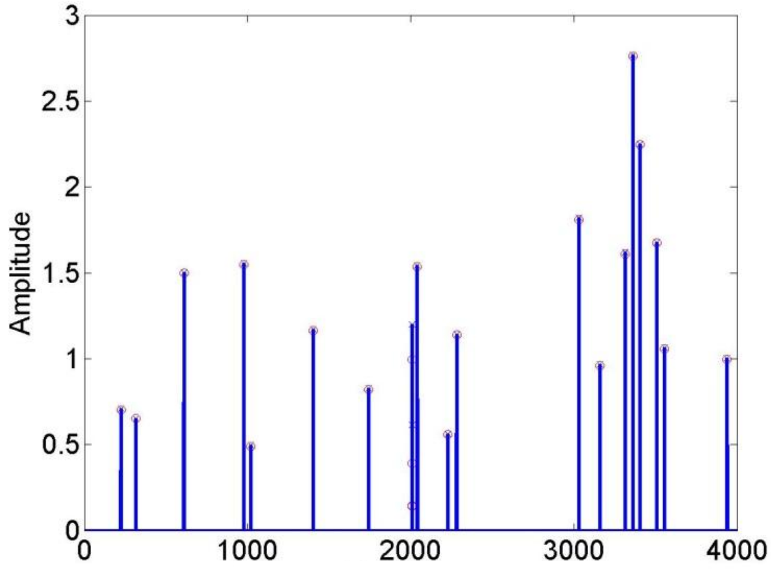
- Simulation: Comparison of a direct and a mosaiced recovery

N = 4000, M = 2000

S = 20, #Segments = 5



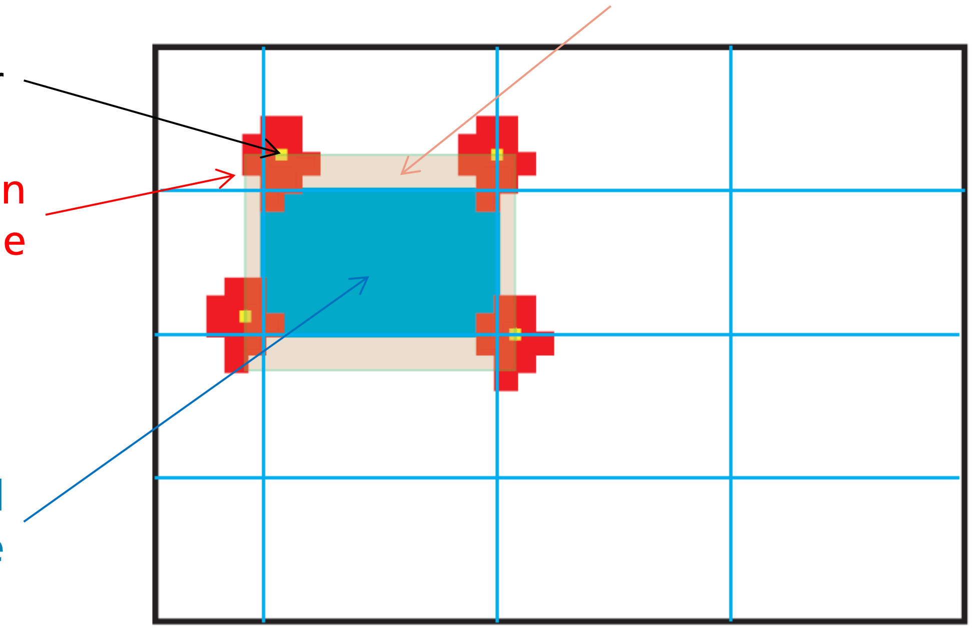
Direct recovery

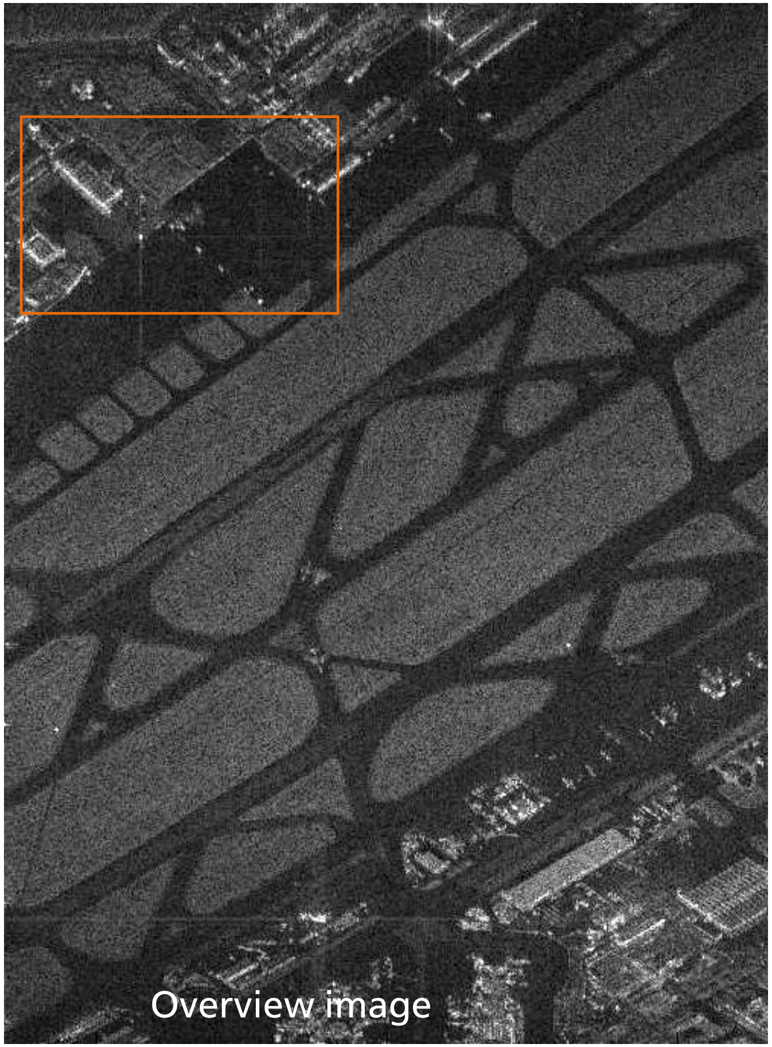


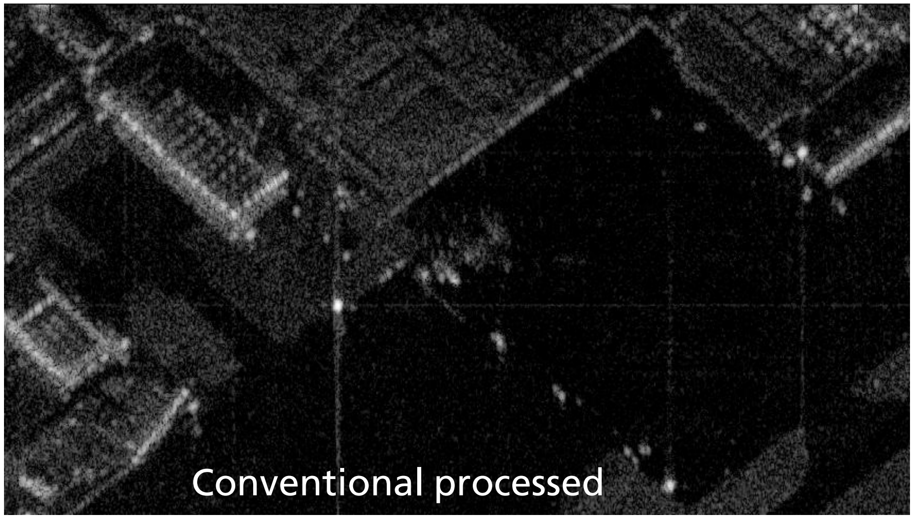
Mosaiced recovery

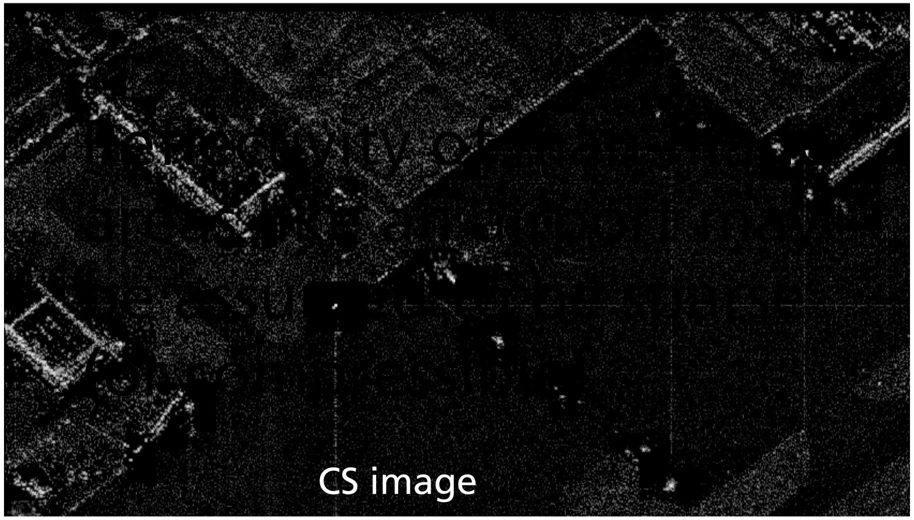
CHALLENGE IV: LARGE SCENES / RAWDATA
Principle of pre-focusing / mosaicing (2D)

- Real data recorded by Fraunhofer AER-II
- Size of the processed SAR-Image: 1991 x 751 = 1 495 241 Pixel
- Number of Mosaic-Pieces: 35 x 35 = 1225

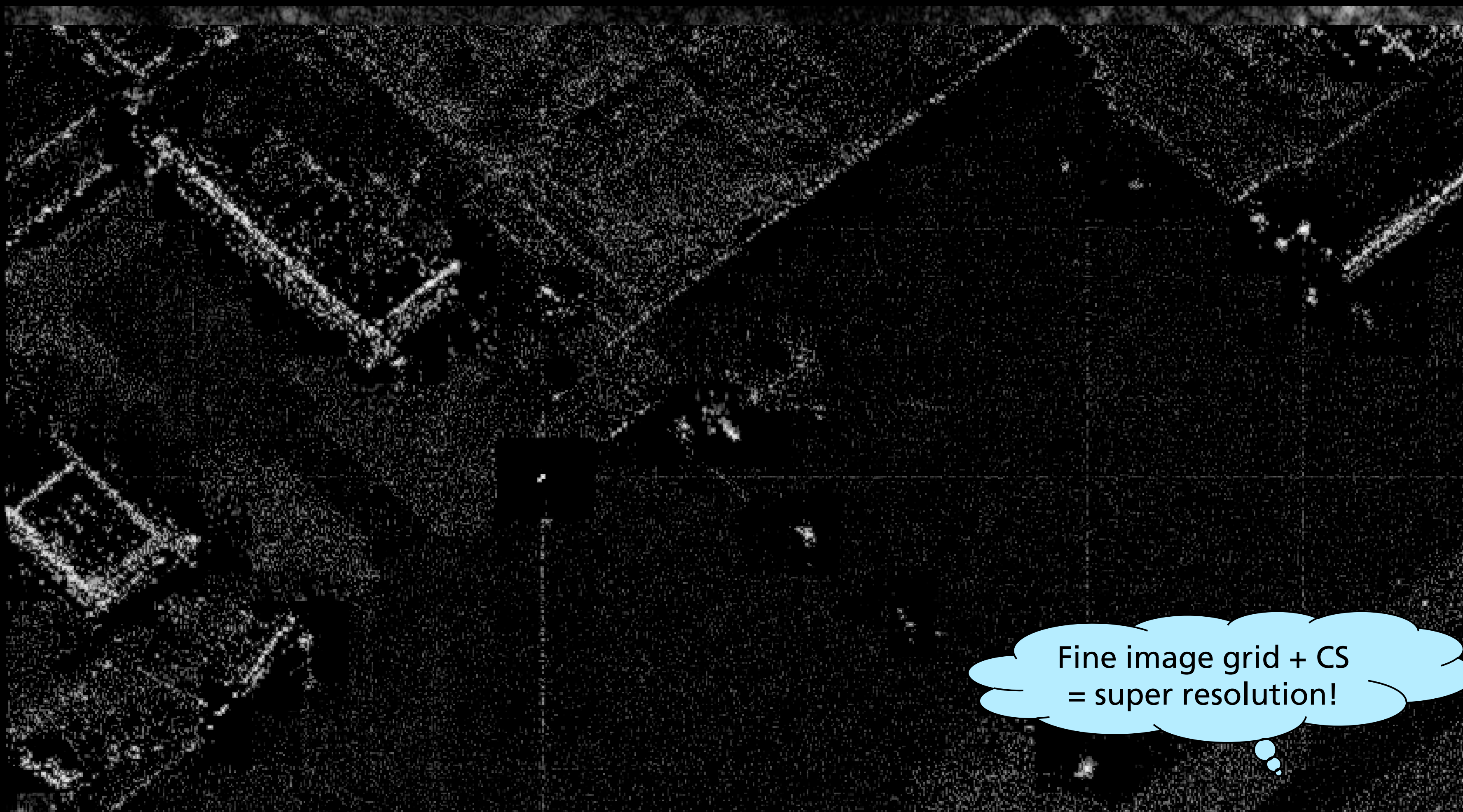



Overview image


Conventional processed


CS image

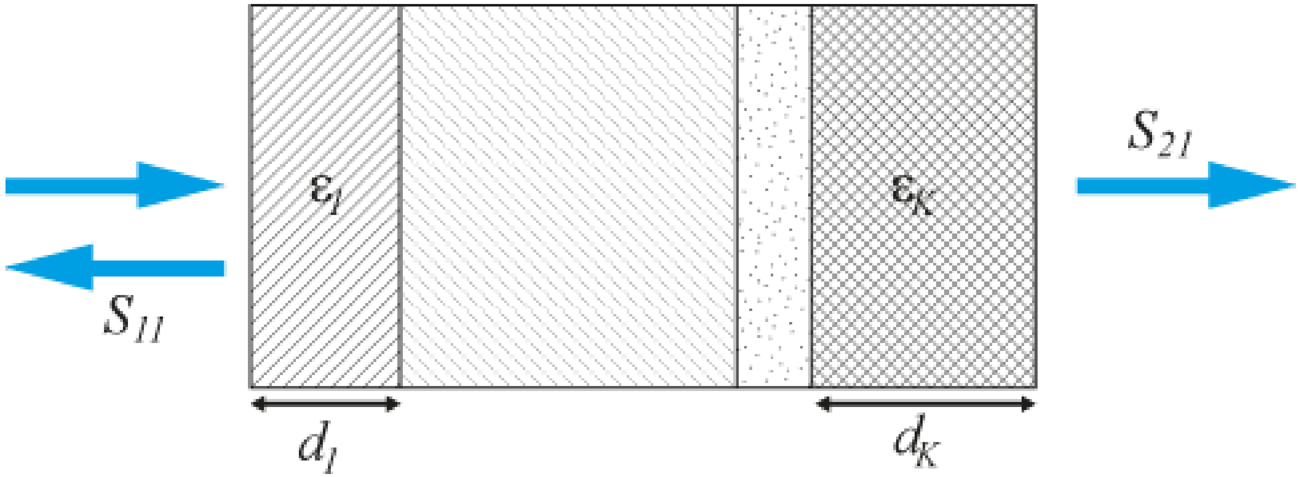
MOSAICING FOR CS-PROCESSING OF A SAR-IMAGE



Challenge V: NON-LINEAR SENSING

CHALLENGE V: NON-LINEAR SENSING

Example: ε-layer retrieval



We regard a material probe composed of K homogeneous lossless plane plates with different permittivities with relative dielectric constants $\epsilon_1,\ldots,\epsilon_K$ which are assumed to be constant over the measured frequency range and thicknesses $d_1,\ldots,d_K$ which are unknown.

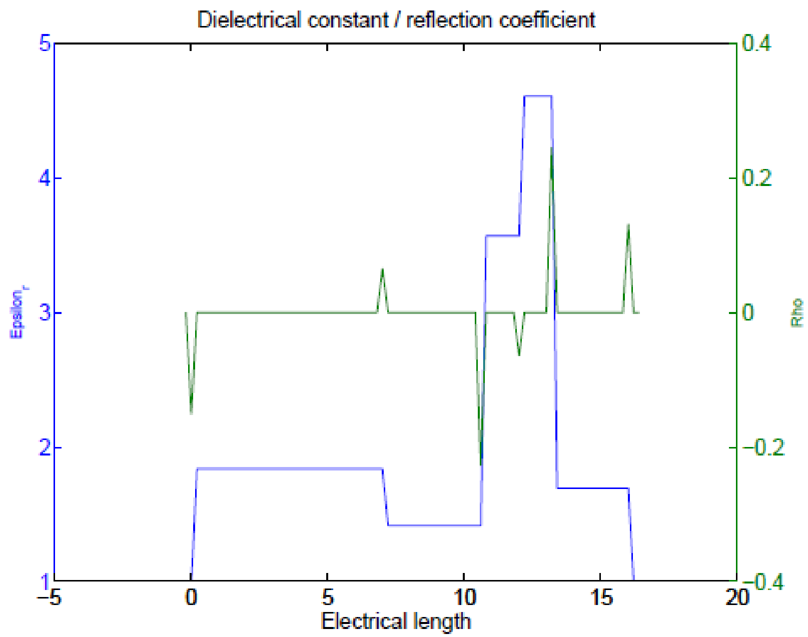
There are only a few layers.

The S-parameters are measured over a range of frequencies.

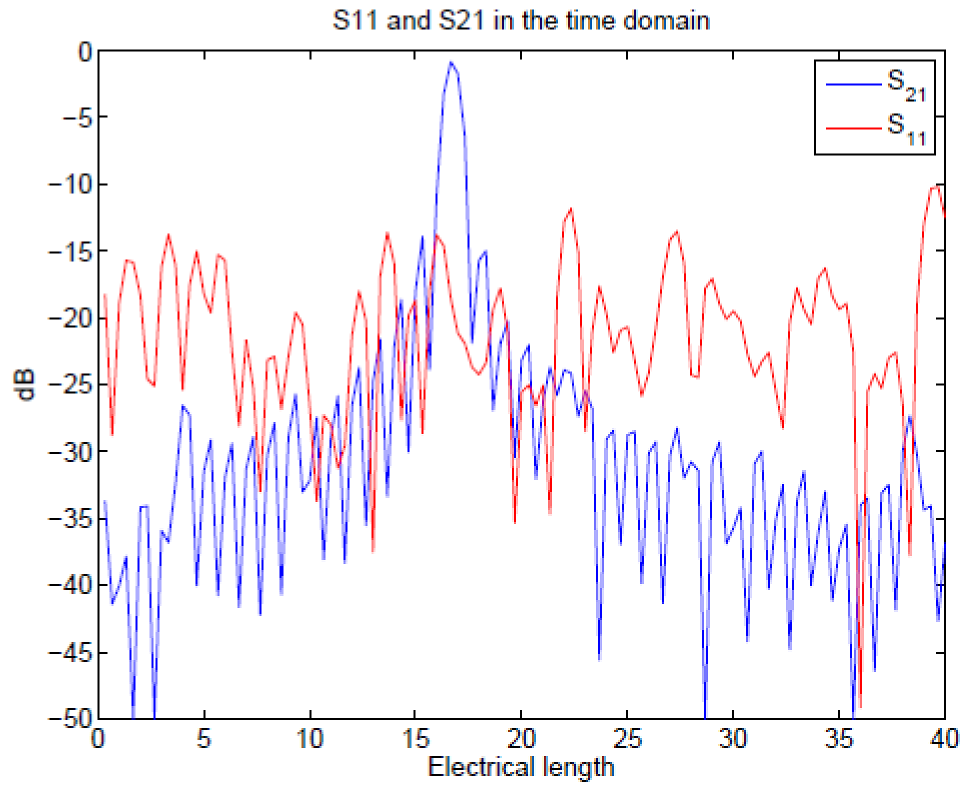
Determine $\epsilon_1,\ldots,\epsilon_K$ and $d_1,\ldots,d_K$!

CHALLENGE V: NON-LINEAR SENSING

Example: ε-layer retrieval (simulation)



Original e-layers and local reflection coefficients



Measured S-parameters in time domain

CHALLENGE V: NON-LINEAR SENSING
Example: ε-layer retrieval

CS-solution: The model of the probe is divided into *N* thin slices of equal relative electrical lengths $\Delta\mathcal{L}$, within which a constant ε is assumed.

Chain matrix for ε-jump:

ρ=reflection coefficient at a slice transition

$$\mathbf{C}_{jump}(\rho) = \frac{1}{\sqrt{1-\rho^2}} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}$$

$$\rho = \frac{\frac{1}{\sqrt{\epsilon_{n+1}}} - \frac{1}{\sqrt{\epsilon_n}}}{\frac{1}{\sqrt{\epsilon_{n+1}}} + \frac{1}{\sqrt{\epsilon_n}}}$$

Chain matrix for passing the slice:

$$\mathbf{C}_{sl}(f) = \begin{pmatrix} q(f) & 0 \\ 0 & q^*(f) \end{pmatrix}$$

$$q(f) = \exp\left\{-j2\pi\frac{f}{f_0}\Delta\mathcal{L}\right\}$$

CHALLENGE V: NON-LINEAR SENSING

Example: ε-layer retrieval

Chaining the chain-matrices:

$$\mathbf{C}(f)(\boldsymbol{\rho}) = \mathbf{C}_{jump}(\rho_{N+1})\mathbf{C}_{sl}(f)\ldots\mathbf{C}_{sl}(f)\mathbf{C}_{jump}(\rho_2)\mathbf{C}_{sl}(f)\mathbf{C}_{jump}(\rho_1)$$

$$S_{11}(f,\boldsymbol{\rho}) = -\frac{C_{21}(f)}{C_{22}(f)}, \quad S_{21}(f,\boldsymbol{\rho}) = \frac{1}{C_{22}(f)}.$$

Idea of CS-recovery of the internal reflection coefficients:

Vector of measurements over the frequencies:

$$\begin{aligned} \mathbf{z} &= \varphi(\rho) + \mathbf{n} \\ &\approx \varphi(\hat{\rho}) + \nabla\varphi(\hat{\rho})(\rho - \hat{\rho}) + \mathbf{n} \\ \mathbf{y}(\hat{\rho}) &:= \mathbf{z} - \varphi(\hat{\rho}) + \nabla\varphi(\hat{\rho})\hat{\rho} \\ &\approx \nabla\varphi(\hat{\rho})\rho + \mathbf{n} \\ &= \mathbf{A}(\hat{\rho})\rho + \mathbf{n} \end{aligned}$$

CHALLENGE V: NON-LINEAR SENSING

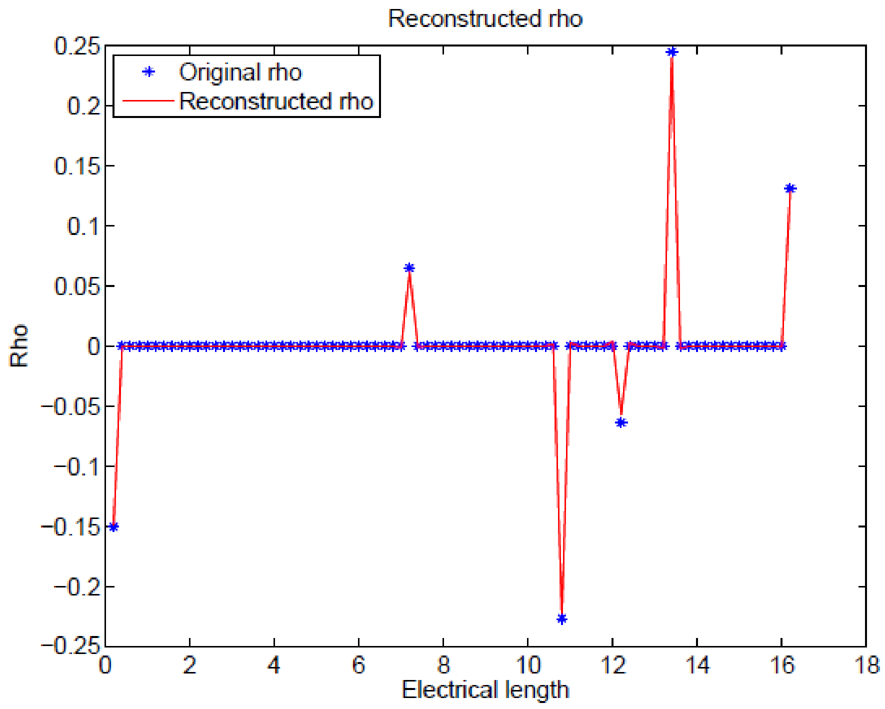
Example: ε-layer retrieval

Iteration: $\hat{\rho}_0 = \mathbf{0}$ $it = 0$

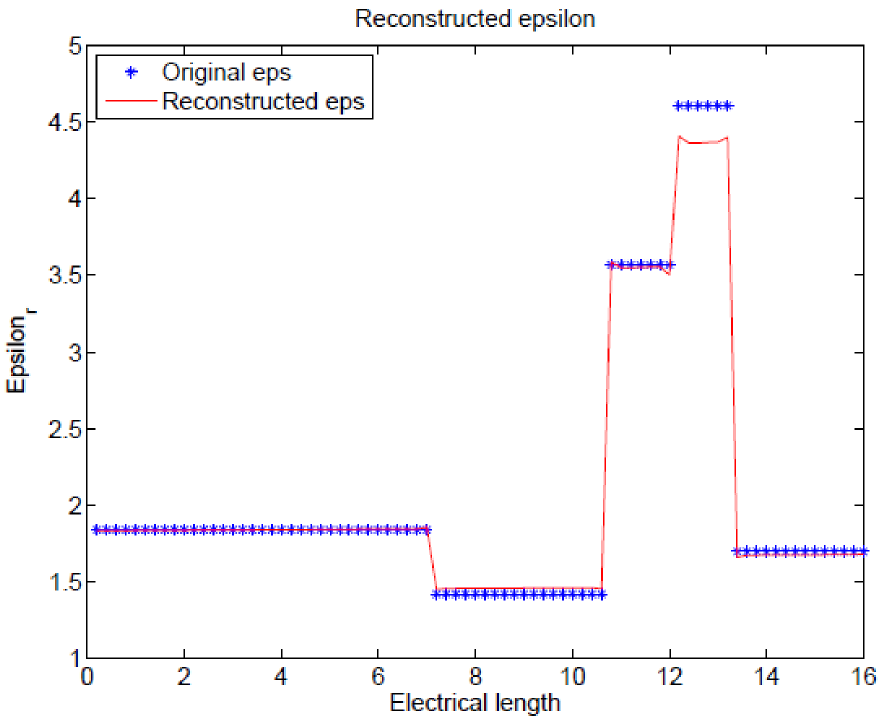
- calculate $\mathbf{A}(\hat{\rho}_{it})$
- solve $\mathbf{y}(\hat{\rho}) = \mathbf{A}(\hat{\rho}_{it})\hat{\rho}_{it+1} + \mathbf{n}$ via CS
- $it = it + 1$
- until a stop criterium is met.

CHALLENGE V: NON-LINEAR SENSING

Example: ε-layer retrieval



Internal reflection coefficients

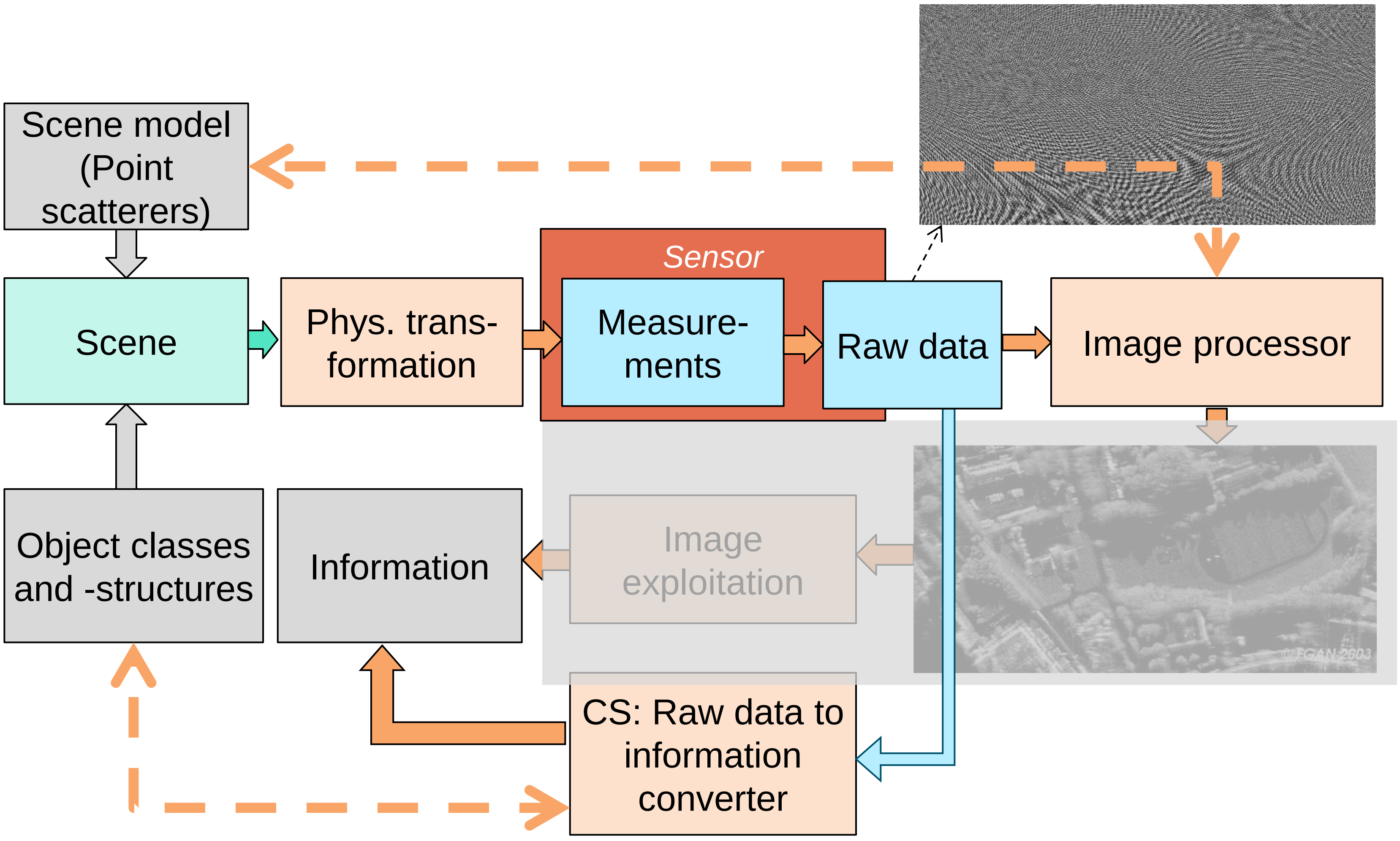


Reconstruction of ε-layers

Challenge VI: HIGHER LEVEL INFORMATION RETRIEVAL

CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

View behind the curtain – Raw data to information converter



CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL
Extraction of scatterers at straight lines in ISAR data

- The scatterers on man made targets are often arranged along straight lines

- Traditional approach:
  - Form an image
  - Apply methods like Hough-transform to identify straight lines
- Our approach:
  - Find a small number of lines explaining the measurements due to scatters placed on these lines largely!
  - A case for block sparse recovery

CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

Extraction of scatterers at straight lines in ISAR data

A finite set of potential lines has to be provided as well as a set of points along each line.

Model for the measurements, arranged as a column vector:

Number of lines

Number of grid points on lane ν

coefficients (reflectivity)

$$\mathbf{z} = \sum_{\nu=1}^{N_l} \sum_{\mu=1}^{M(\nu)} \mathbf{s}_{\nu\mu} x_{\nu\mu} + \mathbf{n}$$

Noise

Measurement

Signal vector for point μ on line ν

CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

Extraction of scatterers at straight lines in ISAR data

Model for the measurements, arranged as a column vector:

$$\hat{\mathbf{x}} = \operatorname{argmin} \sum_{\nu=1}^{N_l} \sqrt{\sum_{\mu=1}^{M(\nu)} |x_{\nu\mu}|^2}$$

$$\text{subj. to } \|\mathbf{z} - \sum_{\nu=1}^{N_l} \sum_{\mu=1}^{M(\nu)} \mathbf{s}_{\nu\mu} x_{\nu\mu}\|_2 \leq \sigma.$$

l1 norm of rms vector

Amplitude rms of line n (l2 norm)

Rest not explained by sparse model

CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

Extraction of scatterers at straight lines in ISAR data

Goal (sparsity in the occupied planes):
Find as few as possible lines explaining the measurement with a remaining error at noise level!
Grid of potential lines and points required!

Mixed ℓ_1/ℓ_2- norm approach:
Minimize

$$\sum_{\nu=1}^{Np} \sqrt{\sum_{\mu=1}^{M(\nu)} |x_{\nu\mu}|^2} \text{ subj. to } \|\mathbf{z} - \sum_{\nu=1}^{Np} \sum_{\mu=1}^{M(\nu)} \mathbf{s}_{\nu\mu} x_{\nu\mu}\|^2 \leq \sigma^2.$$

l1 norm of rms vector

Amplitude rms of line n (l2 norm)

Rest not explained by sparse model

CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

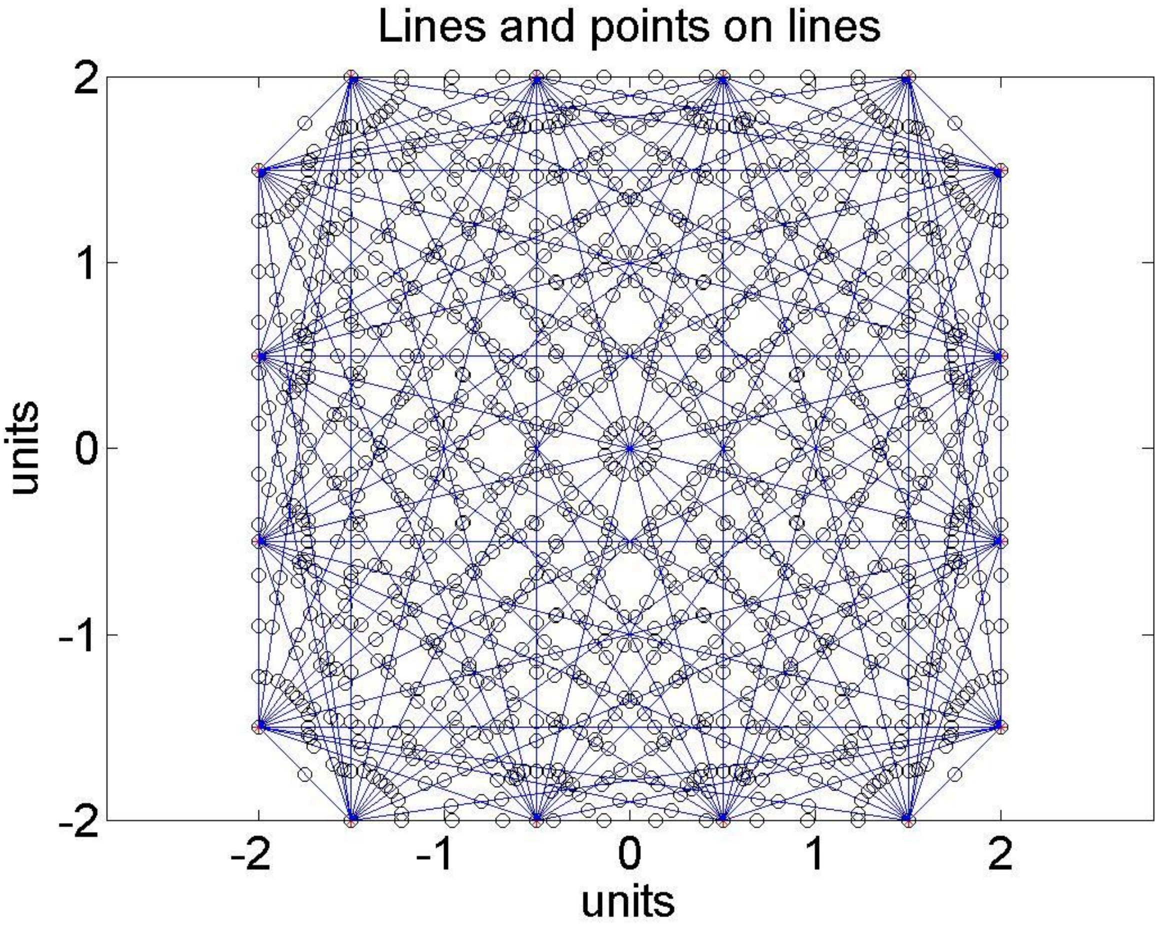
Extraction of scatterers at straight lines in ISAR data

Alternative: Block Orthogonal Matching Pursuit (BOMP)

- it = 1
- Find line with maximum accumulated energy
- Iterate
  - Calculate remainder for the measurement projected to the space spanned by the signals for the points of all planes found until now.
  - Find plane with maximum energy with regard to the remainder
  - it=it+1
- Until the rest can be explained by noise

CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

Extraction of scatterers at straight lines in ISAR data

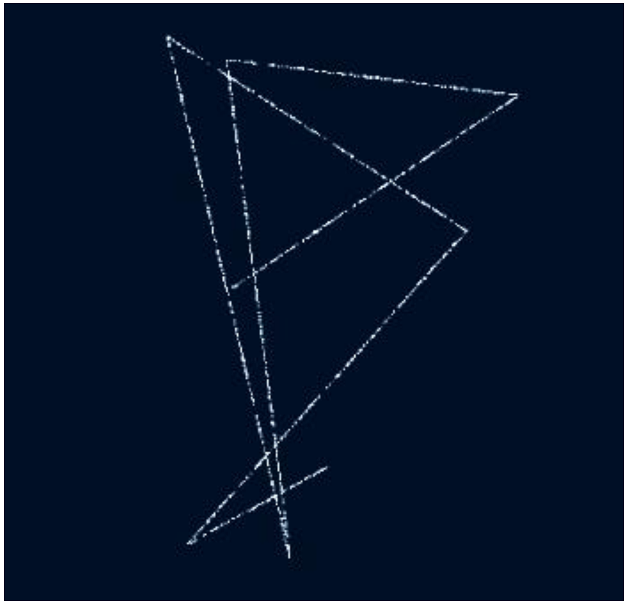


Principle of line-grid and point-grids on the lines

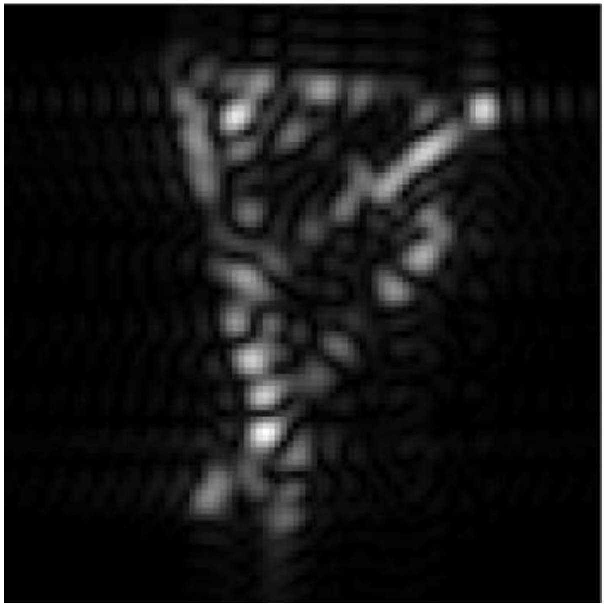
CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

Extraction of scatterers at straight lines in ISAR data

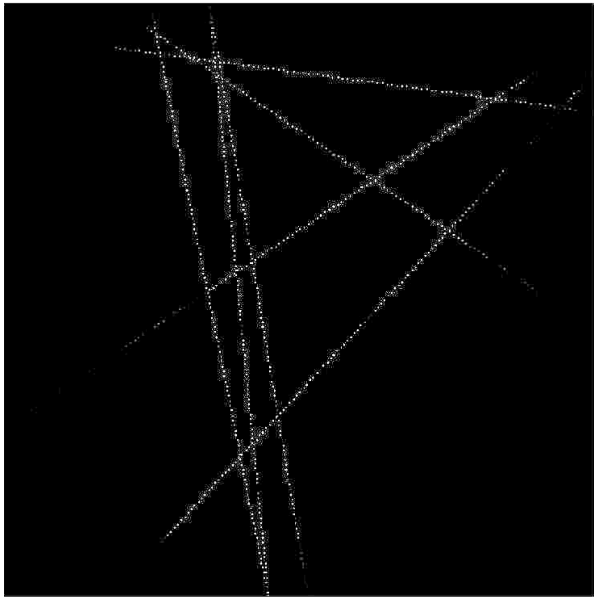
Simulation



Original scene



Fourier reconstruction

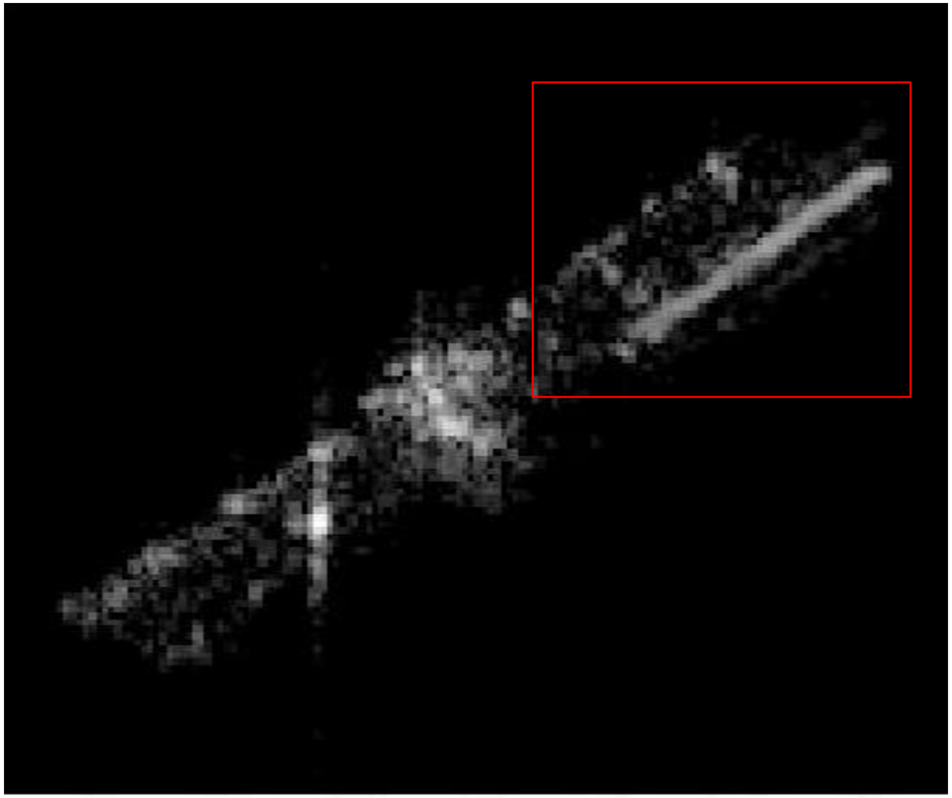


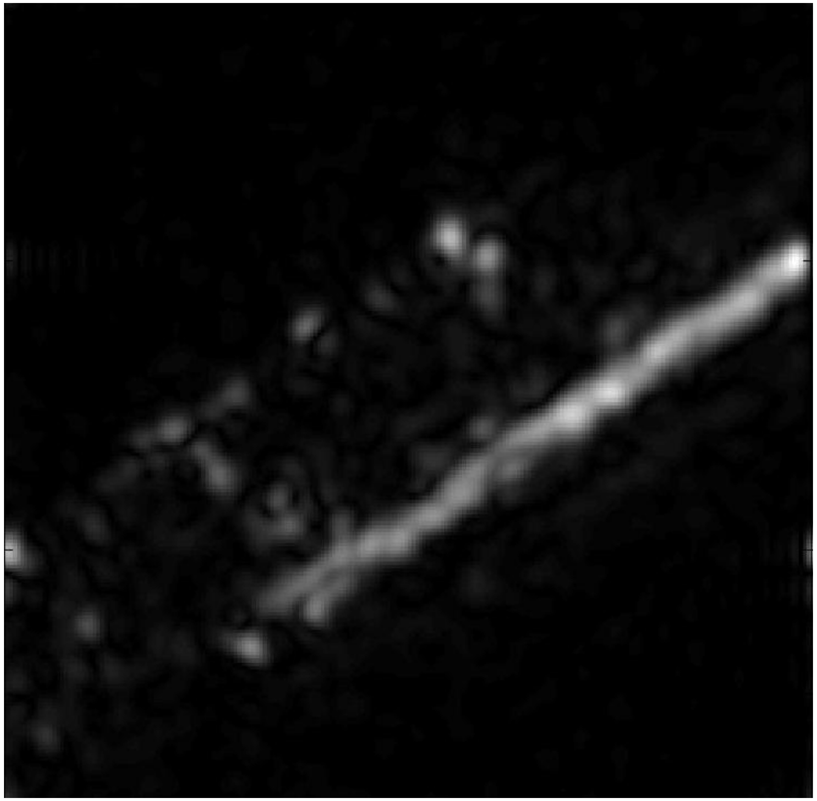
CS-lines reconstr.

CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

Extraction of scatterers at straight lines in ISAR data

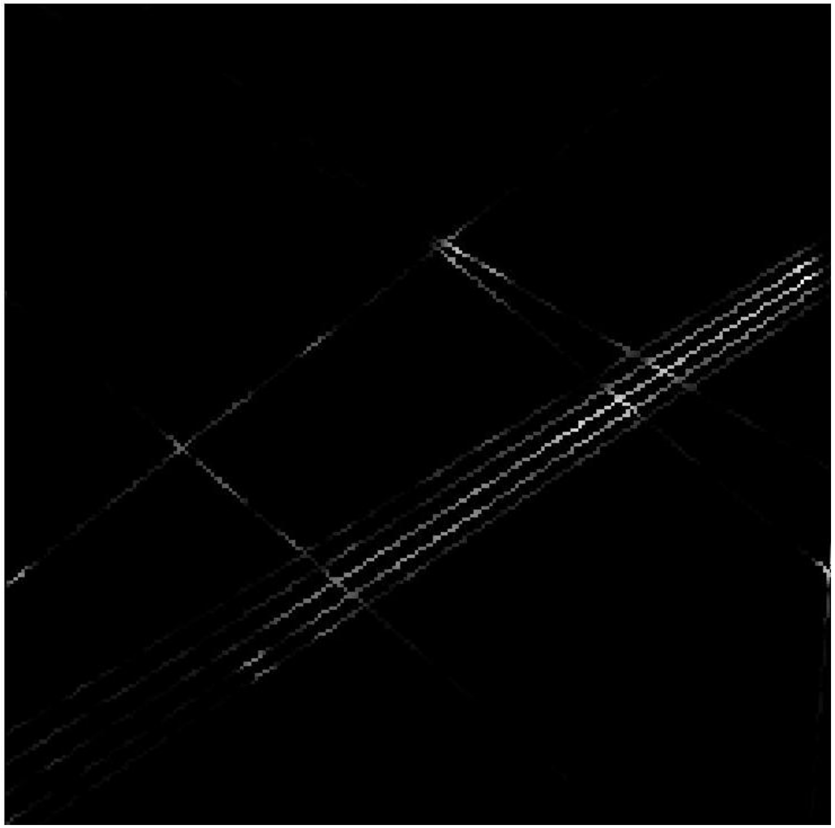
ISAR-Image of a satellite, recorded by the FHR-radar TIRA

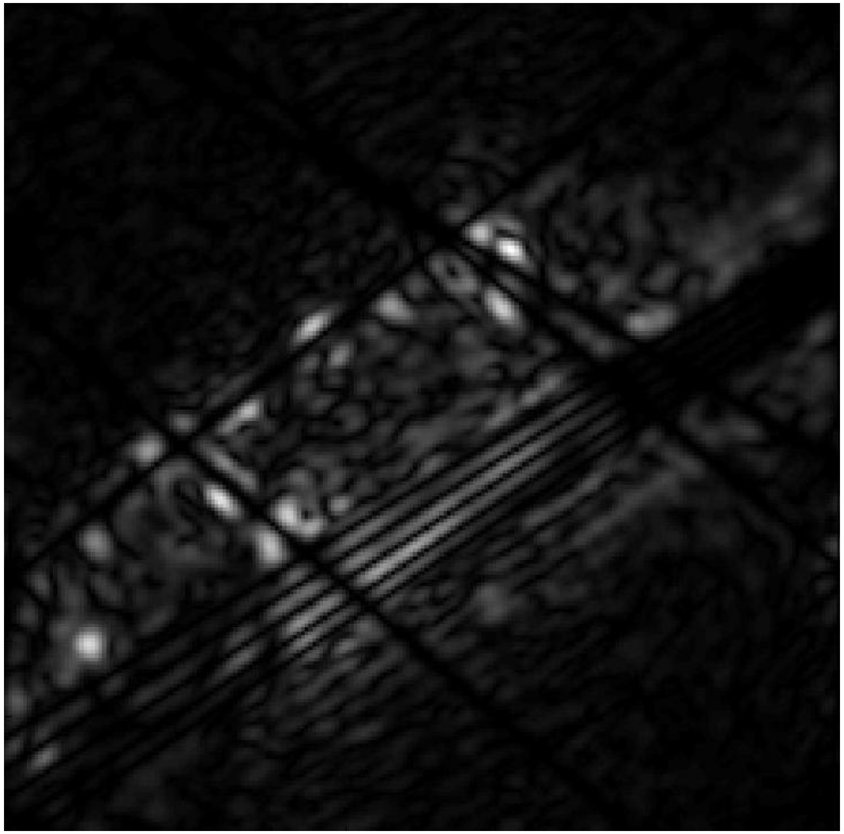


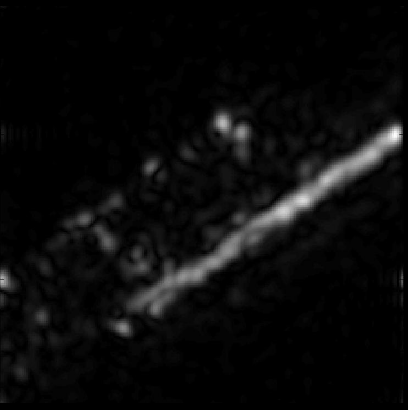
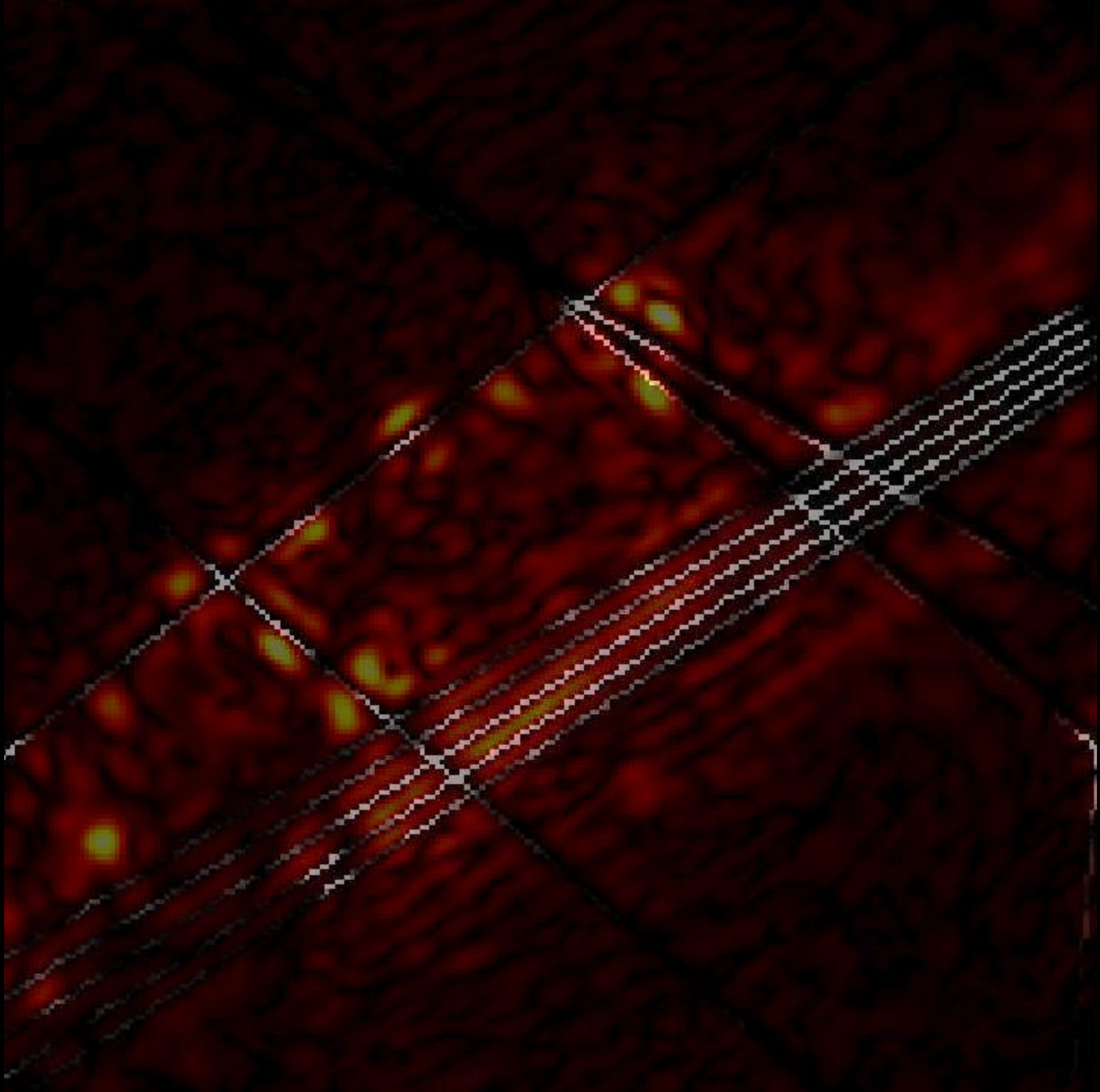


CHALLENGE VI: HIGHER LEVEL INFORMATION RETRIEVAL

Extraction of scatterers at straight lines in ISAR data





Thank you for listening!