Adaptive Progressive Edge-Growth Construction of Low Density Sensing Matrices

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Phase-Shift-Based ToF

- **Measure phase shift instead of time:**
  - Modulated light, typically in the NIR band is emitted to the scene.
  - Modulation frequency in the radio frequency range (20-120MHz).
  - Reflected light reaches the camera with a delay that is proportional to the total distance from the illumination system to the scene point and then to the camera.
  - Intelligent pixels, featuring two or more channels (i.e., multitap pixels) are used to internally correlate the incoming signal with several reference signals.
  - Reference signals with different phase delays are used to sample the cross-correlation function.
  - The relative phase shift can be computed from few measurements (cross-correlation samples).

\[
d = \frac{c}{4\pi f_{\text{mod}}} \theta_d
\]

Since the speed of light is constant, the distance from the sensor to the scene points can be estimated from the phase delay: \( \theta_d = \theta_2 - \theta_1 \).
**Motivation: Pulsed ToF**

- **Short pulses instead of CW:**
  - Instead of continuously emitting a periodic signal, in pulsed ToF a single pulse is emitted and its echo is received.
  - After a period of inactivity the process is repeated until the desired SNR is attained.
  - **Reducing the pulse width yields better depth resolution**, at the time it allows attaining improved SNR without increasing the average illumination power. ✅
  - In a conventional pulsed system, the depth measurement range is fully determined by the pulse width. **Too short widths yield unacceptably short ranges. ❌**
  - One would desire attaining the depth resolution corresponding to the shortest pulse width allowed by the hardware, while keeping an arbitrarily large range. 🧐

The triangular cross-correlation function eliminates the need for a costly arctangent transformation. The depth can be estimated linearly from the raw data.

\[
d = c \frac{Q_B}{T Q_A + Q_B}
\]
CS for Pulsed ToF

- **Goals:**
  - High depth resolution
    - Short pulses → Limits: drivers and light sources
    - Small discretization steps in time domain
  - Large depth range
    - Many discrete steps
- **What is favorable?**
  - Extreme sparsity of the pulse echo(es) in time domain
- **What is unfavorable?**
  - Intractable signal dimensionality
    - E.g., desired range: 10m at 1mm resolution yields \( n = 10^4 \) dimensions
  - Unknown signal support → Needle in the hay
    - Dense sensing matrices ensure that we don’t miss the support, but…
    - The SNR of the measurements tends to zero as \( n \to \infty \)
Random Sensing Matrices

- **Gaussian:**
  - Elements drawn from \( i.i.d. \) normal random variables, e.g., of zero mean and \( 1/m \) variance. Good CS matrices, from an RIP perspective.

- **Bernoulli:**
  - Bernoulli with elements +1 and −1, drawn from \( i.i.d. \) Bernoulli random variables with \( p = q = 0.5 \). Good CS performance, close to that of Gaussian matrices. Binary.
  - Bernoulli with elements +1 and 0, drawn from \( i.i.d. \) Bernoulli distributions with \( p = q = 0.5 \). Reportedly bad CS performance\(^1\), but binary and sparse.

\[\text{Gaussian sensing matrix} \quad \text{Bernoulli sensing matrix}\]

Reduction of storage requirements by, at least, the bit depth used for quantization of the non-binary matrix elements.

\(^1\) V. Chandar, “A negative result concerning explicit matrices with the restricted isometry property,” Tech. Rep., 2008.
Random Sensing Matrices

- **Performance Evaluation:**
  - Signal dimensionality $n = 1024$. Number of measurements $m$ as a function of the sparsity $s$. $10 \leq s \leq 100$, $1 \leq \frac{m}{s} \leq 5$.

- Coherence Evaluation:

- Sparse Recovery Evaluation:
Low-Density Binary Matrices

- **Objectives:**
  - Low density, i.e., as few nonzeros per row/column as possible
  - High regularity, i.e., the density can be globally defined
  - Low coherence

- **Progressive deterministic construction:**
  - Sequential addition of edges to the corresponding Tanner graph
  - Criterion: (local) *girth* maximization at each edge addition
  - Problem: a sequence of optimal decisions does not ensure attaining the global optimum, in terms of *girth* maximization
  - Example:

```
1 1 0 1 0 1
0 0 1 1 1 0
0 1 0 0 1 0
1 0 1 0 0 1
```

\[ m = 4 \text{ check nodes} \]
\[ n = 6 \text{ symbol nodes} \]

\[ g_l = 6 \]
\[ n = 6 \text{ columns} \]
\[ m = 4 \text{ rows} \]

Tanner Graph

LDPC Matrix
Adaptive Deterministic Construction (Algorithm)

Algorithm 1 Adaptive Progressive Edge-Growth (APEG)

Initialize: $\Omega_x^{(1)} = \emptyset$, $\Pi_i^{(0)} = \emptyset \forall i$, $1 \leq i \leq m$

1: for $i = 1; i := i + 1$ to $i = m$ do
2: for $k = 1; k := k + 1$ to $k = d_\epsilon$ do
3: if $k = 1$ then
4: Candidate set: $\Omega_{ci}^{\text{cand}} = \hat{\Omega}^{(i)}$
5: else
6: Expand tree up to depth $l | \hat{\Omega}_{ci}^l \neq \emptyset$ and $\hat{\Omega}_{ci}^{l+1} = \emptyset$
7: Candidate set: $\Omega_{ci}^{\text{cand}} = \hat{\Omega}_{ci}^l \cap \hat{\Omega}^{(i)}$
8: end if
9: Select symbol node index: $j = \arg \min_{j \in \Omega_{ci}^{\text{cand}}} d_{sj}$

10: Add new edge: $\Pi_i^{(k)} = \Pi_i^{(k-1)} \cup (c_i, s_j)$
11: end for
12: Sensing kernel: $\hat{\phi}_i \in \mathbb{R}^n \mid \phi_{i,j} = 1 \Leftrightarrow (c_i, s_j) \in \Pi_i$
13: Measure: $y_i = \hat{\phi}_i^\top \vec{x}$
14: if $y_i < \varepsilon$ then
15: Update forbidden support: $\hat{\Omega}_x^{(i+1)} = \hat{\Omega}_x^{(i)} \cup \text{supp}(\vec{\phi}_i)$
16: else
17: Preserve forbidden support: $\hat{\Omega}_x^{(i+1)} = \hat{\Omega}_x^{(i)}$
18: end if
19: end for

Forbidden Support Set

Maximum Tree Expansion (level $l$):
$\hat{\Omega}_{ci}^l \neq \emptyset$, $\hat{\Omega}_{ci}^{l+1} = \emptyset$

Candidate Set: $\Omega_{ci}^{\text{cand}} = \hat{\Omega}_{ci}^l \cap \hat{\Omega}^{(i)}$

Edge Addition Loop ($k$)

$\Omega_{ci}$

$\Omega^{(i)}$

$\Omega_{ci}^l$

$\Omega_{ci}^{l+1}$

$\Omega_x$

$\phi_{l,j} = 1$

New Edge: $(c_i, s_j)$

Measure: $y_i = \vec{\phi}_i^\top \vec{x}$

Update $\Omega^{(i)}$ if necessary

Measurement Loop ($i$)
Adaptive Deterministic Construction

- How does the sensing matrix look like?
  - Progressive Edge Growth (PEG) Baseline (no adaptiveness):
  - Adaptive PEG (APEG) Construction:

Strong overlap between sensing kernels support and signal support!
Adaptive Deterministic Construction

- Sparse Recovery Evaluation (normalized error):
  - Progressive Edge Growth (PEG) Baseline:
  - Adaptive PEG (APEG) Construction:

\[ d_c = 20 \]  \[ d_c = 40 \]  \[ d_c = 60 \]
Conclusions

- Gaussian and \( \pm 1 \) Bernoulli random matrices exhibit excellent CS recovery performance.
- A construction method of low-density binary sensing matrices has been proposed that is both deterministic and adaptive.
- Sensing matrices constructed using our APEG method exhibit better recovery performance than random and non-adaptive deterministic matrices.
- Densities between 1% and 5% have been observed to be sufficient when dealing with \( n = 1024 \) dimensions.
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Thank you for your attention.

Did you like the idea?
Then, download the sample code and test the performance of the APEG-LDPC matrices yourself!
https://uni-siegen.sciebo.de/index.php/s/mDKhRTClQoA77I9