Sub-Nyquist Radar with Optimized Sensing Matrices – Performance Evaluation Based on Simulations and Measurements

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Motivation

- Pulse radar can be considered **sparse in time**: Typically few distinct target objects over range

\[ s(t) \]

\[ r(t) \]

\[ x(n) \]
Motivation

• Bandwidth/ range resolution of modern digital radars has gradually increased

• Nyquist sampling of received echo signals produces large amounts of data while goal is to extract small number of targets ⇒ Classic approach more and more inefficient

• Compressed Sensing (CS) can reduce sampling rates below Nyquist limit while still capturing the essential received information:
  - Generalized sampling (rather than classical ADCs)
  - Non-linear recovery algorithms (rather than conventional signal processing)
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- Nyquist sampling of received echo signals produces large amounts of data while the goal is to extract a small number of targets ⇒ Classic approach more and more inefficient.
- Compressed Sensing (CS) can reduce sampling rates below Nyquist limit while still capturing the essential received information:
  - Generalized sampling (rather than classical ADCs)
  - Non-linear recovery algorithms (rather than conventional signal processing)
- Preliminaries:
  - Received signals are sparse w.r.t. some basis  ✔ (time domain)
  - Generalized sampling should be conducted in an incoherent domain e.g. frequency domain
- Practical issues:
  - Received data contain noise and (possibly) clutter components
  - Target objects may appear extended rather than point-like (due to physical size and/or off-grid effects)
Outline

- Sub-Nyquist Radar Scheme with Generalized Sampling in Fourier domain
  - Optimized Design based on Sparse Rulers
  - Practical Realization using Bandpass Filters

- Performance Analysis based on Simulation Results
  - Perfect Support Reconstruction
  - Relaxed Support Reconstruction

- Application to Real-World Radar Data
  - Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar
  - Reconstruction Results

- Conclusions and Future Work
Sub-Nyquist Radar with Generalized Fourier Domain Sampling

General Sub-Sampling Scheme

\[ \Phi \cdot \begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} y \end{bmatrix} \]

- \( \Phi \): Sensing matrix/ generalized sampling matrix
  (sub-sampling factor \( q := M/N \ll 1 \))
- OMP: Orthogonal Matching Pursuit for non-linear recovery

\[ \text{OMP} \]
Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Generalized Nyquist Sampling in the Fourier Domain

\[ F \cdot x = y \]

- \( F \): Discrete Fourier Transform (DFT) matrix \( \Rightarrow \) Fourier domain incoherent to time domain
Sub-Nyquist Radar with Generalized Fourier Domain Sampling

Idea

- Sub-sampling in Fourier domain by selecting a sub-set of $M << N$ rows of $F$
- CoSeRa 2015 paper considered selection of
  (a) individual rows
  (b) sub-blocks of $B$ subsequent rows ($c$ sub-blocks, $M := c \cdot B$)

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- In practical system sub-sampling would be conducted in analog domain
- Option (b) seems more attractive for hardware implementation, since it can be realized with few bandpass filters (BPFs), e.g. $c < 10$
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- Care must be taken concerning the spacing of the BPFs in frequency domain
- It turns out that the overall bandwidth extent (*aperture A*) has significant impact
Sub-Nyquist Radar with Generalized Fourier Domain Sampling Idea (cont.)

- $S_B F \cdot$  
- $x = y$

$S_B$: Block selection matrix ($M \times N$), $\Phi := S_B F$
Sub-Nyquist Radar with Generalized Fourier Domain Sampling Idea (cont.)

- For good recovery performance, spacing of BPFs should be 'irregular'
- Baranski et al. proposed corresponding scheme with $c=4$ BPFs; spacing optimized heuristically

For good recovery performance, spacing of BPFs should be _irregular_.

Baranski et al. proposed a corresponding scheme with c=4 BPFs; spacing optimized heuristically.


In CoSeRa 2015 we showed:
- Placing BPFs according to the marks of an _optimal sparse ruler (OSR) _offers good recovery performance for _arbitrary _sub-sampling q = M/N and numbers of BPFs c
- Superior to random placement of the BPFs
Sub-Nyquist Radar with Generalized Fourier Domain Sampling
Practical Realization using Bandpass Filters

- **Analog domain**: Sampling via $c$ parallel BPFs
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Performance Analysis based on Simulation Results
Perfect vs. Relaxed Support Reconstruction

- **Task:** Recover delays (ranges) $t_k \approx n_k \cdot \Delta$ of K point targets based on measurement equation $y = \Phi \cdot x = S_B \cdot F \cdot x$
  
  ⇒ Support reconstruction problem (as $x$ is approximately sparse)
  
  ($\Delta$: grid-size that would result from Nyquist sampling, $n_k$ integer = range cell index)
Performance Analysis based on Simulation Results
Perfect vs. Relaxed Support Reconstruction

• Task: Recover delays (ranges) $t_k \approx n_k \cdot \Delta$ of $K$ point targets based on measurement equation
  $$y = \Phi \cdot x = S_B \cdot F \cdot x$$
  ⇒ Support reconstruction problem (as $x$ is approximately sparse)
  ($\Delta$: grid-size that would result from Nyquist sampling, $n_k$ integer = range cell index)

• Two cases considered:
  (a) Perfect support reconstruction  ⇒ Detections of OMP must exactly fit the range cell indices $n_k$
  (b) Relaxed support reconstruction  ⇒ Detections of OMP may be within tolerance window $\{n_k - \varepsilon, \ldots, n_k + \varepsilon\}$
  ($\varepsilon$: denoted as scope)
Performance Analysis based on Simulation Results
Perfect Support Reconstruction

- **Perfect support reconstruction**: *Worst-case coherence* $\mu_{\Phi}$ of measurement matrix $\Phi$ known to be a good substitute for resulting probability of detection $P_d$
- Worst-case coherence is *maximum correlation* $\rho(n,n')$ between any two columns $n, n'$ of $\Phi$ ($n' \neq n$)
- For considered sub-sampling in Fourier domain it suffices to consider only correlations $\rho(n,1)$, $n = 2, ..., N$
Performance Analysis based on Simulation Results

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- In CoSeRa 2015 we showed: Correlations $\rho(n,1)$ can easily be calculated via DFT of selection vector $s_B$ (corresponding to matrix $S_B$)
  
  ⇒ Efficient calculation of $\mu_\Phi$ possible, e.g. based on a FFT of $s_B$
  
  ⇒ Very useful e.g. for optimizing the aperture value $A$ of the sub-Nyquist radar scheme (for given parameters $N, M, c, B$)

$$\rho_\Phi(n, 1) \propto \| [F \cdot S_B]_n \| \quad \rightarrow \quad \mu_\Phi = \max_{n \in \{2, \ldots, N\}} \rho_\Phi(n, 1)$$
Performance Analysis based on Simulation Results

Perfect Support Reconstruction

- **Example**: Optimization of aperture value $A$ for $c=4$ BPFs spaced according to OSR

Sub-sampling factor $q = 6.4\%$ ($M = 640$, $N=10000$)

$c = 4$ BPFs (OSR)

$K = 6$ point targets

$\text{SNR} = 26\, \text{dB}$

⇒ Excellent correspondence
Performance Analysis based on Simulation Results

Relaxed Support Reconstruction

- Relaxed support reconstruction ($\epsilon > 0$): Strong correspondence between worst-case coherence $\mu_{\Phi}$ and resulting detection performance does not hold anymore
- In fact, when increasing $\epsilon$ smaller aperture values $A$ become optimal
- Reasonable, as we essentially allow for a coarser range resolution, which can generally be achieved by smaller overall signal bandwidth (corresponding to a smaller aperture value $A$)
Performance Analysis based on Simulation Results
Relaxed Support Reconstruction

- Simulation results for detection probability $P_d$ for different numbers of target objects $K$ (optimized aperture $A$)

$P_d$-value of 93% achieved for sufficiently high SNRs

Even for $K=12$ targets $P_d$-value of 93% achieved for sufficiently high SNRs
Performance Analysis based on Simulation Results

Relaxed Support Reconstruction

- Simulation results for detection probability $P_d$ for different sub-sampling factors $q$ (optimized apertures $A$)

\[ 10 \log_{10}(SNR_{MR}) \text{ [dB]} \]

$P_d$ vs. $10 \log_{10}(SNR_{MR})$ for different sub-sampling factors $q$ and optimized apertures $A$.

Results show well-known relation between parameters $N$, $M$, $K$ (i.e., for good performance $N \gg M \gg K$)

Sub-sampling factor $q = 1.6\%, 3.2\%, 6.4\%, 12.8\%$ (N=10000)

$c = 4$ BPFs (OSR)

Scope $\varepsilon = 1$

$K = 6$ point targets
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Application to Real-World Radar Data
Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

• Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar
  – **Purpose:** Capture realistic clutter/ noise scenarios, off-grid effects & physically extended targets
Application to Real-World Radar Data
Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

- Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar
  - Processing done completely in digital domain by sub-sampling high-rate ADC data (retroactively)

Air-to-sea radar data set recorded above German North Sea on July 4, 2014 @ (54.199°N, 6.757°E, altitude 1400 m)
Application to Real-World Radar Data
Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

- Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar

→ Exemplary reconstruction result (c = 4 BPFs, OSR; sub-sampling factor q = 2.7%)
  - Target detections from OMP (K = 2 iterations)
Application to Real-World Radar Data
Air-to-Sea Data from Maritime Mode of Airbus EBS SmartRadar

- Sub-Nyquist radar scheme with Fourier domain sub-sampling and BPF placement according to OSRs was applied to real-world air-to-sea radar data collected with Airbus EBS SmartRadar

→ Generally, reconstruction results were stable for sufficient SNR/SCR and target object spacing
Conclusions and Future Work

Summary

• Investigated performance of a practicable sub-Nyquist radar system based on a set of parallel BPFs for extracting range information of sparse target objects

• Positions of BPFs in frequency domain optimized based on OSR approach

• Performance of OMP for non-linear recovery step
  – Perfect range recovery: Direct correspondence between worst-case coherence of measurement matrix and probability of missed detection; useful for optimizing aperture
  – Relaxed range recovery: Direct correspondence does not hold anymore; smaller aperture values tend to be optimal compared to perfect range recovery

• Simulation results: Sub-Nyquist radar system achieves detection probabilities > 90% for various numbers of target objects and sub-sampling factors, as long as SNR is sufficiently high (on the order of 12...20 dB)

• Sub-Nyquist radar scheme successfully applied to real air-to-sea radar data recorded with Airbus EBS SmartRadar pod system
Conclusions and Future Work

Topics for Future Work

• More detailed investigations concerning application of sub-Nyquist radar scheme to real radar data
• New methods for optimizing the aperture value in case of relaxed range recovery (so far done by means of computer search)
• Extension of sub-Nyquist radar scheme to (practically relevant) scenario where number of target objects is not known a-priori