Sparse signal separation and imaging in Synthetic Aperture Radar

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Talk Outline

• Exploiting sparse and structured representations
  • Undersampling/compressed sensing
  • Blind Deconvolution
  • Signal Separation

• Applications of sparsity in SAR
  • Low Frequency SAR
  • Range Correction and Autofocus
  • Ground Moving Target Decomposition

• Data, Sparsity and Computation
Sparse Representations and Decompositions
Signal Model:

(Approximate) $k$-sparse signal model

Encoder:

Generalized sampling (typically random projection) that hopefully “preserves” information.

Decoder:

Nonlinear mapping to invert the linear projection on the signal set, e.g. L1, OMP, IHT, Message Passing, etc.
**Generic CS**

Generic reconstruction algorithm:

**Relaxation**: replace $l_0$ with $l_1$ (c.f. Iterative Soft Thresholding):

$$\hat{x} = \arg\min_x \|x\|_1 \text{ subject to } \Phi x = y$$

**Theorem**: RIP $\Rightarrow$ guaranteed sparse recovery

+ many others: IHT, OMP, CoSAMP, AMP, etc…
Sparsity based Deconvolution/Deblurring

[Levin et al. “Image and Depth from a Conventional Camera with a Coded Aperture” SIGGRAPH 07]

Sparsity based blind deconvolution/deblurring...

and depth through focusing

Blur kernels as a function of depth

Estimated depth map
Sparsity and Separating Decompositions


Sparse representations can enable a meaningful decomposition through morphological differences...

\[
\{\hat{\alpha}_0, \hat{\alpha}_1\} = \arg\min_{\alpha_0, \alpha_1} \|C\alpha_0\|_1 + \|D\alpha_1\|_1, \text{ s.t. } \alpha_0 + \alpha_1 = x
\]
Sparsity and Separating Decompositions

[D. & Daudet “Sparse Audio Representations using the MCLT” 2006]

Sparse decomposition into Dual-resolution components, enables separation of individual notes
New directions & challenges in sparsity & CS

- Fundamental Statistical bounds;
- Better algorithms: structured, Bayesian, message passing;
- Data driven representations;
- Continuous/off-the grid models/multi-resolution imaging;
- Blind deconvolution/calibration;
- Sparse signal separation;
- Compressed detection/signal processing
- Hardware/computationally efficient solutions;
Applications in Synthetic Aperture Radar
SAR acquisition can be thought of as approximately sampling in k-space.
SAR System model

Idealized dechirped SAR phase history model:

\[ Y = \Phi_F(X) = \left\{ \sum_{k=1}^{K} \sum_{l=1}^{L} X_{kl} \exp \left( \frac{-j 4\pi f_m u_{kl}(\tau_n)}{c} \right) \right\}_{m,n} \]

where

- \( X \) is the scene reflectivities (discretized);
- \( \tau_n \) is the time of the \( n \)th pulse;
- \( u_{kl}(\tau_n) \) is the distance from platform to target relative to scene centre (can incorporate velocities, DEM, etc.);
- \( f_m \) denotes the range frequencies, \( m = 1, \ldots, M \) associated with the dechirping process.
Airborne Low Frequency SAR
Low Freq. Synthetic Aperture Radar

Why image with UHF/VHF?

- Foliage penetration (FoPEN) Radar
- Ground penetration Radar (GPR)

Major Issues:
1. Interference between SAR systems and radio, television and communications systems.
2. Radio Frequency Interference (RFI)
3. Calibration/autofocus
Traditional imaging techniques lead to poor image formation generating high sidelobes due to missing data.
Sparsity in SAR images

Interaction of Reflectors in a Range Cell

- **Random interference**: Speckle dominates images due to many random reflectors in a range cell - not compressible.

- **Coherent interference**: Coherent reflectors (often targets of interest) localized high intensity reflectors - compressible in spatial domain.
Compressed Sensing SAR Image Formation

Decompose image into compressible and uncompressible components...

$$\hat{X}_S = \arg\min_{X_S} \|X_S\|_1$$

subject to

$$\|Y - \Phi_F(\hat{X}_S + X_{bg})\|_F \leq \epsilon$$

$$\hat{X}_{bg} = \arg\min_{X_{bg}} \|Y - \Phi_F(\hat{X}_S + X_{bg})\|_F$$
Compressed Sensing Image Formation

- Significant improvement in imaging of bright targets!
- Degradation in background speckle compared with fully sampled image
Radio Frequency Interference

Additional RFI can be detected in dead time between pulses and the RFI statistics estimated.

Traditional solution is to filter out radar returns
   – introduces large sidelobes

Instead incorporate RFI suppression through weighted fidelity term:

\[
\hat{X} = \arg\min_X ||X||_1 \\
\text{s.t. } ||Y - \Phi_F(X)||_{Q_N^{-1}} \leq \epsilon
\]

where $Q_N$ is dechirped/deskewed the RFI covariance matrix (approximated well using a diagonal matrix) [Kelly et al 2013]
Autofocus

Inaccuracies in propagation delay estimates introduce unknown phase errors, $\phi_{\epsilon_n}$. These defocus targets and degrade reconstruction.

For small delay errors, $\epsilon_n \leq \lambda/8c$ (equiv. range errors $\delta R_n \leq \lambda/16$):

$$\phi_{\epsilon_n} \approx \omega_0 \epsilon_n - \alpha \epsilon_n^2$$

where

- $\epsilon_n$ is the delay error at the $n$th transmit pulse;
- $\omega_0$ is the carrier frequency;
- $\alpha$ is the chirp rate;

The adjusted model approximates as:

$$Y = \Phi_F(X) \text{diag}\{e^{j\phi}\}$$

Classical autofocus, e.g. Phase Gradient Autofocus, assumes far field, fully determined and separable - not appropriate for undersampled data.
AutoFocus

For undersampled SAR a better solution is:

\[
\text{minimise } \|X\|_1 \quad \text{subject to: } \|Y \text{ diag}\{d_n\} - \Phi_F(X)\|_F \leq \sigma
\]

\[|d_n| = 1, n = 1, \ldots, N\]

- Fast block-relaxation algorithms exist [Kelly et al 2012/14] (virtually no additional cost)
- No far field/ small aperture assumptions

However no theoretical guarantees
Multi-Pass Autofocus
Multi-pass Autofocus

Multiple flight passes offer opportunity for limited elevation information:

- Interferometric SAR
- TomoSAR
- Compressive Volumetric SAR

However first need to be able to coherently combine different passes
Multi-pass Autofocus

Coherently combining multipass data may result in relative delay errors that are too large for classical autofocus techniques: $\epsilon_n > \lambda/8c$

An improved approximation is:

$$Y_{m,n} = \Phi_F(X) \times \exp\left\{-j \left(2\pi f_m \epsilon_n + \frac{\alpha \epsilon_n^2}{2}\right)\right\}$$

Note the extra phase term.

Phase error is now a linear function of range frequency, $f_m$. 

![Graph showing phase error as a function of frequency]
Multi-pass Autofocus

Extended Autofocus Algorithm can be viewed as a Structured Phase Retrieval Problem. Proposed Algorithm, inspired from Gershberg-Saxton algorithm: alternate between enforcing sparsity and phase constrained data fidelity.

Iterate the following steps:

1. Element-wise Soft Thresholding: \( X^{[k]} = S_\lambda \left( \Phi_H^H \left( \Gamma_\epsilon^{[k]} \odot Y \right) \right) \)

2. Estimate phase errors: \( \epsilon^{[k+1]} = \arg\min_{\epsilon} \left\| \Gamma_\epsilon \odot Y - X^{[k]} \right\|_F \)

where

- \( \epsilon \) is the vector of delay errors;
- \( \Gamma_\epsilon \) encodes the phase errors as a linear function of range frequency;
- \( \odot \) denotes elementwise multiplications
Multi-Pass Autofocus Algorithm

For 2 passes with a 20cm relative range error in X-band ($\lambda \approx 30cm$) Phase Gradient Autofocus (PGA) no longer works while proposed technique does.

![Original phase history](image1)

![PGA corrected phase history](image2)

![Proposed algorithm](image3)

![Convergence](image4)

2-pass reconstruction of car (GOTCHA data) manages to resolve car pillars (arrows)
Ground Moving Target Decomposition
Exploiting Sparsity in SAR+GMTI

Conventional SAR imaging assumes a static scene. Dynamic targets therefore appear displaced and defocused.

Multiple channels can be used to identify moving targets using phase differences across the different channels (e.g. DPCA, ATI, etc.)

Sparsity based Dynamic Imaging

1. Image Formation with full dynamic-static decomposition using sparsity:

   \[ X = X_s + X_d \]

   (without displacement correction or refocussing)

2. Full target velocity estimation through sparsity-based refocussing

Where's the car? (AFRL “GOTCHA” GMTI data)
Including Elevation Information

First we need to do some pre-processing... Unfortunately there is no digital elevation map (we got ours from US Geological Survey)
Exploiting Sparsity in SAR+GMTI

Step one: Image Formation with Dynamic-Static Decomposition:

Proposed model:

\[
\begin{align*}
\text{minimise} \quad & \frac{1}{2} \sum_i \| \tilde{Y}_i - \Phi^0_F(X_s + X_d \odot P^{i-1}) \|^2_F \\
\text{such that:} \quad & \|X_d\|_0 \leq s \\
& |P_{kl}| = 1, k = 1, \ldots, K, l = 1, \ldots, L \\
& \text{supp}(X_d) = \text{supp}(P - 1)
\end{align*}
\]

(1)

\(\tilde{Y}_i\) is the balanced phase history for the \(i\)th channel;

\(X_s\) and \(X_d\) are static and dynamic (without displacement correction) components,

\(P\) is the phase correction matrix for the dynamic component \(X_d\) and \(\Phi^0_F\) is the forward model assuming zero velocity.
Exploiting Sparsity in SAR+GMTI

“Morphological Decomposition” into static and dynamic components (using small sub-apertures)...

Radial velocity and target displacement can now be estimated from phase correction matrix, P.
Exploiting Sparsity in SAR+GMTI

Step 2: Full target velocity estimation using sparsity-based refocussing

(imaging with the correct velocity will produce a sparser image)

\[
\min_v \|X_{\text{Target}}(v)\|_1
\]

such that

\[
X_{\text{Target}}(v) = \min_x \|\Phi^0_F(X_d) - \Phi^V_F(X)\|_F^2
\]

\[
v = \gamma(v^{(z)}, v^{(r)})
\]

where

- \(X_d\) is the dynamic component restricted to target neighbourhood
- \(\Phi^V_F\) is the forward model with the corrected radial velocity;
- \(\gamma\) enforces consistency with radial velocity, \(v^{(r)}\), and target movement on the DEM
Target Velocity Estimates

x-,y-,z- velocities all accurately estimated from SAR data
Open Challenge…

Given a dynamic decomposition we should now be able to form a large aperture (high resolution) image of the moving target using velocity estimation and displacement correction (Inverse SAR)….
Data, Computation and Sparsity
Traditionally attention has focused on estimation accuracy with little attention to computation.

What do we actually want?

- Iterative vs non-iterative (Deep NN reconstruction)?
- Randomized vs deterministic?
- What are the appropriate computational models?

Notion of Time-Data Complexity
[Chandrasekaran & Jordan 13]
C-SENSE: Exploiting Low Dimensional Models in Sensing, Computation and Signal Processing

Now recruiting…
References

Low Frequency SAR

- S. I. Kelly, M. Yaghoobi, and M. E. Davies, Auto-focus for Compressively Sampled SAR, CoSeRa 2012.
- S. I. Kelly and M. E. Davies, RFI suppression and sparse image formation for UWB SAR. 14th International Radar Symposium (IRS), 2013

Sparse Multipath Autofocus


Sparsity based SAR + GMTI

- D. Wu, M. Yaghoobi and M. E. Davies, Sparsity Driven Moving Targets and Background Separation via Multi-Channel SAR. Preprint, 2016.